Extragalactic Astrophysics: Question Sheet 4

1. Show that the time between two subsequent *strong* collisions between stars can be written as

$$t_{\rm strong} \sim 4 \cdot 10^{12}\,{\rm yr} \left(\frac{v}{10\ {\rm kms}^{-1}}\right)^3 \left(\frac{m}{M_\odot}\right)^{-2} \left(\frac{n}{1\ {\rm pc}^{-3}}\right)^{-1}, \tag{1}$$

where m is the mass of the stars, n their number density and v their relative speed.

2. Consider a spherical halo of dark matter, of mass $M_{\rm DM} = 10^{12} M_\odot$ and radial extent $r_{\rm DM} = 200$ kpc. The halo is composed of massive black holes (BHs) with mass $m_{\rm BH} = 10^4 M_\odot$. Embedded within the halo is a stellar cluster of mass $10^9\,M_\odot$, extending out to $r_c = 1$ kpc. Both distributions are assumed to be constant density for simplicity. Each star is assumed to have a mass of $m_s = 1 M_\odot$. You have already encountered the relaxation time in the lectures. This is a good measure of the time for the objects in a system to undergo significant interaction with each other. For an extended distribution, however, the relaxation time may vary by several orders of magnitude in different regions. It is therefore useful to define a single measure of relaxation time that characterises a system. This is done by defining the half-mass relaxation time t_{rh} at r_h, the radius that encloses half of the total mass. Write down an expression for the half-mass relaxation time of the stellar cluster and use it to work out whether the stellar + BH system within r_h can be said to be collisional or collisionless.

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