Control loops and control theory

I. **Feedback loop and transfer function**

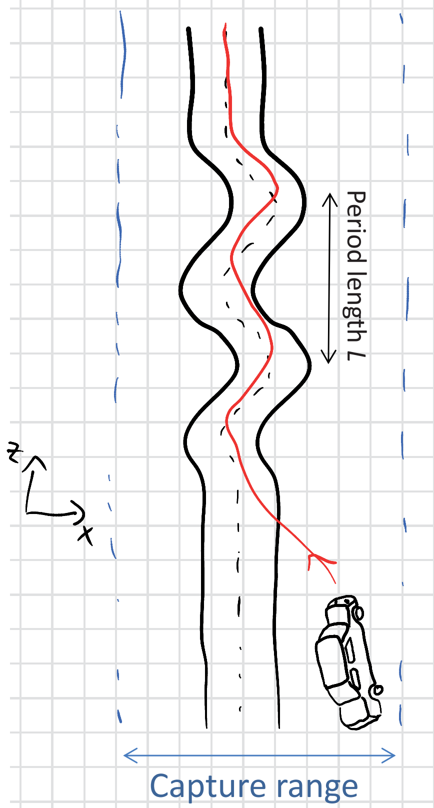
II. **PI controllers**

III. **Stability criteria**

IV. **PI controller + fast loop**

V. **1Hz Clock laser example**

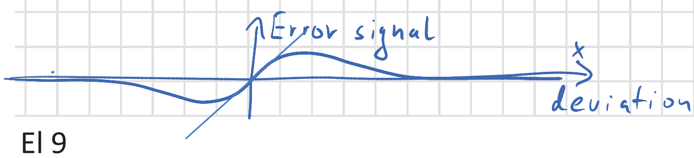
Control loop example: driving



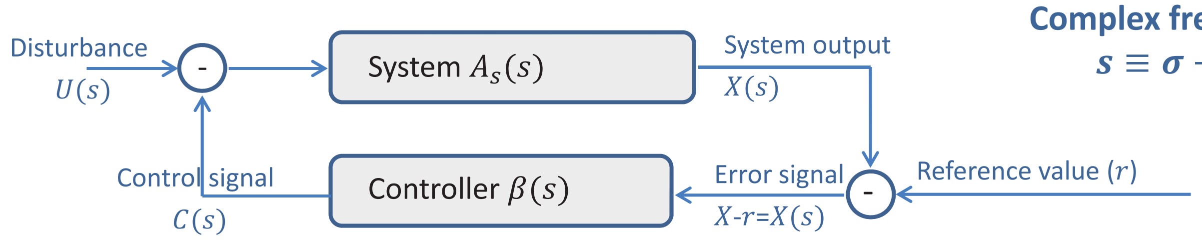
For the driver, the (periodic) oscillations in the road have an apparent frequency of $f = v/L$.

1. At which frequency f_k (variable speed v) do you expect the car/driver to lose control of the car? (perfect grip)

2. Which factors contribute to this loss of control?



Feedback loop



Complex frequency
$\boldsymbol{s \equiv \sigma + i\omega}$

Example: Proportional controller $\beta(s)$=A_p
Assumption: no delay, constant gain A_p

System (*Strecke*) is slow, has delay τ
⇒ phase response delayed, by 120° at f_k. For this delay, effective negative feedback becomes impossible.

Hence, total gain $\beta(f_k) \cdot A_p(f_k)$ should be smaller than one, to avoid resonance/introduce damping.

Drawbacks:
Limited the gain at low frequencies.
Exponentially slow approach to reference=0.
Constant offset $x(\infty) - r \neq 0$ if $r \neq 0$.

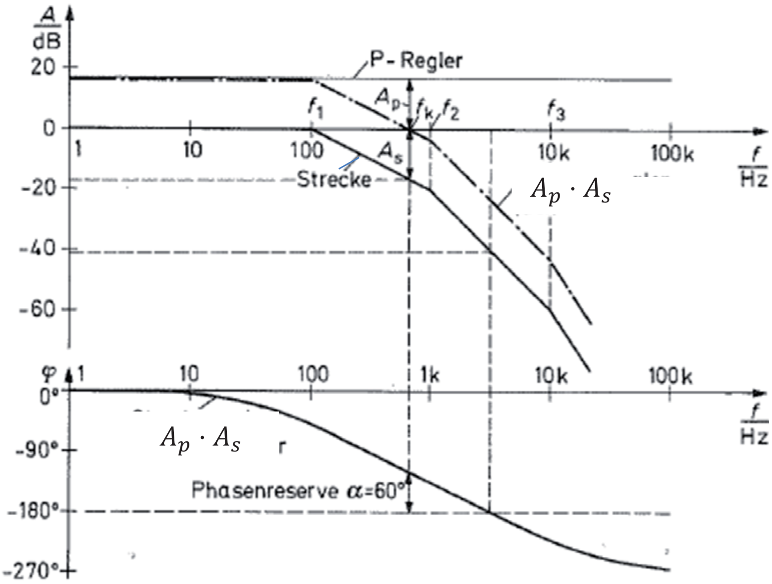
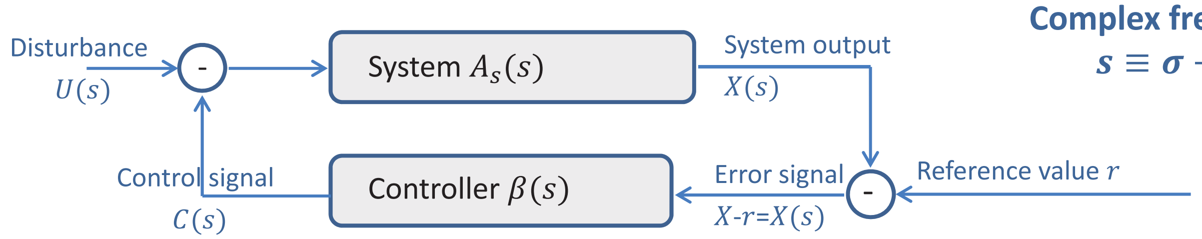


Abb. 19.2. Beispiel für das Bode-Diagramm einer Strecke mit P-Regler

Tietze+ Schenk, Chap 19

Open loop gain $A_s \cdot \beta$ and transfer function $F(s)$



Complex frequency
$\boldsymbol{s \equiv \sigma + i\omega}$

Response of system to input? Without feedback:

$$x(t) = \int_0^{t'} u(t') \cdot a_s(t-t')\, dt' \equiv u * a_s \qquad * \equiv \text{Convolution}$$

$$X(s) = U(s) \cdot A_s(s)$$ Convolution → Multiplikation. Solve in frequency domain, final IFT

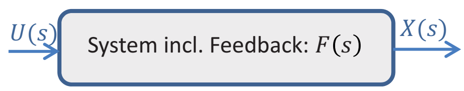
Closed loop ($r \equiv 0$):

$$X(s) = \underbrace{U(s) \cdot A_s(s)}_{\text{disturbance}} \underbrace{-X(s) \cdot \beta(s) \cdot A_s(s)}_{\text{negative feedback}}$$

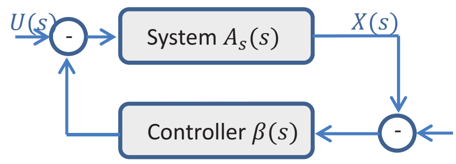
$$X(s) = \frac{A_s}{1+\beta \cdot A_s} \cdot U(s)$$

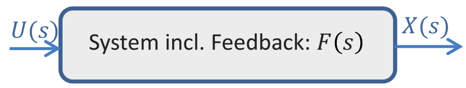
$$F(s) = \frac{A_s}{1+\beta \cdot A_s}$$ is called transfer function or impulse response, because with δ-impulse

$$u(t) = \delta(t-0) \quad \Rightarrow U(s) = 1;$$
$$X(s) = F(s) \cdot 1 \quad \Rightarrow x(t) = f(t)$$



Transfer function $F(s)$ and stability





$$F(s) = \frac{A_s}{1 + \underbrace{\beta \cdot A_s}}$$

A_s ← System response

Suppression of disturbances by feedback

The larger $\beta(s)$, the better suppression of disturbances at that frequency s

Stability: $\boldsymbol{\beta(s) \cdot A_s(s)}$ should never become $= -1$
because then positive feedback, instable

Goals?

- **Stability,** i.e. $\beta(s) \cdot A_s(s) \neq -1 \quad \forall s$
- **Large gain $\boldsymbol{\beta}$ at low freqencies**
  - Why: Most disturbances are typically acoustic and hence at frequencies < 2 kHz

Two viewpoints:

1. Study $\beta(s) \cdot A_s(s)$ in Bodediagrams, avoid $\varphi(\beta \cdot A_s) = -180°$ AND $|\beta \cdot A| > 1$
   - More intuitive. This lecture.
2. Study poles and zeros of $F(s)$ in complex $s = \sigma + i\omega$ plane. Mathematically more powerful (residue theorem).
   - Several stability criteria e.g. poles must be in σ<0 halfplane. Routh-Hurwitz criteria
   See 2nd lecture.

Laplace transform

Complex frequency
$\boldsymbol{s \equiv \sigma + i\omega}$

Laplace transform = Fourier transform generalised to complex s:

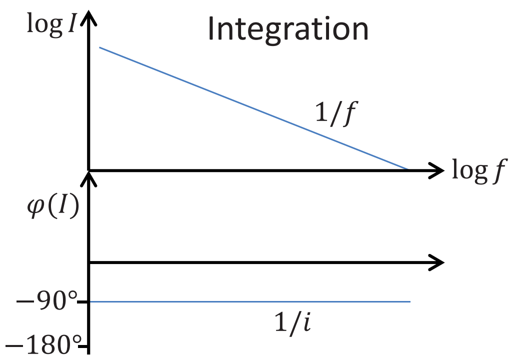
$$F(s) = \int_0^\infty e^{-s\cdot t} f(t)\, dt \quad \Leftrightarrow \quad f(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{s\cdot t} F(s)\, ds$$

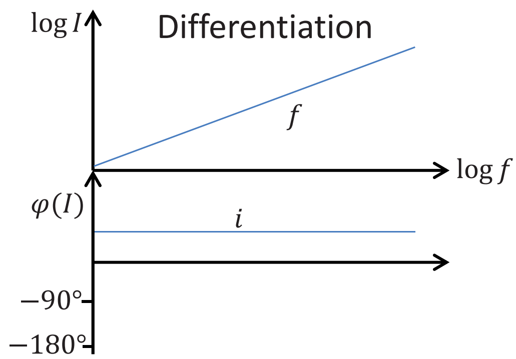
Integration & Differentiation $\Leftrightarrow$ Division & Multiplication by s

$$j(t) = \int_0^t f(t')\, dt' \quad \Leftrightarrow \quad J(s) = \frac{F(s)}{s} \qquad J(\omega) = \frac{F(\omega)}{i\,\omega}$$

$$d(t) = \frac{df(t)}{dt} \quad \Leftrightarrow \quad D(s) = s \cdot F(s) \qquad D(\omega) = i\,\omega\, F(\omega)$$

Proof: $\mathrm{J}(s) = \int_0^\infty \underbrace{e^{-s\cdot t}}_{u'} \underbrace{\int_0^t f(t')dt'}_{j(t)}\, dt = \underbrace{u \cdot i|_0^\infty}_{=0} - \int_0^\infty \underbrace{u(t)}_{\frac{e^{-st}}{s}} \cdot J'(t)\, dt = \frac{F(s)}{s}$ Integrate by parts



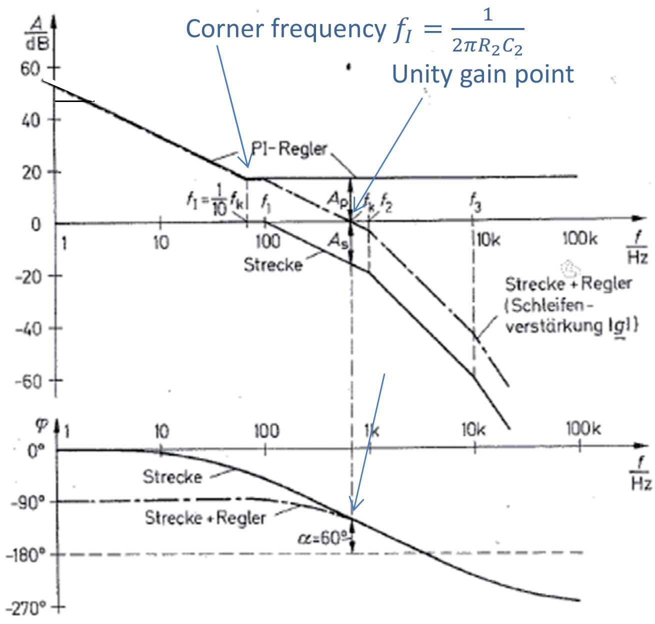


PI-Controller

$$A_{PI}(\omega) = A_p\left(1 + \frac{2\pi\, f_I}{i\,\omega}\right)$$

Proportional gain at high frequencies

Integral gain at frequencies $f < f_i$

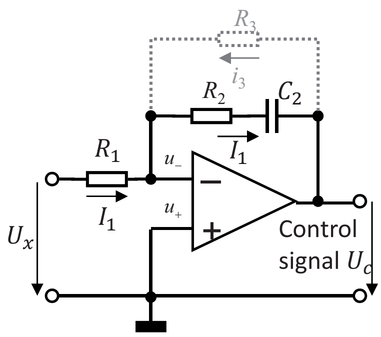


Advantages:

- Much higher gain at low frequencies f:
  - Limited by corner frequency f_I: $\frac{A_{PI}}{A_P} = \frac{f_I}{f}$
- Zero offset $x(\infty) - r = 0$ from ref. value $r \neq 0$.

⇒ PI controller much better than P controller!

But: Extra 90° delay. No problem, only 120° is.



Current $I_1 = U_x/R_1$ charges C_2, changing U_C.
Until deviation from desired reference=x-r=U_x is zero, i.e. perfect lock.

Math: Gain= $\frac{Z_2}{Z_1} = \frac{R_2+(i\omega\, C_2)^{-1}}{R_1} = \frac{R_2}{R1}\left(1 + \frac{(R_2\, C_2)^{-1}}{i\,\omega}\right) = A_p\left(1 + \frac{2\pi\, f_I}{i\,\omega}\right)$

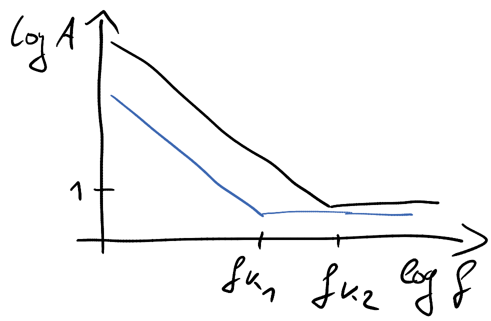
Z_2 is large for low f, where C_2 is effective block. At large f, C_2 becomes conducting, and R_2 dominates
R_3 limits gain at very low frequencies, i.e. allows discharging of C_2

Tietze Schenk, Chap. 19

PI controller tips

General Guideline: Maximum gain at low, acoustic f's!

- More important than Bandwith f_{BW} as figure of merit.
  Def.: f_{BW}: $\varphi(\beta(f_{BW}) \cdot A_s(f_{BW}))$=-120°
- But the higher the bandwidth, the higher f_I ⇒ higher the gain at low frequencies.
  - Check the delays of each relevant part: System, detector, parts of the controller like preamp, actuator (e.g. piezo, AOM). Optimize limiting one.
  - **Increase bandwith by adding second, faster control element, e.g. AOM. →2nd lecture**.

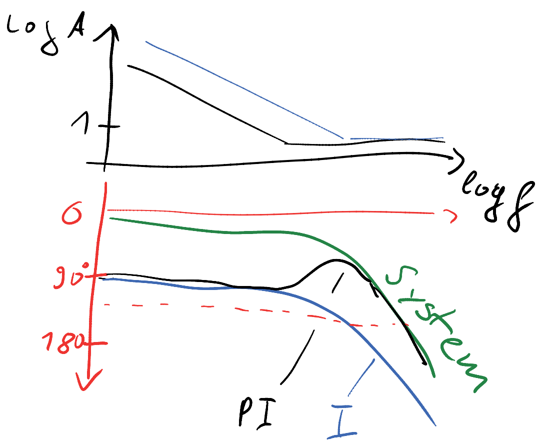


Use only I controller without P

- Yes, your bandwith will go down due to extra 90°. But you are taking advantage of the 1/f behaviour always.
- And you will not have to choose f_I correctly

Phase and amplitude linked by causality/FourierTrafo

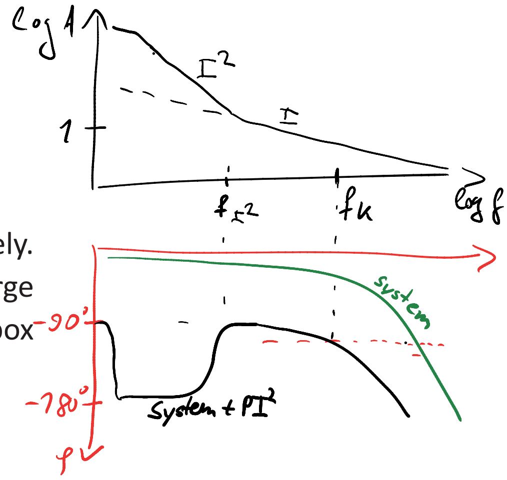
- Can directly deduce $A \Leftrightarrow \varphi$ ($I \Leftrightarrow -90°; D \Leftrightarrow +90°$)
- provided phase shifts caused by lowpass behaviour and not cable delays



Advanced PI controller tips

Do double integration: PI²

- More gain at low frequency
- $-180°$ delay at low I²-frequencies is not a problem, as long as $f_{I^2} \lesssim f_K/4$ (proof later)
- Limit double integration at low f, otherwise inital locking hard.
- Integrator is a lowpass ⇒ can implement 2nd *I* passively. However, 1 integrator must be active, to be able to charge capacitor to setpoint. Implement PI² with normal PI-lockbox (corner f_{I^2}) + extra external lowpass with f_{LP}~100 Hz.

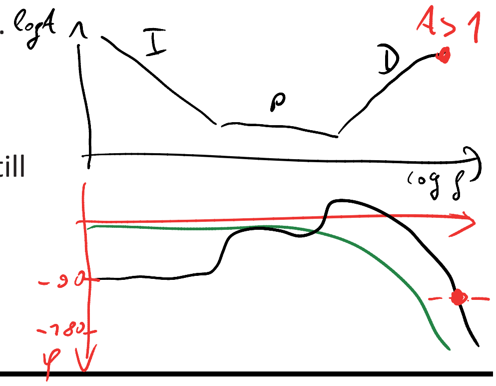


Sharp piezo resonances

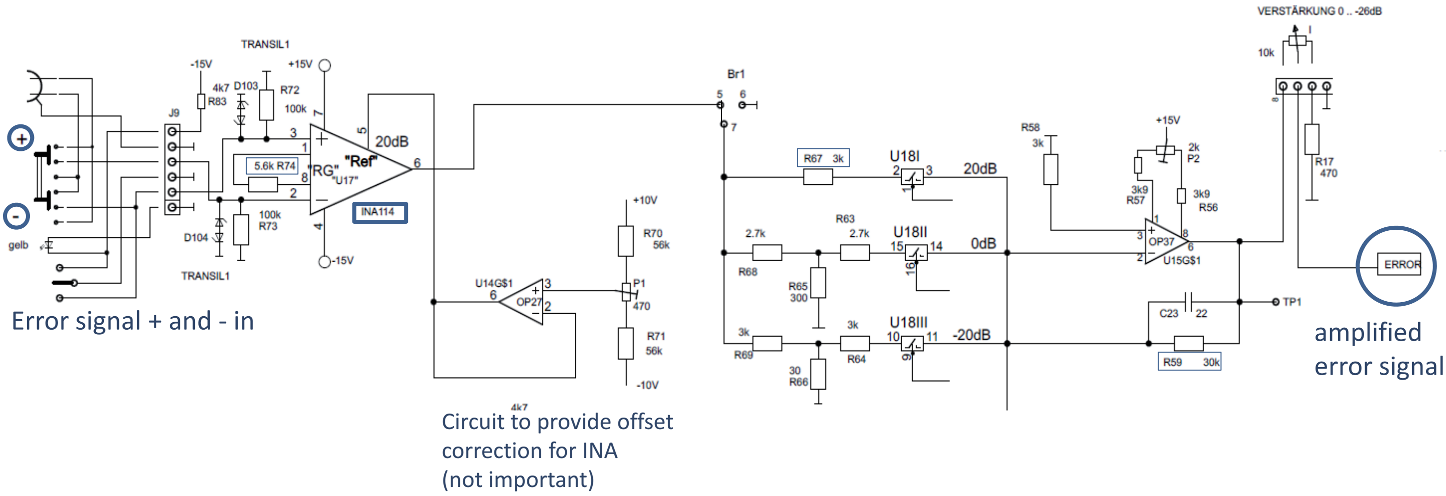
- cause strong phase delays >120° and hence limit bandwidth
- Sometimes helps to suppress gain at $f_{piezo\ res}$ by (higher order) low pass with $f_{LP} < f_{Piezo\ res.}$ Then total gain can be increased.
- -> more gain at low frequency

Normally: Do not use differential gain (PID)

- Gain increases *‚infinitely'* for increasing f. But the $\varphi = -120°$ still arise due to system delay, despite +90°. Here A>1 -> instable
- D-part good when cancelling some I-part in the system, e.g. for current feedback on laserdiodes (which are ~capacitors).
    - Good for very slow loops like temperature controllers



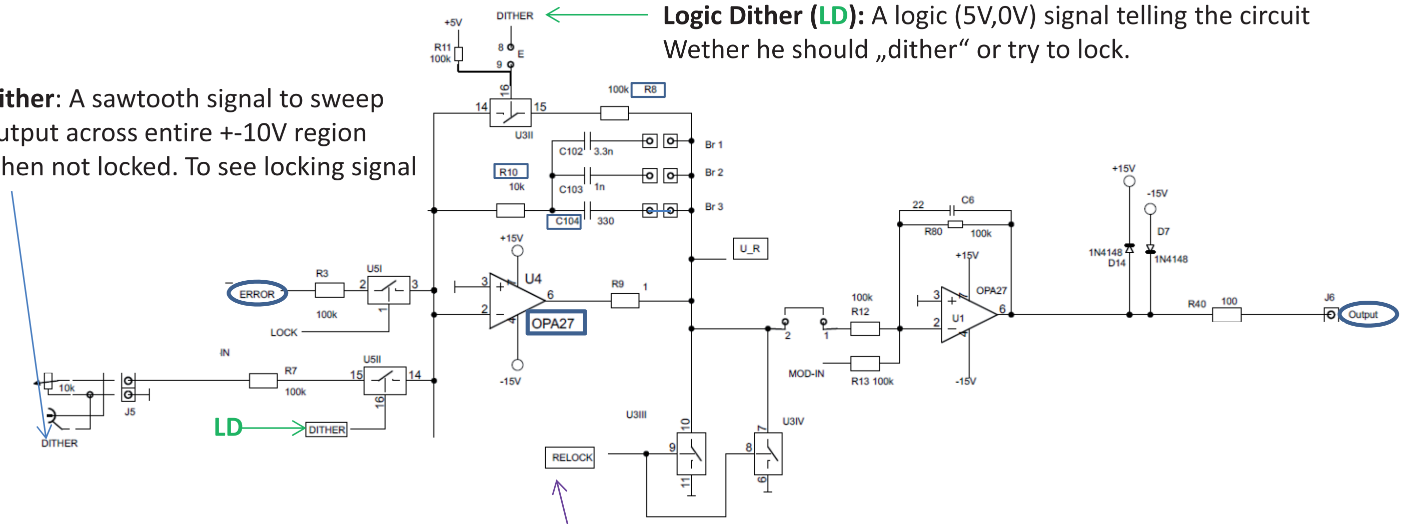
Exercise in circuit diagram reading – PI input amp stage



Integrator + output stage

Logic Dither (LD): A logic (5V,0V) signal telling the circuit Wether he should „dither“ or try to lock.

Dither: A sawtooth signal to sweep output across entire +-10V region when not locked. To see locking signal



Relock (logic): When lock is lost, integrator & output saturates. -> Relock is put to zero (elsewhere), discharging integrator C

Some PI controller models to consider

- PI controller Scheich = Hänsch group electronician: Anton.Scheich@physik.uni-muenchen.de
  - Good, PI part, extra faster P part. You get circuit diagram and can make changes. Not very fast (<1MHz). Cheap ~800€.
- Toptica Lockbox. Similar to Scheich. Prize? Circuit diagram?
- Newport LB1005: 10 MHz fast analog PI lockbox. Very good. Price ~1700€?
- Vescent: 10 MHz fast lockbox with PI^{2}D, good controls, 3500$
- Toptica FALC. Extremely fast lockbox (45 MHz). One slow PID (for e.g. Laser piezo) and one fast (Laser diode current). Ideal for high bandwidth locks: Phase locks with fast feedback to laserdiode.
- TEM Noiseeater: Continous lock (no dither). Good for laser power stabilisation

} analog

- Toptica Digilock: FPGA or DSP based digital lockbox. 2 slow PID (1 MHz for e.g. laser piezo) and one fast (21 MHz, for Laser diode corrent).
  - All values adjustable via computer: f_I, f_D, several filters, gains, relock, Control value like e.g. cavity transmission on which to switch from dither to lock. Diverse extra functions
  - Internal Pound Drever hall function
  - ~4000€. No Potis!
- TEM Laselock digital: Similar to Digilock. ~3400€
- National Instruments CRIO: FPGA based logic with various Analog/Digital In and out perhiphery. Can e.g. realise up to ~16 PI locks with bandwidth ~10 kHz, 16 bit D/A A/D output.
  - Lots of channels: digital Functionality such as sample + hold, logic, freely programmable
  - Realively slow when many channels: Not cheap either: 16 ch ~10‘000€. Digital noise.
    - Good for example for piezo-strain gauge locks, uncritical locks, temperature control, interlocks …