Hadron Physics Summer School 2014

Schloss Rauischholzhausen, September 1 - 5, 2014

Accelerators for *Hadronists*

A short and individual overview over basic principles and types of machines

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Electron Stretcher Accelerator

ELSA

HPSS, September 04, 2014

universitätbonn

Accelerators for *Hadronists*

Outline:

Introduction:	Acceleration: why and how
Circ. Accelerators:	Magnets, magnets, magnets, …
Beam Dynamics:	Wanted and unwanted …
Beam Quality:	Damping and phase space cooling
Limitations:	Space charge, beam-beam, SR, …

Why Accelerators?

Better understanding of:

strong QCD, structure of hadrons, spin structure, mass of the nucleon, …

⇩

need for GeV-beams for probing the nucleon:

Accelerators

Hadronists wish list comprises the following:

- GeV beams of all kind of particles (γ, e, μ, π, p, i, …)
- premium beam quality and performance
- ultimate intensity while having stable beam delivery all the time
- polarized particles of all kinds (preferably antiparticles like e^+ and $\bar{p}$)

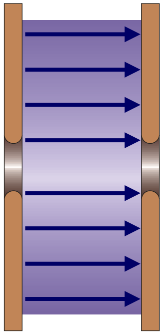
Acceleration

Charged particles are influenced by the Lorentz force: $\vec{F} = e \cdot \vec{E} + e \cdot \left(\vec{v} \times \vec{B} \right)$

Energy gain: $\Delta W_{kin} = \int \vec{F} \cdot d\vec{s} = e \cdot \int E_{\parallel} \cdot ds = e \cdot U$

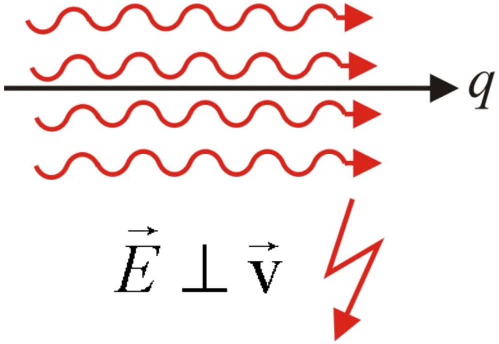
→ We need a longitudinal electrical field $E_{\parallel}$!

Capacitor:



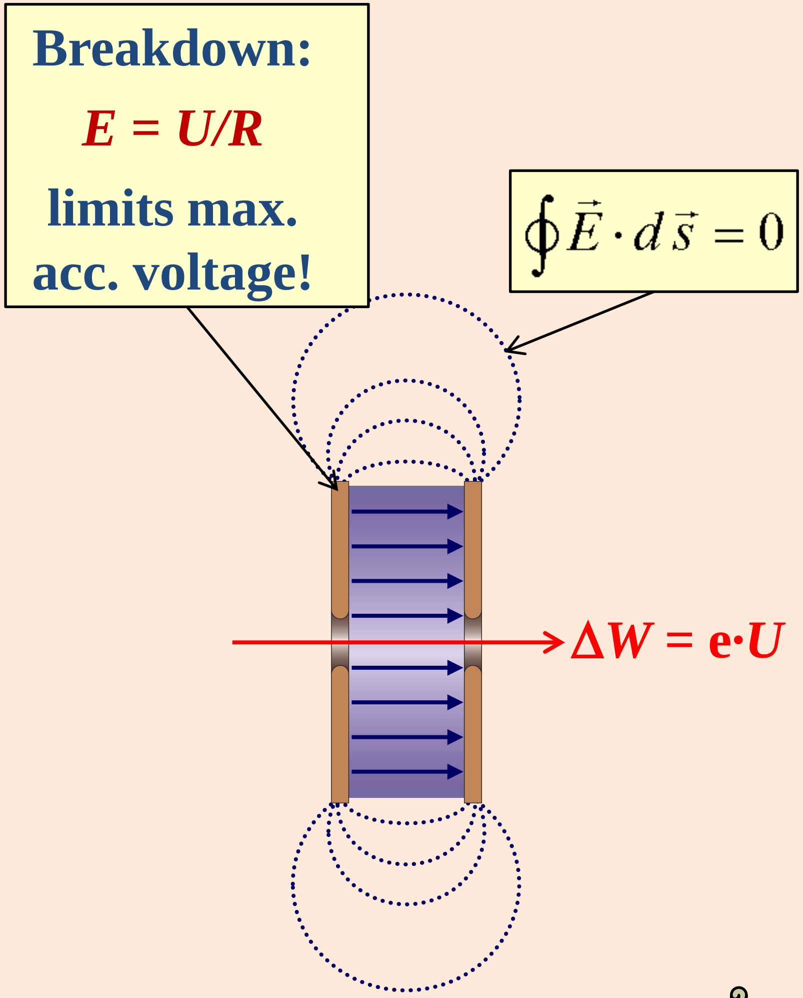
✓

Light beam:
(TEM)



$$\vec{E} \perp \vec{v}$$

Beam Acceleration



Breakdown:

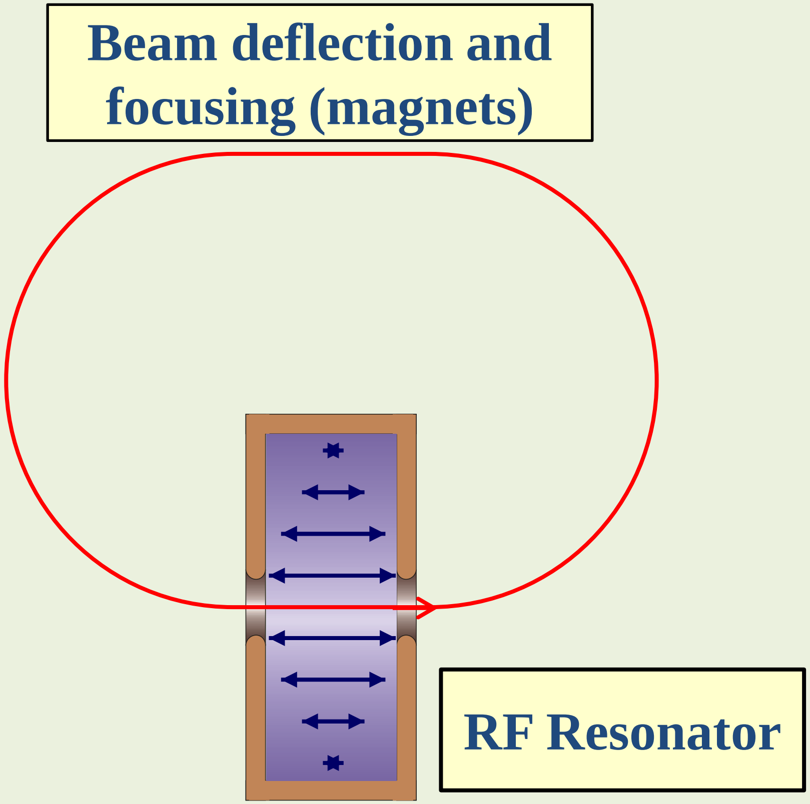
$E = U/R$

limits max. acc. voltage!

$$\oint \vec{E} \cdot d\vec{s} = 0$$

$\Delta W = e \cdot U$

Electrostatic Acceleration



Beam deflection and focusing (magnets)

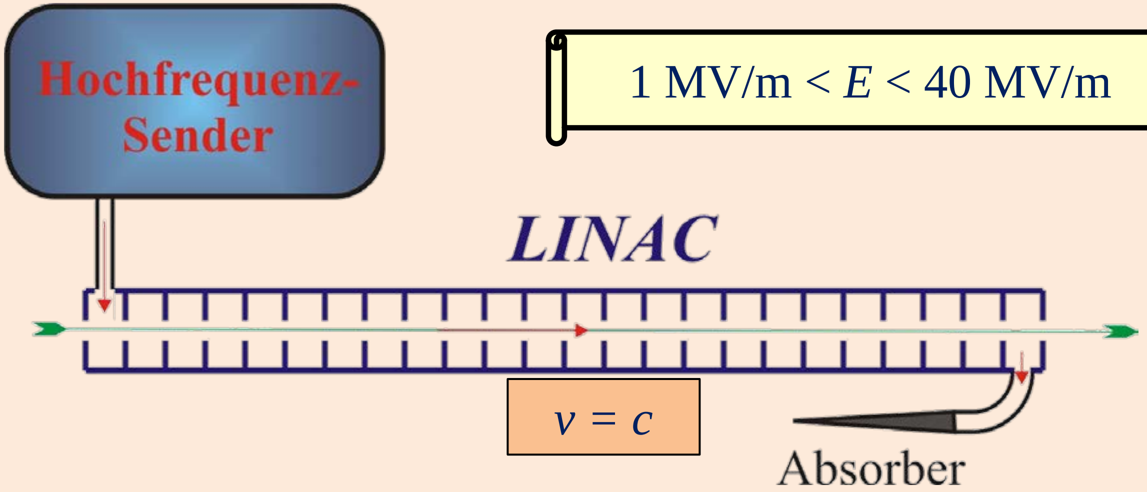
RF Resonator

RF-based Acceleration

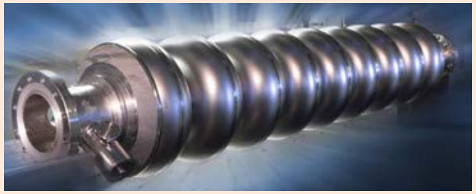
Linear Accelerators

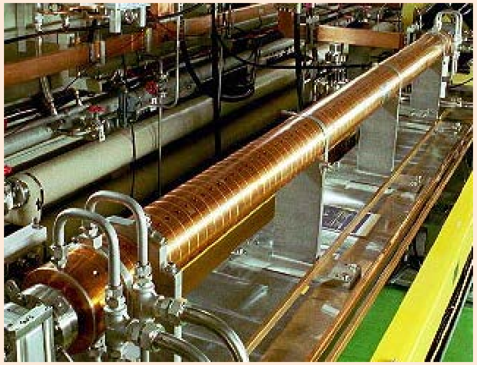
Electrons:

$1\ \text{MV/m} < E < 40\ \text{MV/m}$

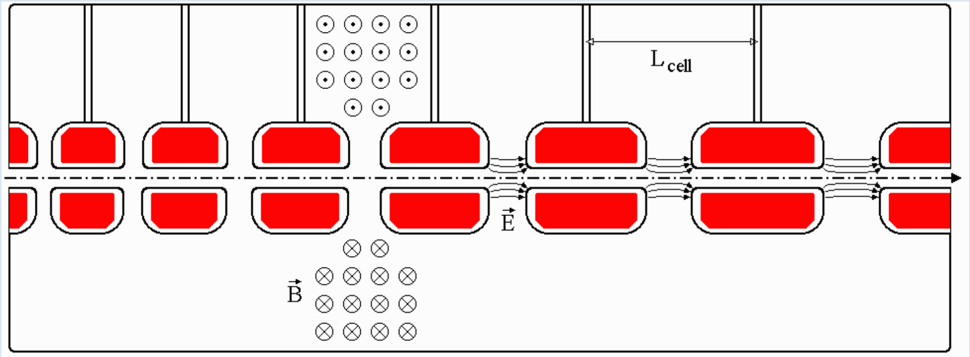


$v = c$

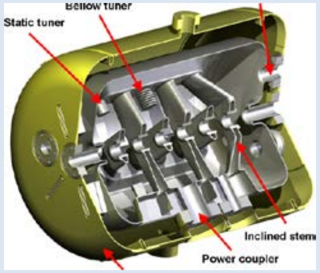


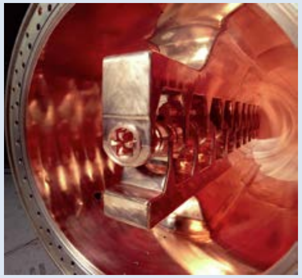


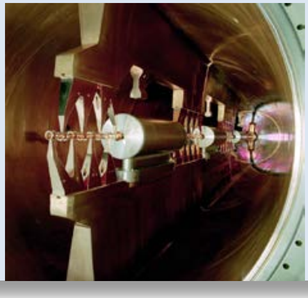
Protons / Ions:

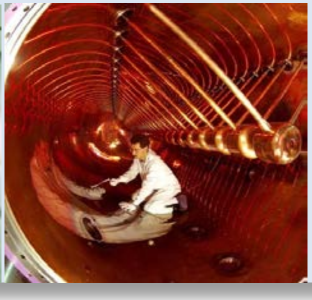


$v << c$





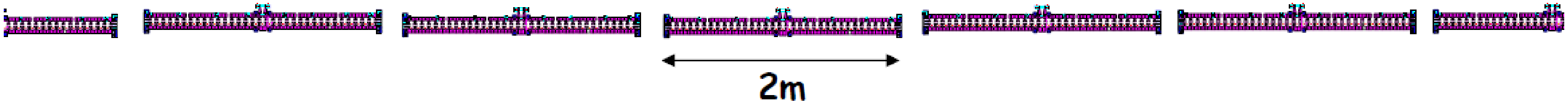




Possible Set-Ups:

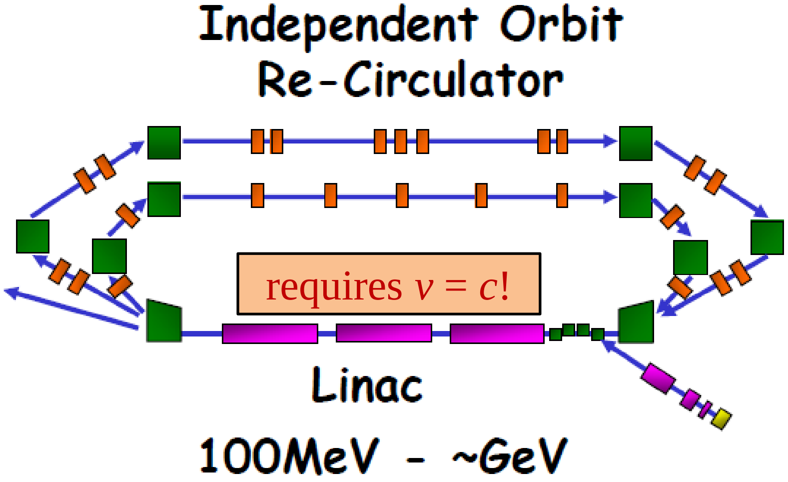
a) cw-LINAC of km length:

nc-copper structure in cw-operation: ~ 1MeV/m @ 15kW/m

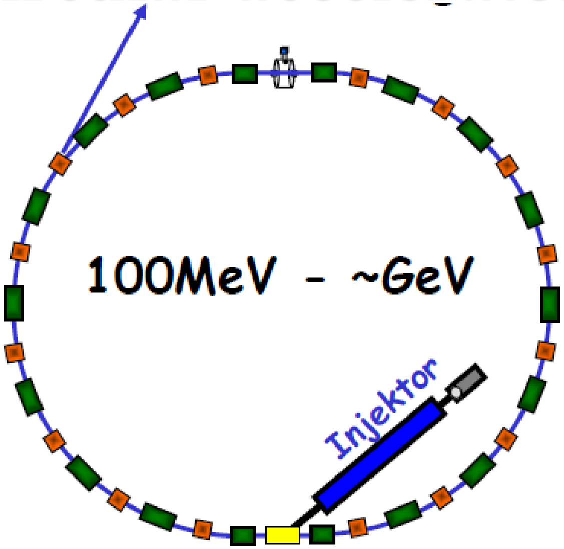


b) recirc. e-LINAC:

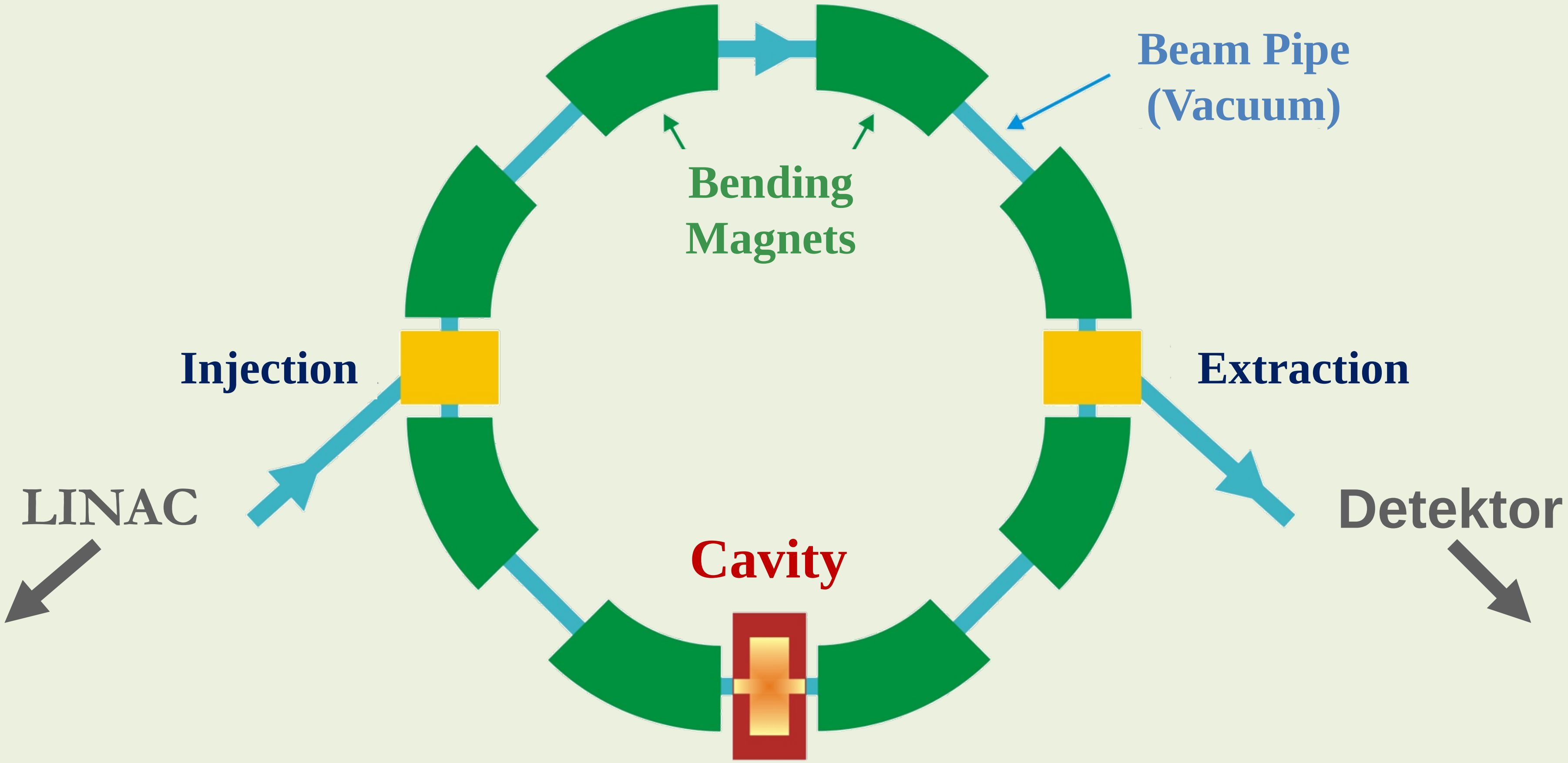
c) circular accelerator



requires $v = c$!



Circular Accelerators:



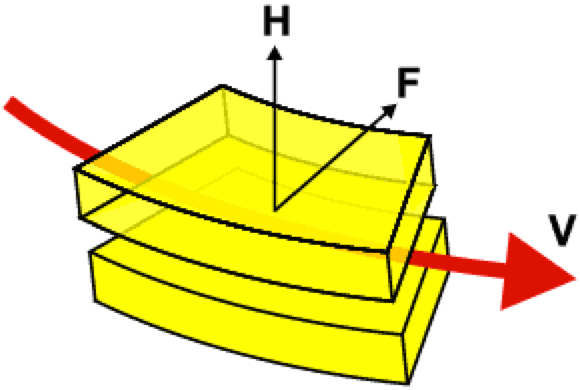
Acceleration

Beam Deflection

Lorentz force: $\vec{F} = e \cdot \left(\vec{E} + \underline{\underline{\underline{\boxed{\vec{\mathrm{v}}}}}} \times \vec{B} \right)$ → **magnetic fields**

circ. orbit ↔ homogeneous magnetic field

$$\boxed{\frac{m\mathrm{v}^2}{R} = e \cdot \mathrm{v} \cdot B_z \quad \Rightarrow \quad \frac{1}{R} = \frac{e}{p} \cdot B_z}$$



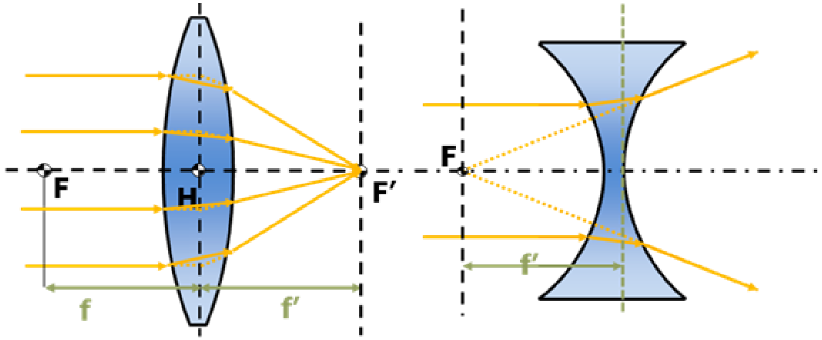
dipole

Important quantities:

- dipole strength: $\boldsymbol{\kappa} = \mathbf{1}/\boldsymbol{R}$, $[\kappa] = \mathrm{m}^{-1}$ (curvature)
- magnetic rigidity: $\boldsymbol{BR} = \boldsymbol{p}/\mathbf{e} \approx \boldsymbol{E}/\mathbf{c}$ (ultra relativistic!)
- beam energy: $p = \frac{e}{2\pi} \cdot \oint B_z \cdot dl$

Beam Focusing

Lens (Light Optics):

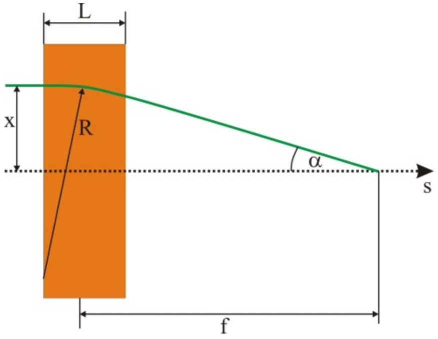


Deflection increases linearly with distance from the optical axis!

$$\vec{F} = e \cdot \vec{v} \times \vec{B} \quad \Rightarrow \quad B_z = g \cdot x, \quad B_x = g \cdot z, \quad g = \frac{\partial B_z}{\partial x}$$

Important Quantities:

- quadrupole strength: $\boldsymbol{k = e/p \cdot g}$, $[k] = \mathrm{m}^{-2}$
- focal length: $\boldsymbol{1/f \approx k \cdot L}$



$$\frac{x}{f} = \tan\alpha = \frac{L}{R} = L \cdot \frac{e}{p} B_z = \frac{e}{p} g\, x L = x k L$$

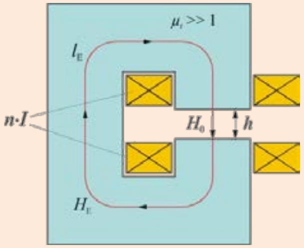
Higher Orders:

Taylor expansion of the magnetic fields:

$$B_z(x) = \underbrace{B_0}_{\text{Dipole}} + \underbrace{\frac{\partial B_z}{\partial x}}_{\text{Quadrupole}} \cdot x + \frac{1}{2}\underbrace{\frac{\partial^2 B_z}{\partial x^2}}_{\text{Sextupole}} \cdot x^2 + \frac{1}{6}\underbrace{\frac{\partial^3 B_z}{\partial x^3}}_{\text{Oktupole}} \cdot x^3 + \ldots$$

Definition of a scalar magnetic potential:

$$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{j} = 0 \qquad \Rightarrow \quad \vec{B} = \vec{\nabla}\Phi$$

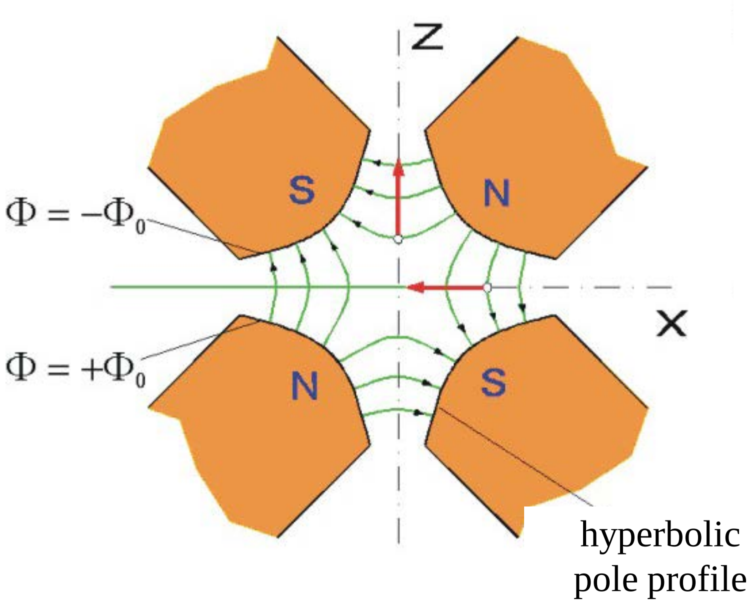


- Dipole: $-e/p \cdot \Phi_1 = \kappa_z x - \kappa_x z$
- Quadrupole: $-e/p \cdot \Phi_2 = -1/2\,\underline{k}\left(x^2 - z^2\right) + k\,x\,z$
- Sextupole: $-e/p \cdot \Phi_3 = -1/6\,\underline{m}\left(x^3 - 3\,x\,z^2\right) + 1/6\,m\left(3\,x^2\,z - z^3\right)$
- …

$\Uparrow$ skew $\qquad \Uparrow$ upright

Beam Focusing

Quadrupole Magnet:



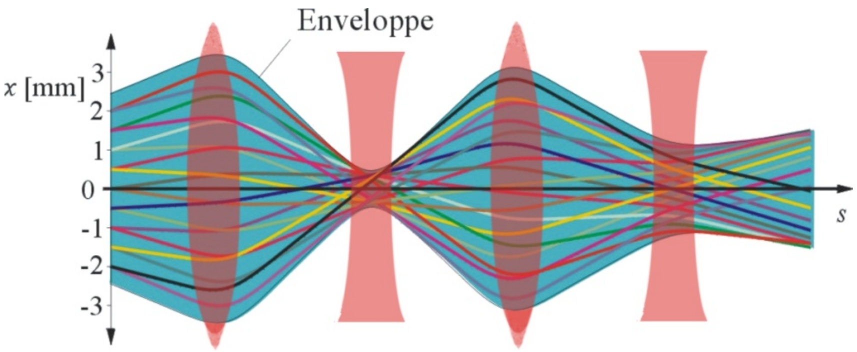
$$\Phi = -g\cdot x\cdot z$$

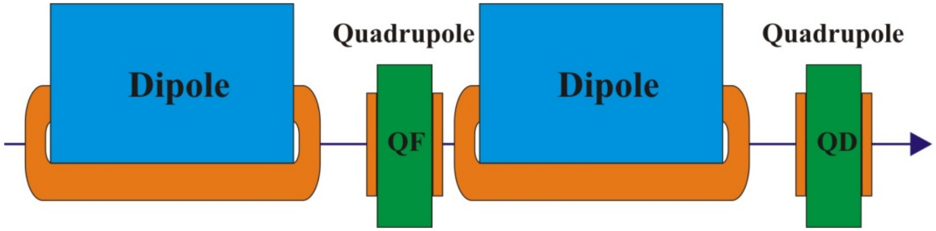
$$\vec{B} = g\cdot\left(x\hat{e}_z + z\hat{e}_x\right)$$

$$\vec{F} = e\cdot\left(\vec{\mathrm{v}}\times\vec{B}\right) = e\mathrm{v}g\cdot\left(x\hat{e}_x - z\hat{e}_z\right)$$

Strong Focusing:

(Courant, Livingston, and Snyder, 1952)

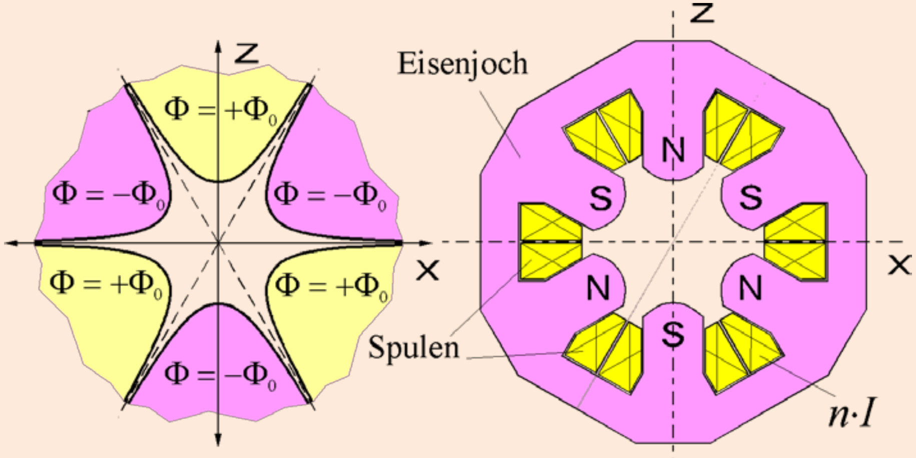




Simplest arrangement: FODO

Chromatic Correction

Sextupole Magnet:

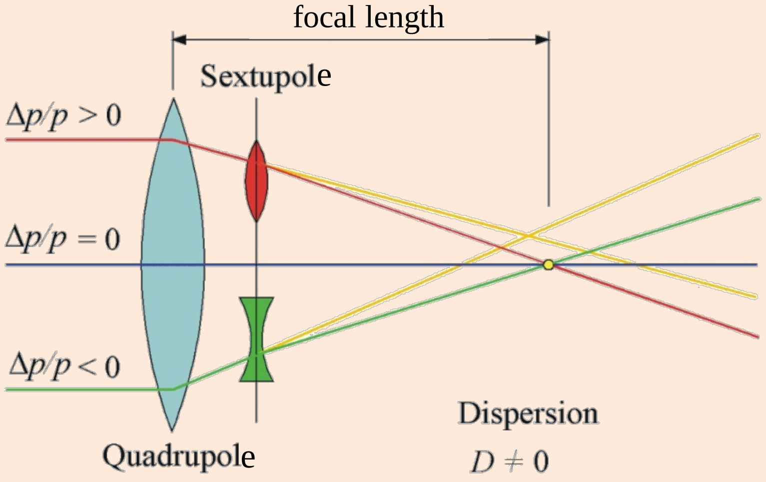


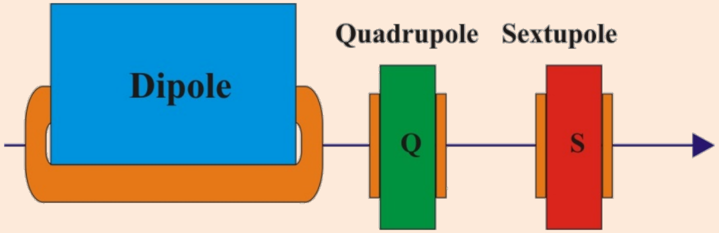
$$\Phi(x,z) = -\frac{1}{6} g' \left(z^3 - 3x^2 z \right)$$

$$\vec{B} = g' \left\{ x z \cdot \hat{e}_x + \frac{1}{2} \left(x^2 - z^2 \right) \cdot \hat{e}_z \right\}$$

$$\vec{F} = evg' \left\{ \frac{1}{2} \left(x^2 - z^2 \right) \cdot \hat{e}_x - x z \cdot \hat{e}_z \right\}$$

Correction of focal length:





Iron Dominated Magnets

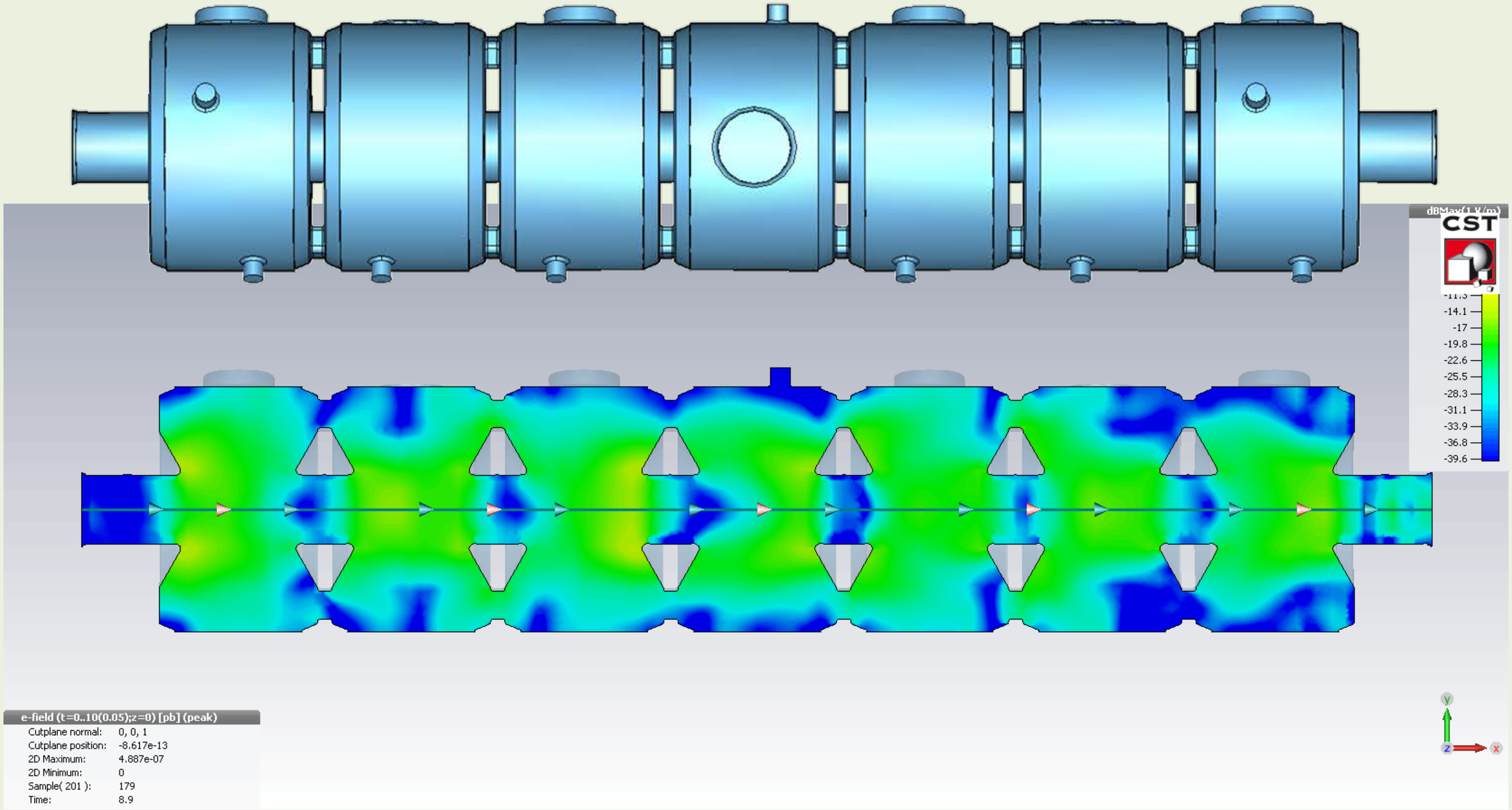
Deflection	Focussing	Chromaticity
Dipoles	Quadrupoles	Sextupoles

Properties defined by pole profiles!

Maximum achievable fields: $B < 2.5$ Tesla

Beam Dynamics

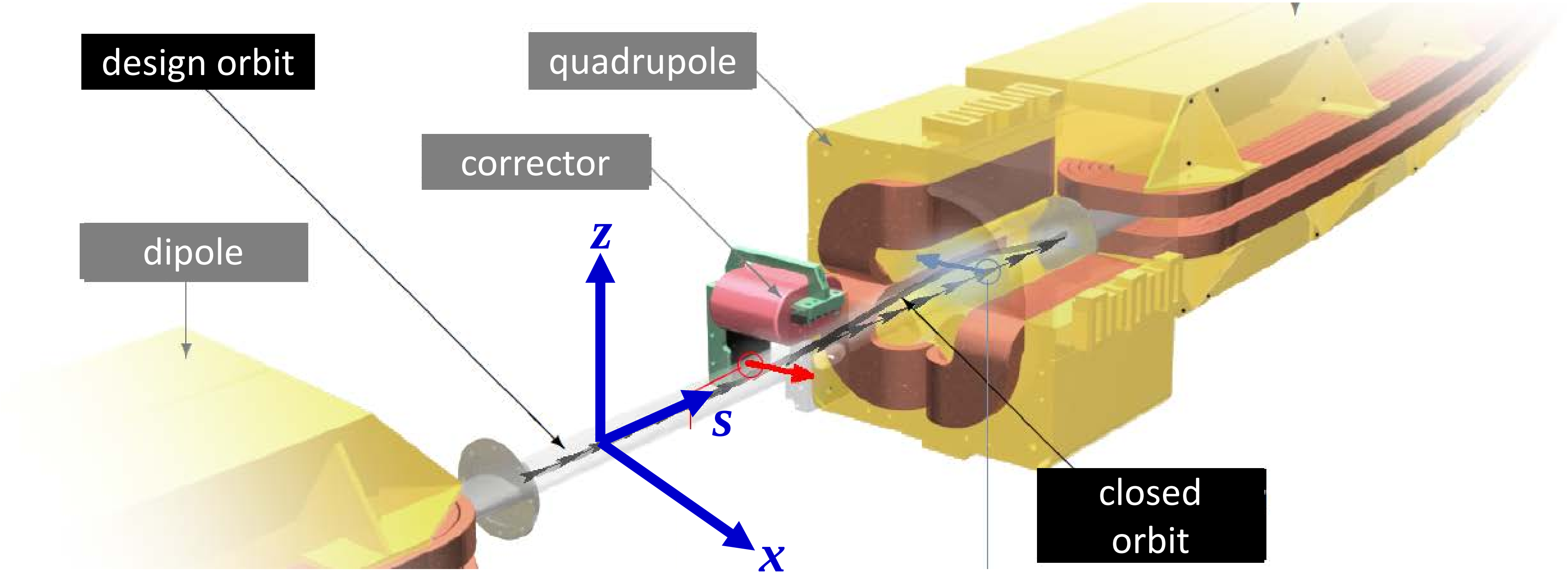
Example: particle bunch in a PETRA 7-cell resonator:



e-field (t=0..10(0.05);z=0) [pb] (peak)

Cutplane normal: 0, 0, 1
Cutplane position: -8.617e-13
2D Maximum: 4.887e-07
2D Minimum: 0
Sample(201): 179
Time: 8.9

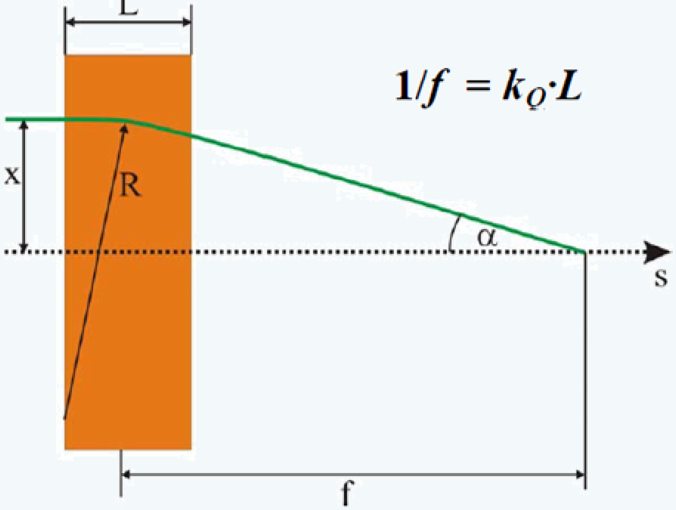
Particle Paths



moving reference frame,
fixed to reference particle

Equation of Motion

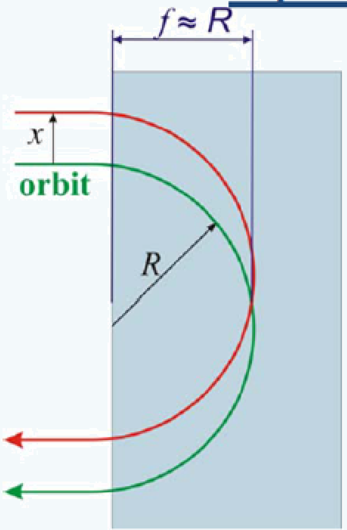
Quadrupole Magnet:



$1/f = k_Q \cdot L$

$$\vec{F}_{mag} = m \cdot (\beta c)^2 \cdot k \cdot (x\hat{e}_x - z\hat{e}_z)$$

Dipole Magnet:



focusing
analog to
quadrupole:

$1/f = k_D \cdot L$

$k_D = 1/R^2$

$$\vec{F}_{mag} = m \cdot (\beta c)^2 \cdot \frac{1}{R^2} x\hat{e}_x$$

⇓

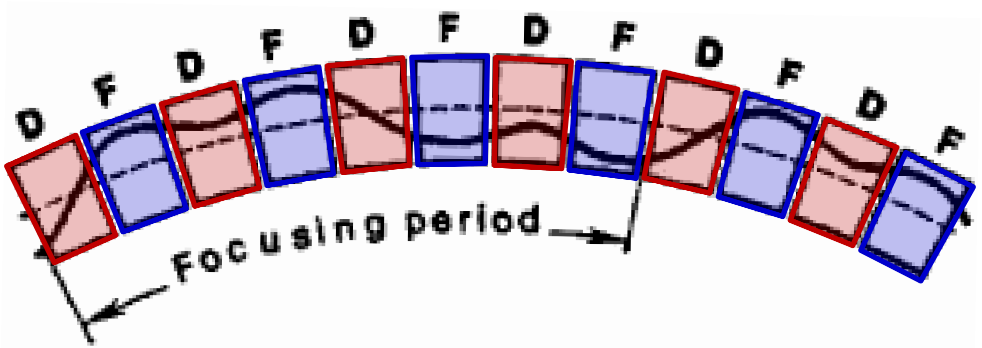
$$m \cdot \ddot{\vec{x}}(t) = m \cdot (\beta c)^2 \cdot \vec{x}''(s) = \vec{F}_{mag}$$

⇒ **Hill's Differential Equation:**

$$x''(s) + \left(\frac{1}{R^2(s)} - k(s)\right) \cdot x(s) = \frac{1}{R(s)} \frac{\Delta p}{p}$$

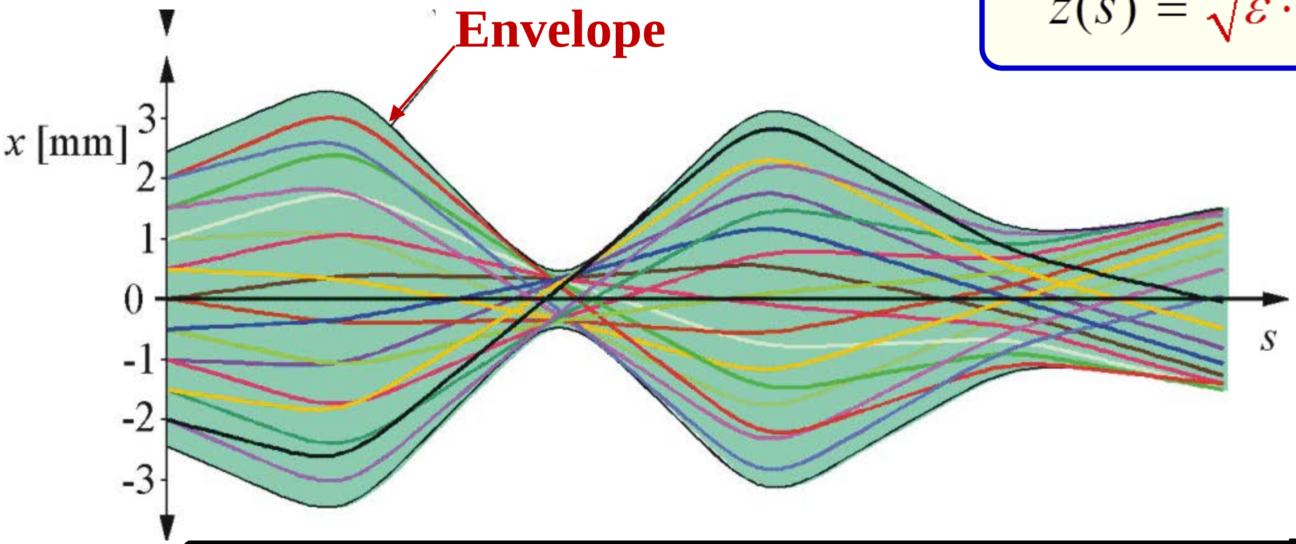
$$z''(s) + k(s) \cdot z(s) = 0$$

Betatron Oscillations



$$z''(s) + k(s)\cdot z(s) = 0$$

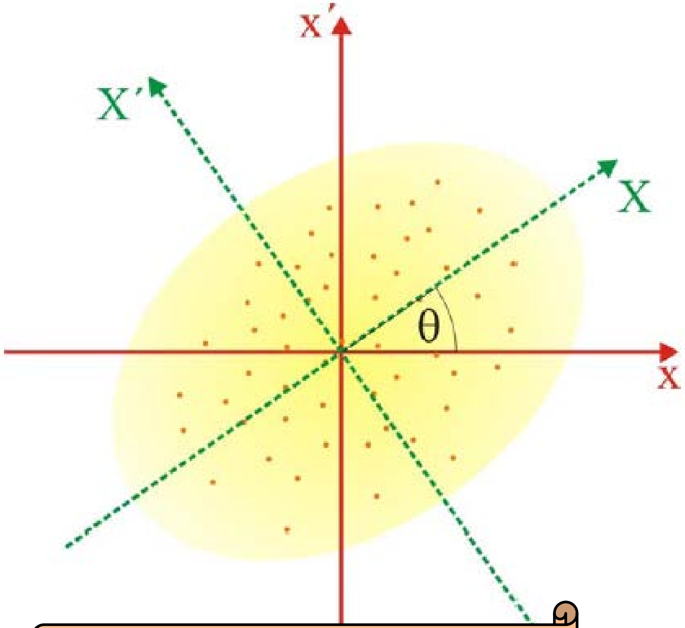
$$z(s) = \sqrt{\varepsilon \cdot \beta_z(s)} \cdot \cos\left(\phi(s) + \varphi_0\right)$$



 # Oscillations per Revolution: Tune $Q_{x,z} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,z}(s)}$

TWISS and Emittance

Particle's Phase Space:



ε = enclosed area / π

TWISS Parameters:

$$\sigma_x = \sqrt{\overline{x^2}} = \sqrt{\beta\,\varepsilon}$$

$$\sigma_{x'} = \sqrt{\overline{x'^2}} = \sqrt{\gamma\,\varepsilon}$$

$$r\,\sigma_x\sigma_{x'} = \overline{x\,x'} = -\alpha\,\varepsilon$$

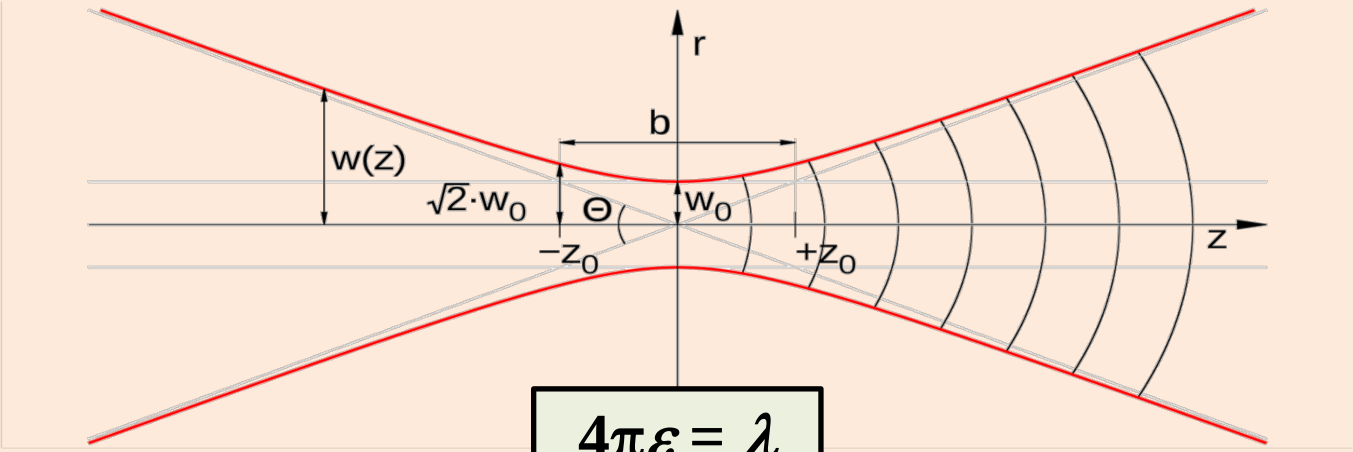
Dispersion:

$$\delta x = D\cdot\frac{\Delta p}{p}$$

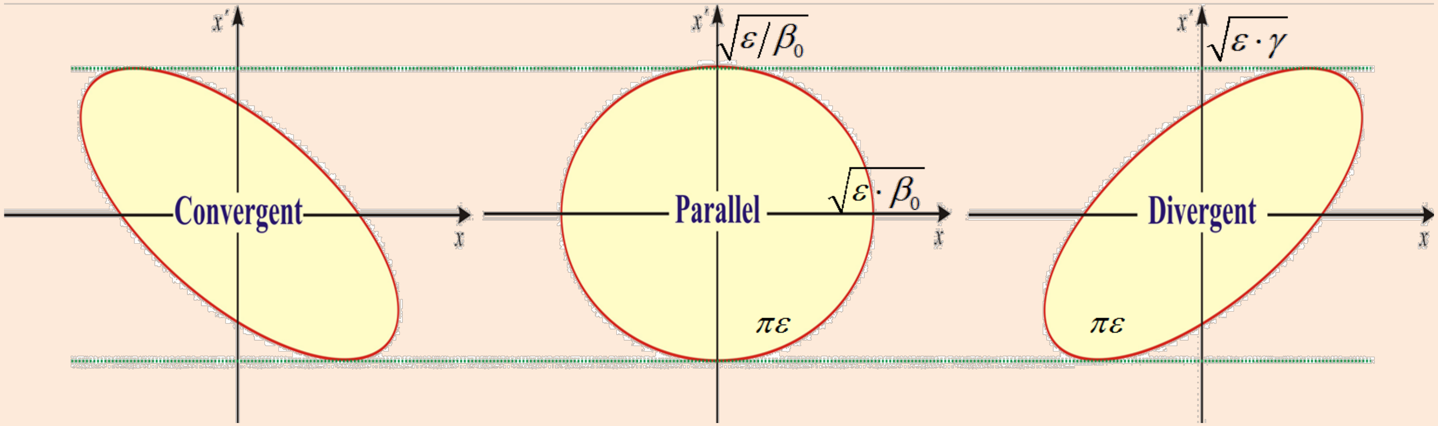
$$\delta x' = D'\cdot\frac{\Delta p}{p}$$

$\gamma\,x^2 + 2\alpha\,x\,x' + \beta\,x'^2 = \varepsilon,$ where $\alpha = -\frac{\beta'}{2}$ and $\gamma = \frac{1+\alpha^2}{\beta}$

TWISS and Emittance

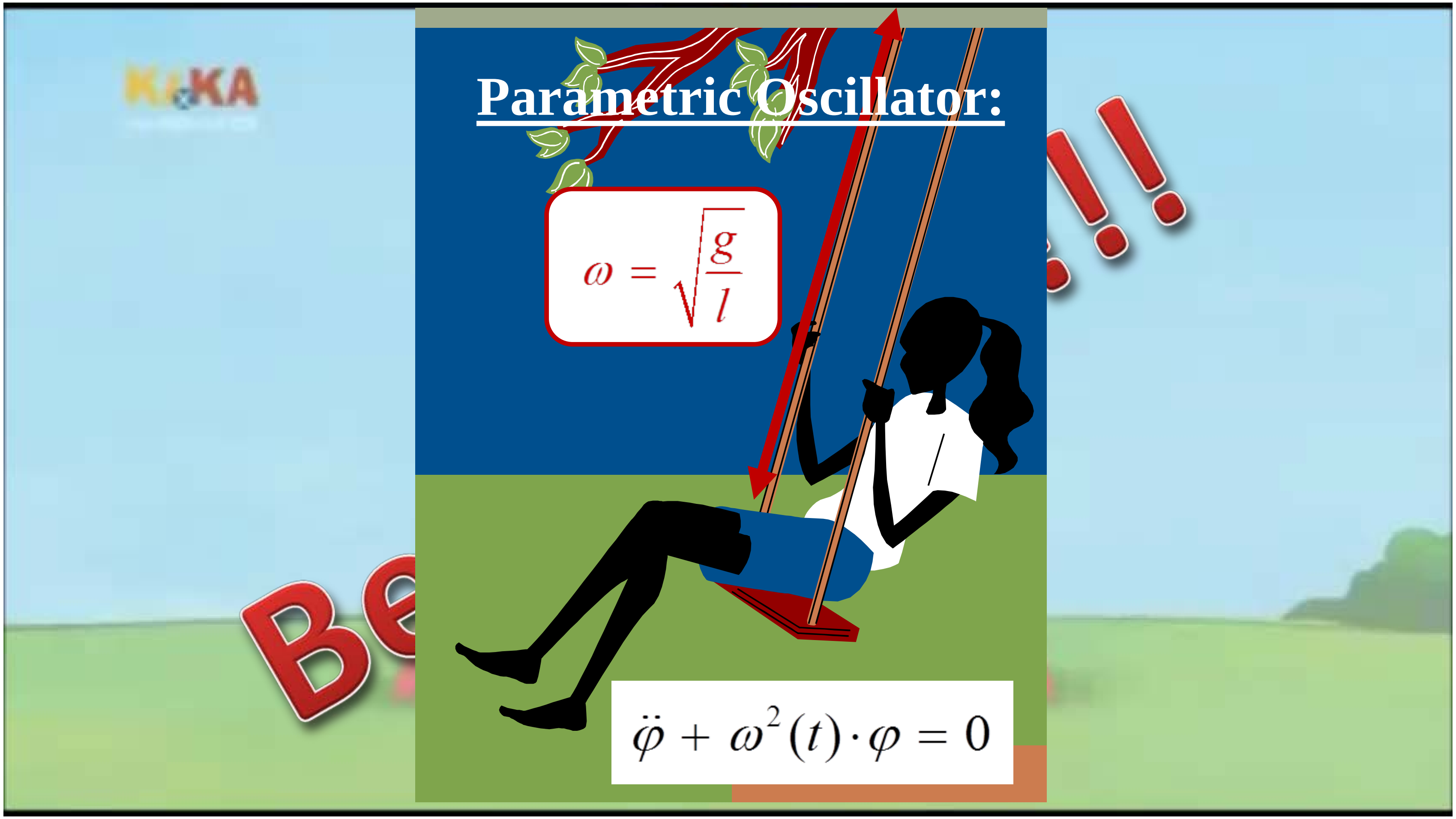


$$\boxed{4\pi\varepsilon = \lambda}$$



$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon, \qquad \text{where} \quad \alpha = -\frac{\beta'}{2} \quad \text{and} \quad \gamma = \frac{1+\alpha^2}{\beta}$$

Solving Hills DGL



Parametric Oscillator:

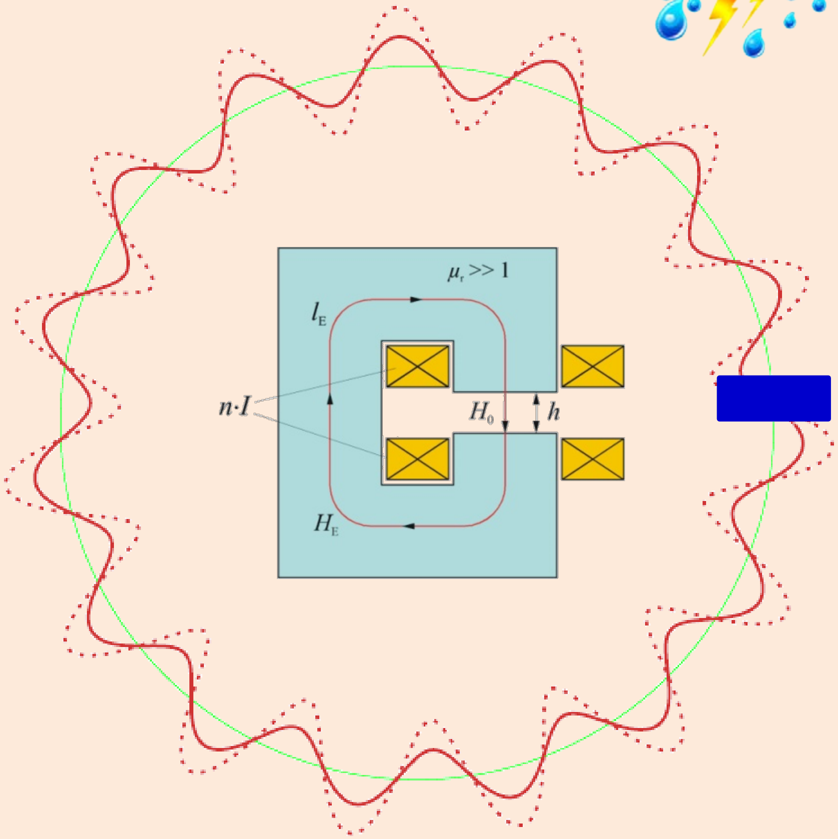
$$\omega = \sqrt{\frac{g}{l}}$$

$$\ddot{\varphi} + \omega^2(t) \cdot \varphi = 0$$

Field Errors

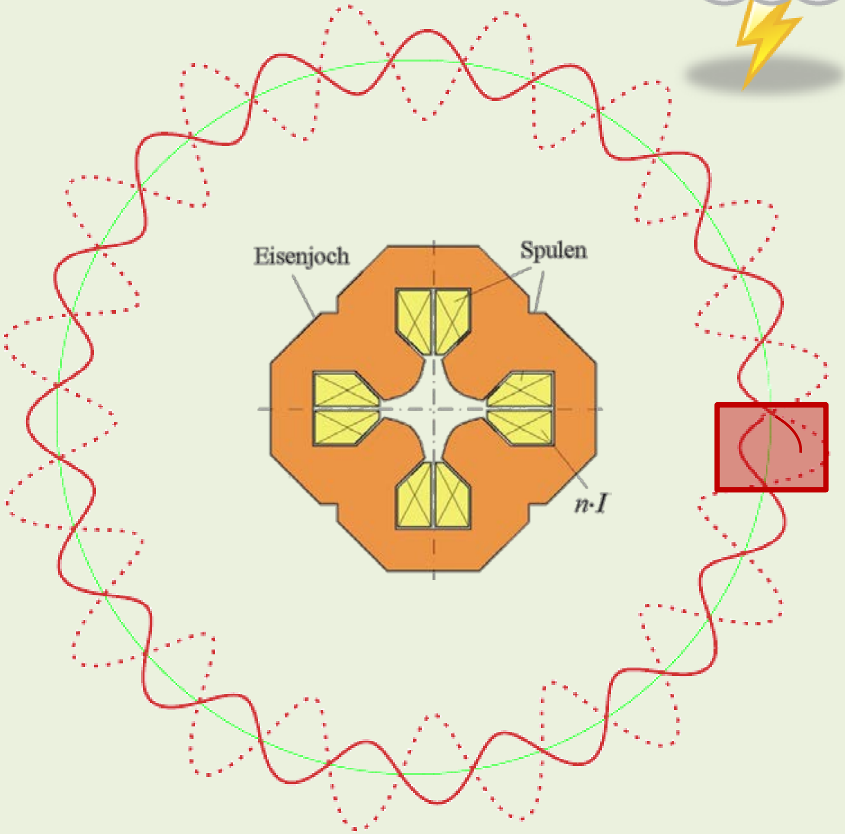
Dipole Magnet

$Q = n$

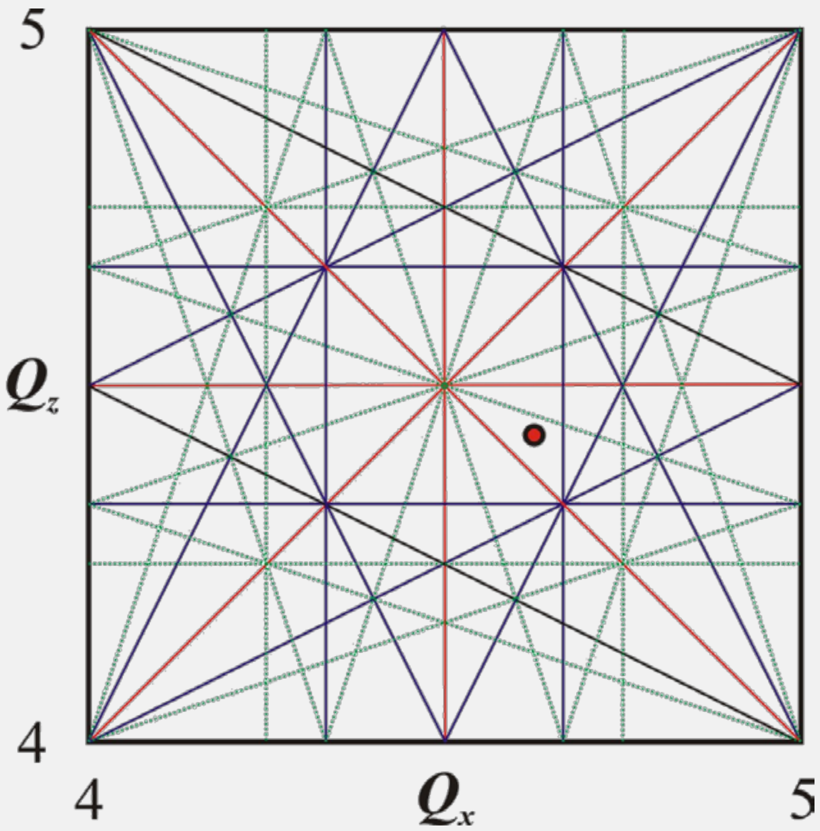


Quadrupole Magnet

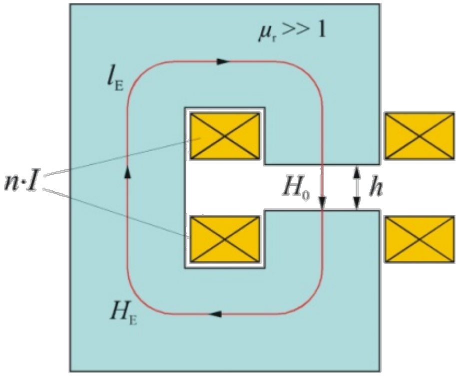
$Q = m/2$

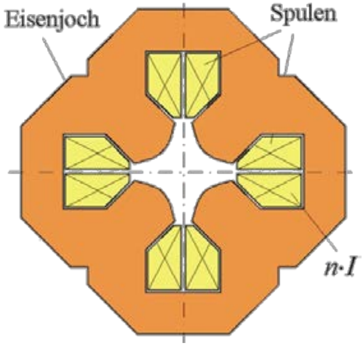


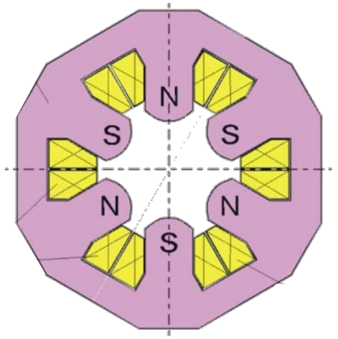
Optical Resonances



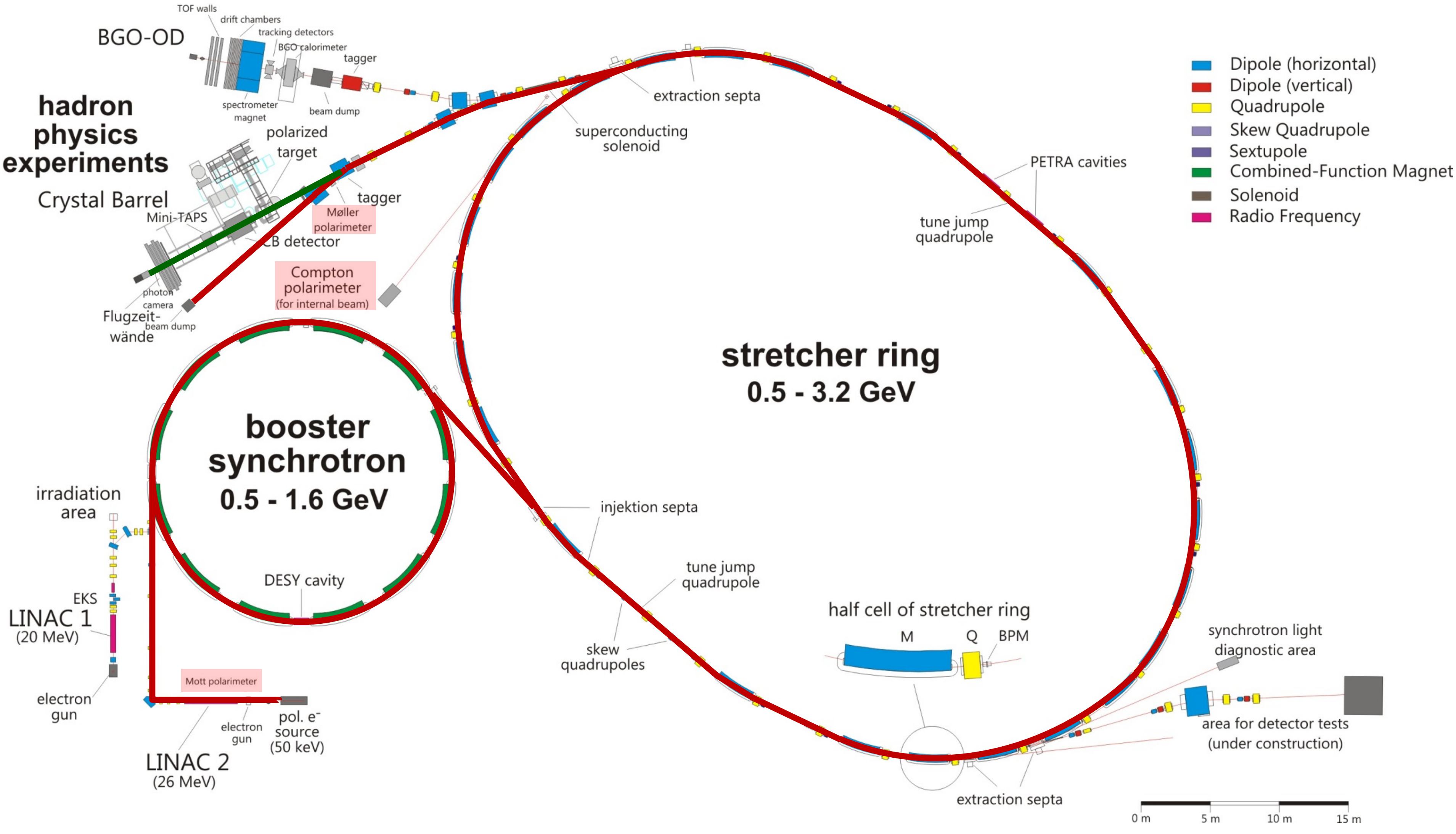
$$j \cdot Q_x + k \cdot Q_z = N$$







Electron Stretcher Accelerator (ELSA)



Improving Beam Quality

Luminosity:

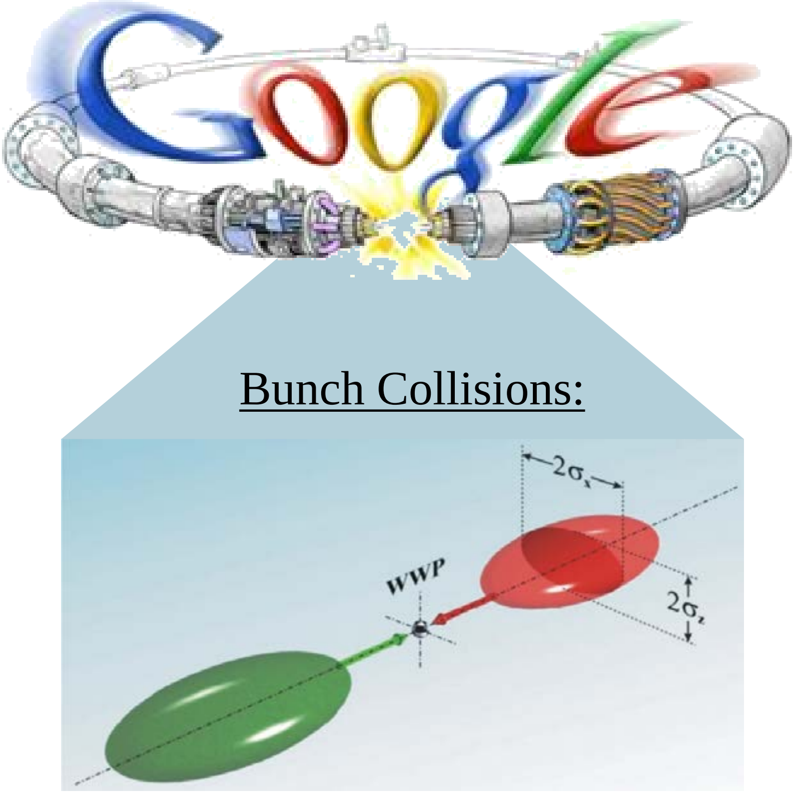
$$\dot{N} = \sigma \cdot \mathfrak{L}$$

e+-e-, p-p Collider:

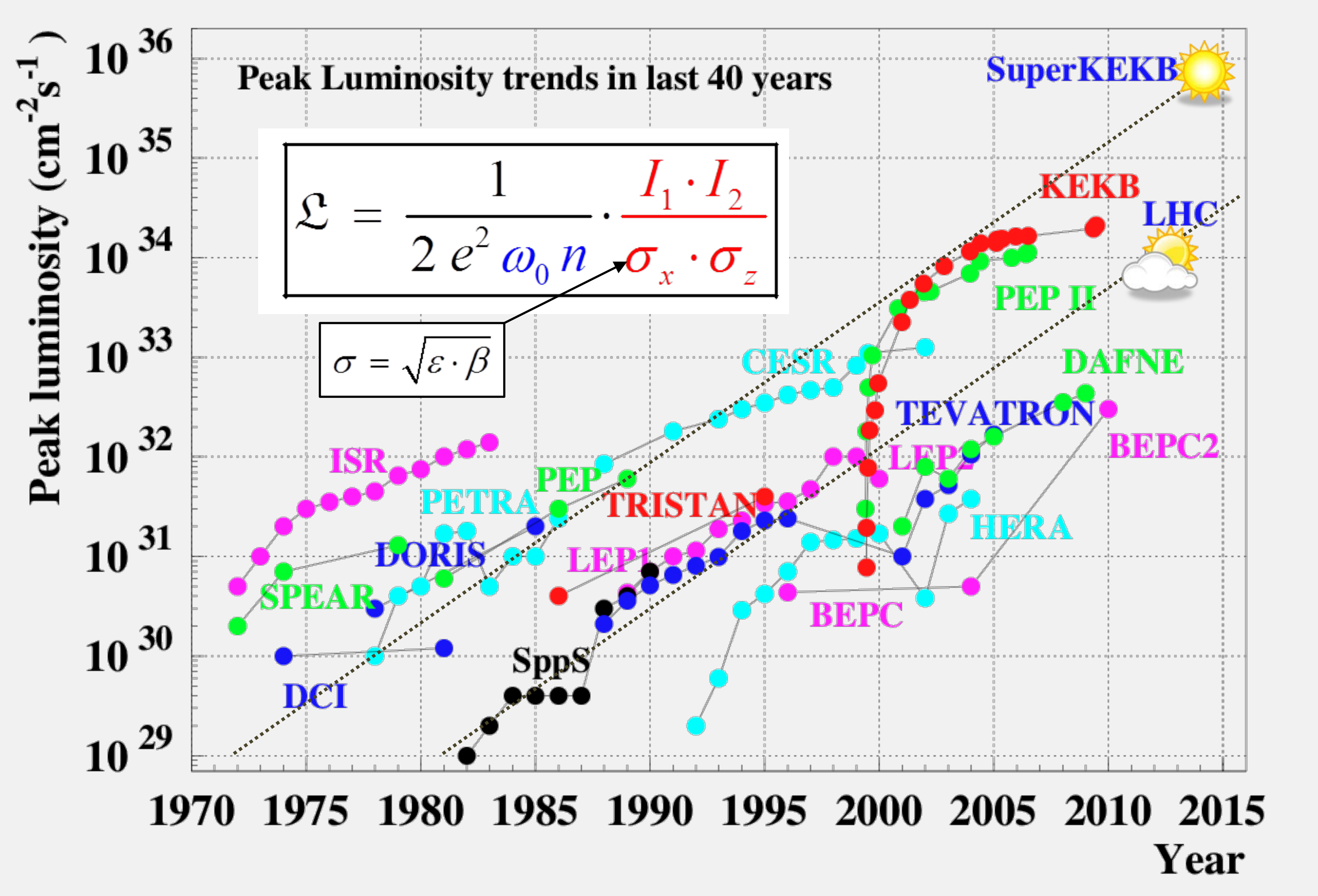
$$\boxed{\mathfrak{L} = \frac{1}{2\, e^2\, \omega_0\, n} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}}$$

$$\boxed{\sigma = \sqrt{\varepsilon \cdot \beta}}$$

- **high beam current**
- **small beam size**



Luminosity



Peak Luminosity trends in last 40 years

$$\mathfrak{L} = \frac{1}{2\,e^{2}\,\omega_{0}\,n} \cdot \frac{I_{1} \cdot I_{2}}{\sigma_{x} \cdot \sigma_{z}}$$

$$\sigma = \sqrt{\varepsilon \cdot \beta}$$

$$1\mathrm{ab}^{-1}/\mathrm{Jahr} \leftrightarrow \mathfrak{L} = 3 \cdot 10^{35} \frac{1}{\mathrm{cm}^{2}\mathrm{s}}$$

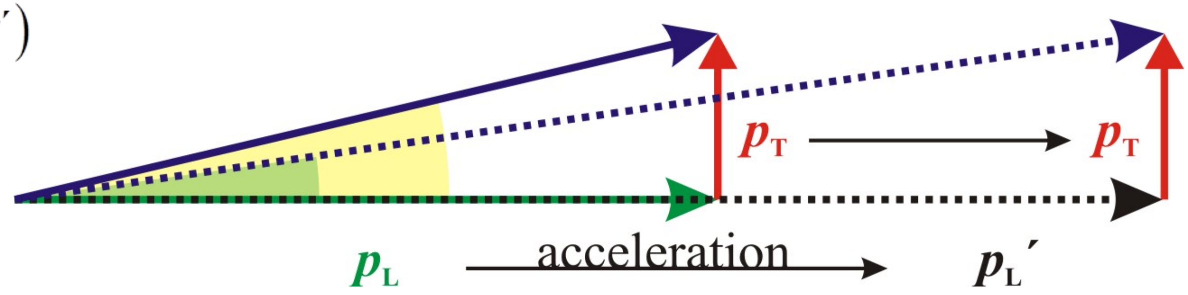
Adiabatic Damping

Proton Beams: negligible influence of synchrotron radiation!

"A proton machine remembers everything done to the beam like an elephant!"

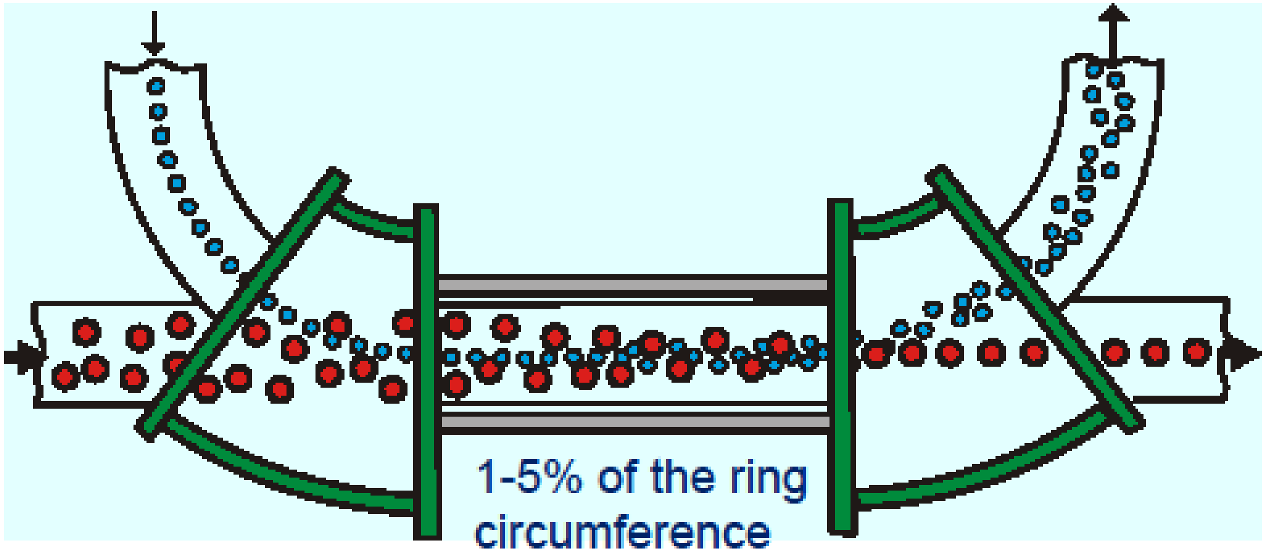
What remains is:

$(\varepsilon \approx \sigma \cdot \sigma')$



Adiabatic damping: $\varepsilon = \frac{p_0}{p} \cdot \varepsilon_0 = \frac{\varepsilon_{norm}}{\beta\gamma}$

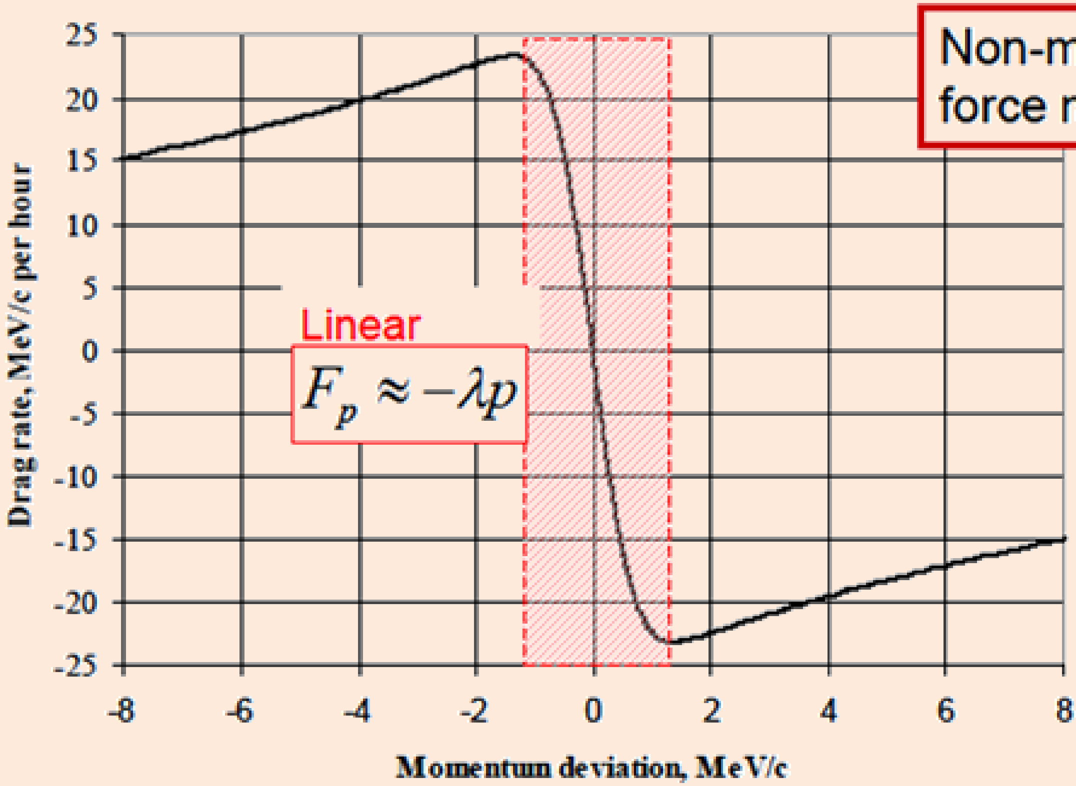
Electron Cooling



Heat exchanger with cooled electrons via Coulomb interaction

Important: $v_e = v_i$

Electron Cooling



Non-magnetized cooling force model

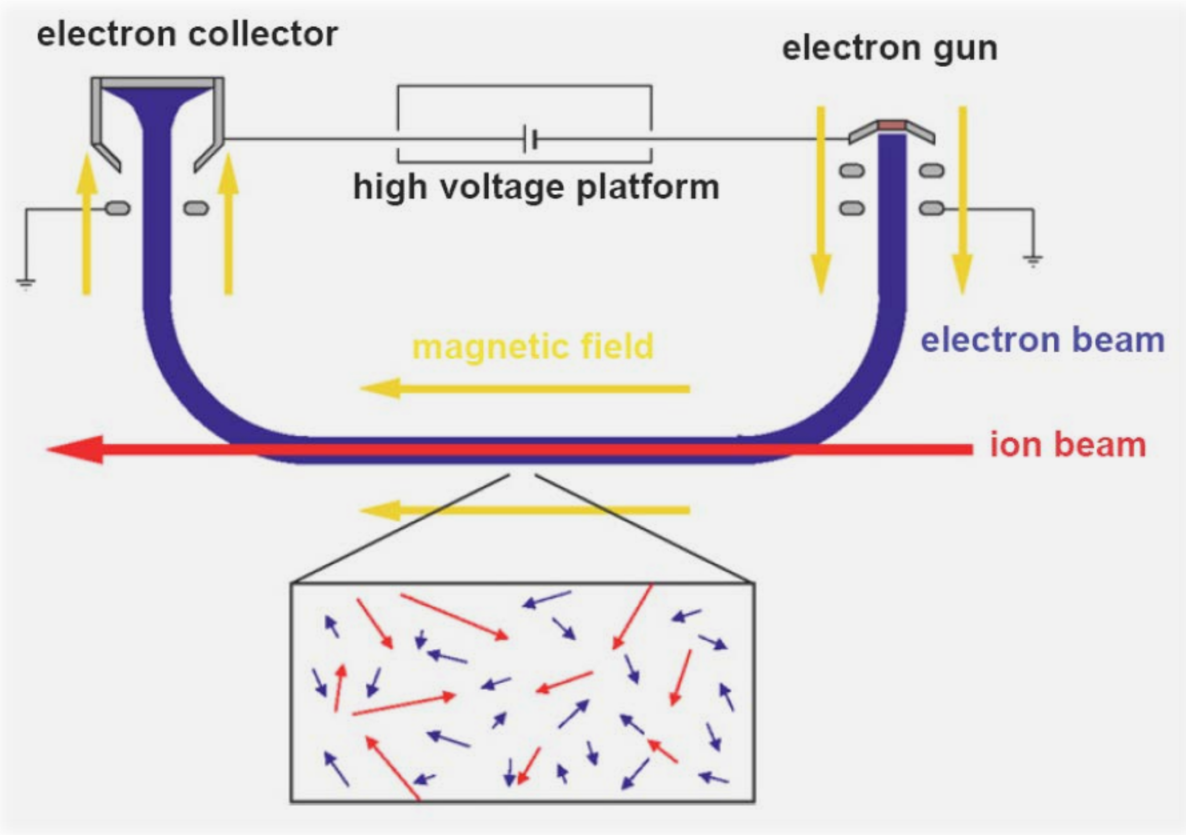
Linear

$F_p \approx -\lambda p$

$$\vec{F}(\vec{v}_i) = -\frac{4\pi n_e e^4 Z^2}{m_e} L_C \int \frac{\vec{v}_i - \vec{v}_e}{\left|\vec{v}_i - \vec{v}_e\right|^3} f(v_e)\cdot d^3 v_e$$

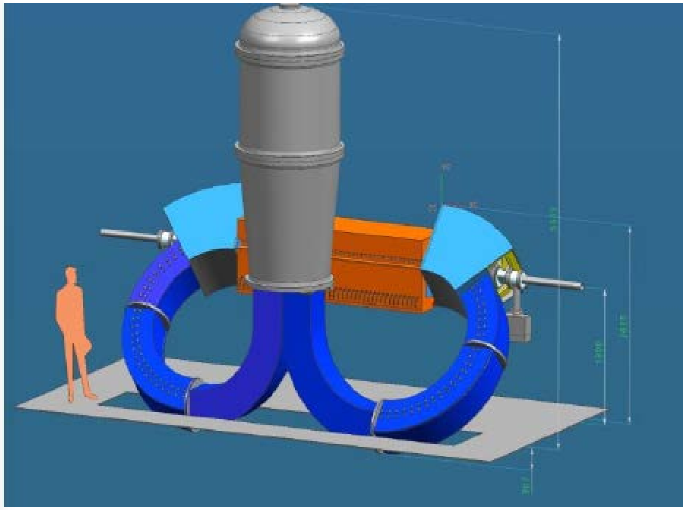
Electron Cooling

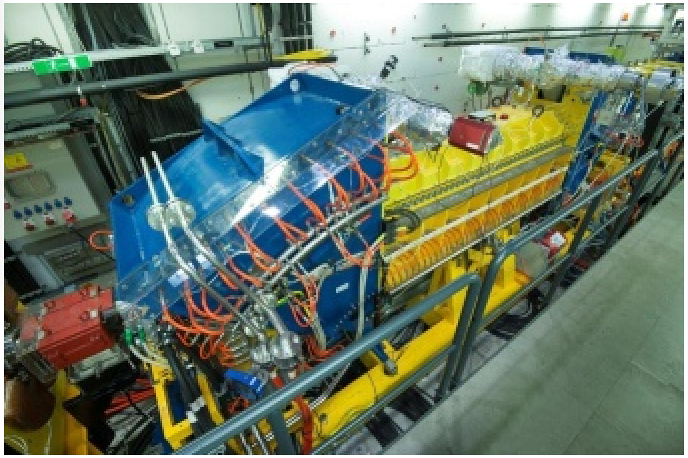
$$\underbrace{k_B T_{\parallel} = \left(k_B T_{cath}\right)^2 / 4E_0}_{(< 1\ \text{meV})} \ll \underbrace{k_B T_{\perp} = k_B T_{cath}}_{(\approx 100\ \text{meV})}$$ → **magnetized cooling!**



Limitation: $\frac{p_e}{p_i} = \frac{m_e}{m_p}$

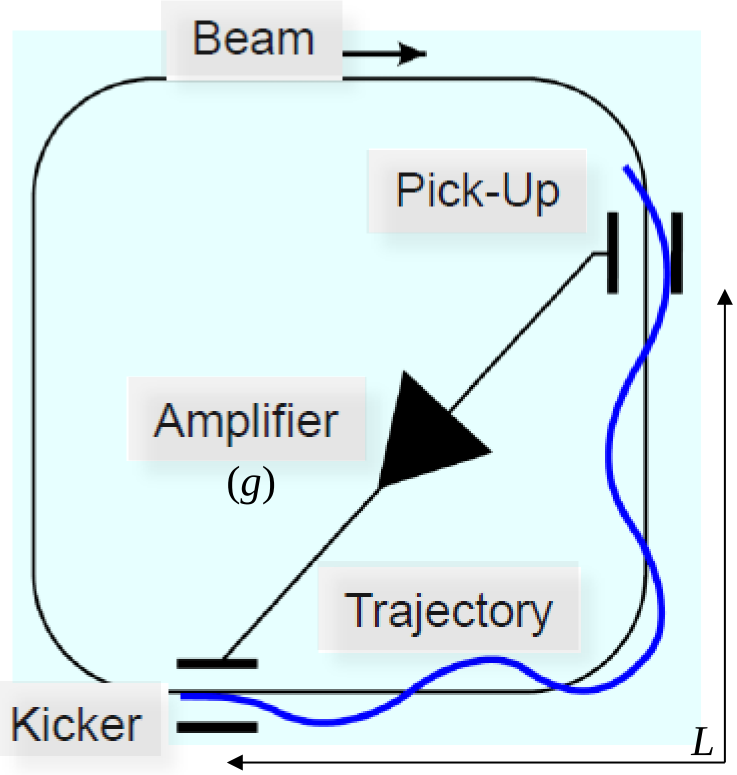
FZJ: 2MeV-Cooler





Stochastic Cooling

- **measure and correct a beam sub-sample**



Important: High BW

$$\mathcal{W} = \frac{1}{2T_{sample}}$$

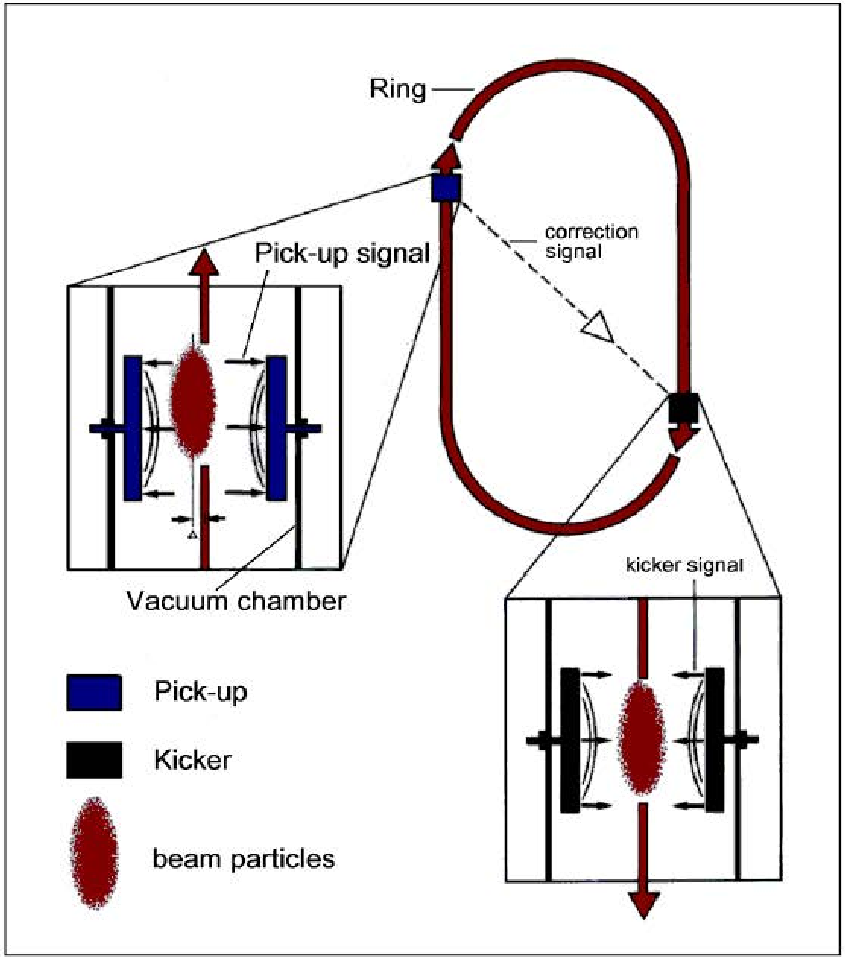
Cooling rate:

$$\frac{1}{\tau} = \frac{\mathcal{W}}{N}\left\{2g\left[1-\left(\frac{L}{C\cdot M}\right)^2\right]-g^2\left[M+U\right]\right\}$$

- **take care of mixing *M* (which is essential):**

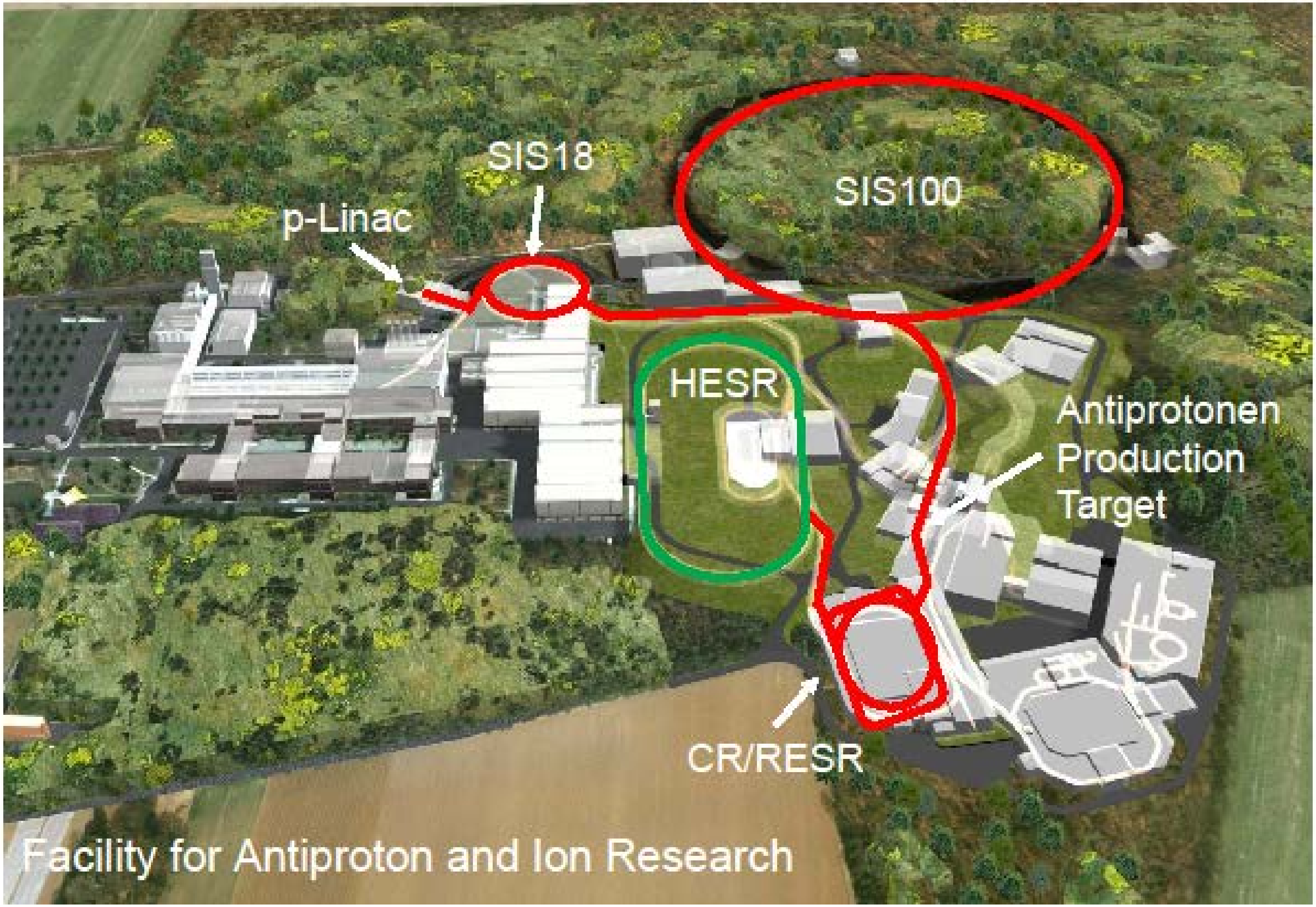
→ Δ(PU–K): small, Δ(K–PU): large

Stochastic Cooling @ COSY

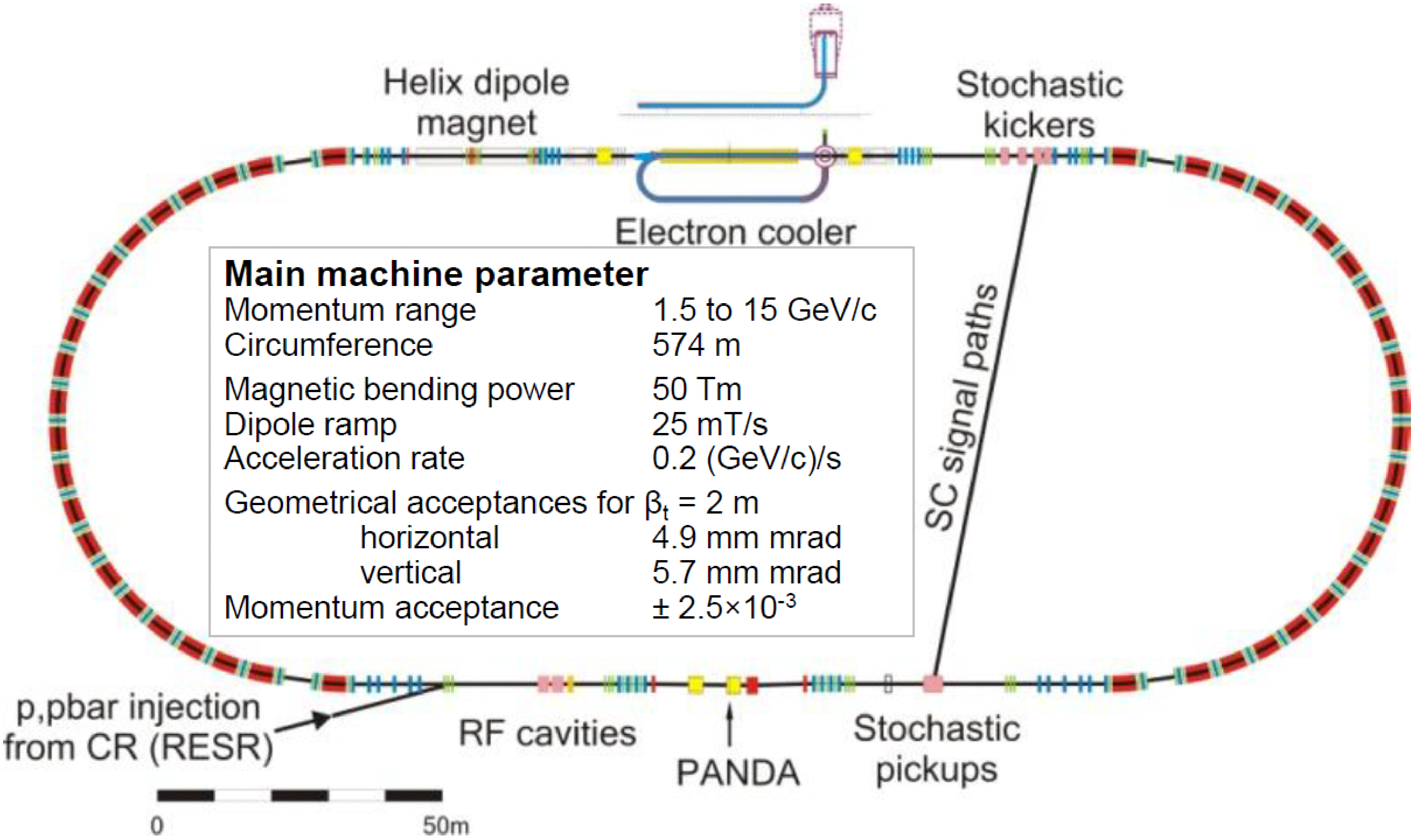


- Transverse and longitudinal
- Frequency range: 1-3 GHz
  2 bands
- RF power: 500 W
  per plane

Phase-Space Cooling at FAIR



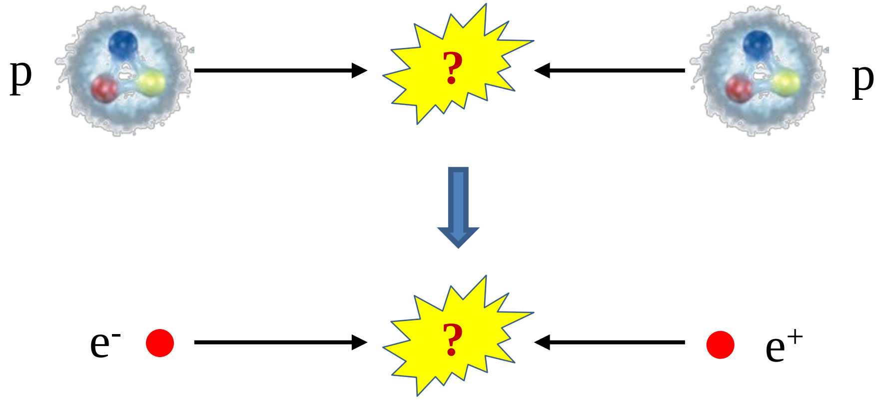
HESR Layout:



Main machine parameter

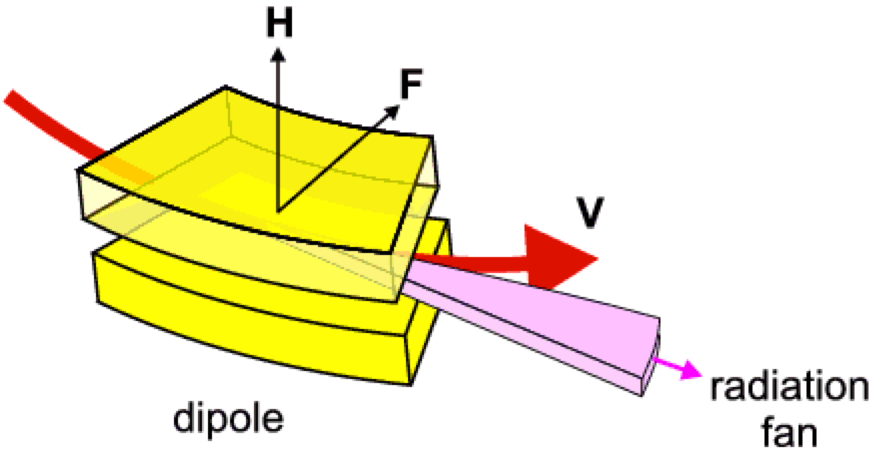
Momentum range	1.5 to 15 GeV/c
Circumference	574 m
Magnetic bending power	50 Tm
Dipole ramp	25 mT/s
Acceleration rate	0.2 (GeV/c)/s
Geometrical acceptances for $\beta_t = 2$ m	
horizontal	4.9 mm mrad
vertical	5.7 mm mrad
Momentum acceptance	$\pm 2.5\times10^{-3}$

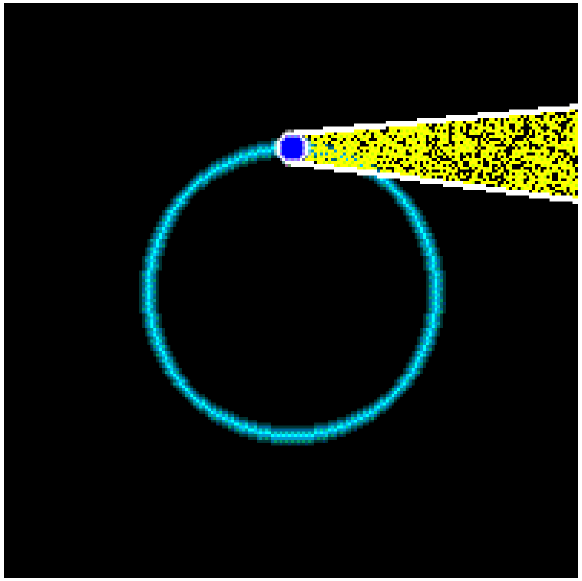
Electrons / Positrons



p ? p

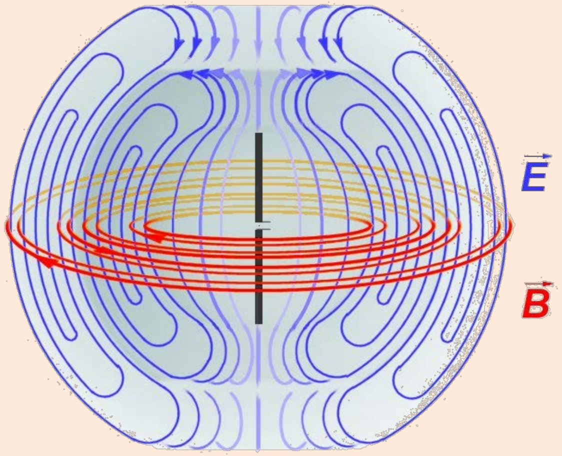
e^- ? e^+





Synchrotron Radiation

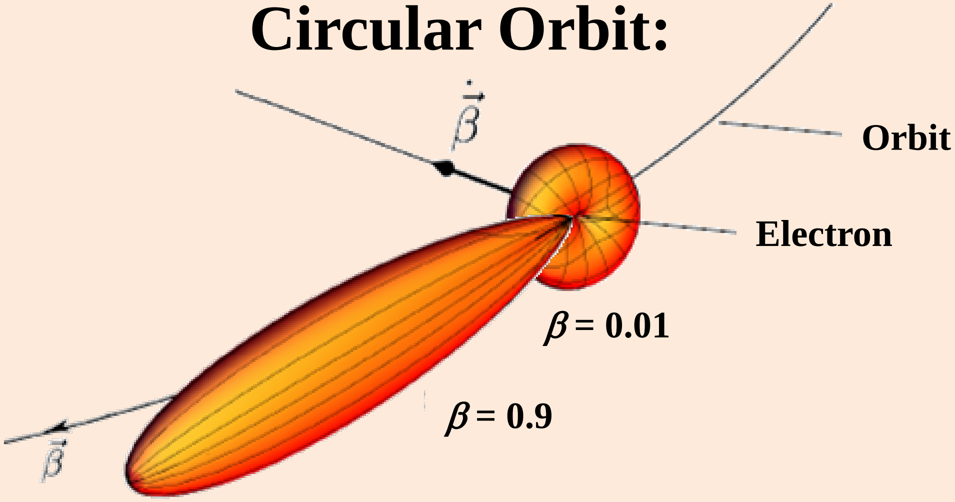
Hertz Dipol:



$$P = \frac{e^2}{12\pi\varepsilon_0 c^3}\cdot\omega^4 d^2$$

$\longrightarrow$

Circular Orbit:

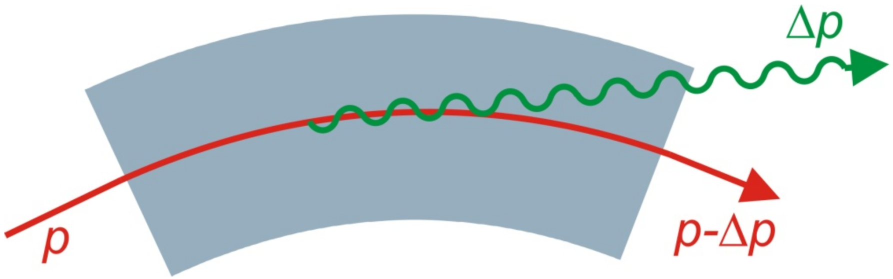


$$P = \frac{e^2 c}{6\pi\varepsilon_0}\cdot\frac{\gamma^4}{R^2}$$

$$U\,[\mathrm{kV}] = 88.5\cdot\frac{E^4\,[\mathrm{GeV}^4]}{R\,[\mathrm{m}]}$$

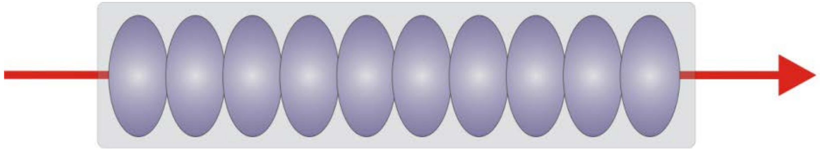
Radiative Cooling

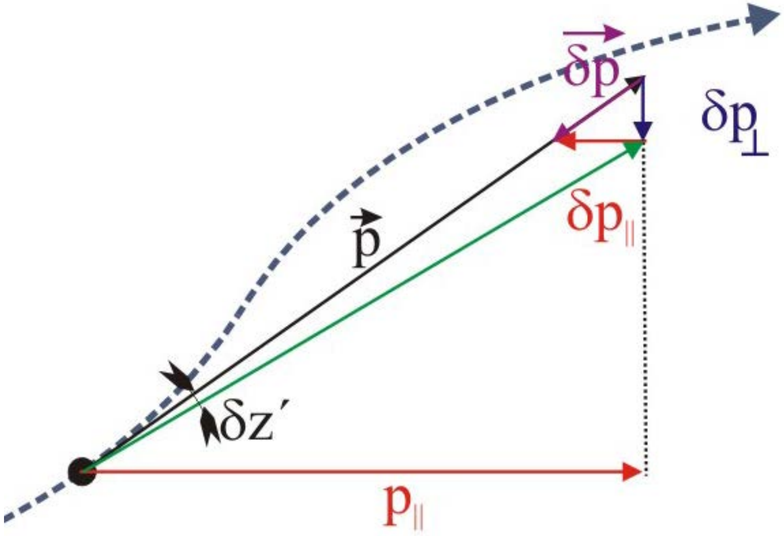
Synchrotron Radiation



+

beam acceleration

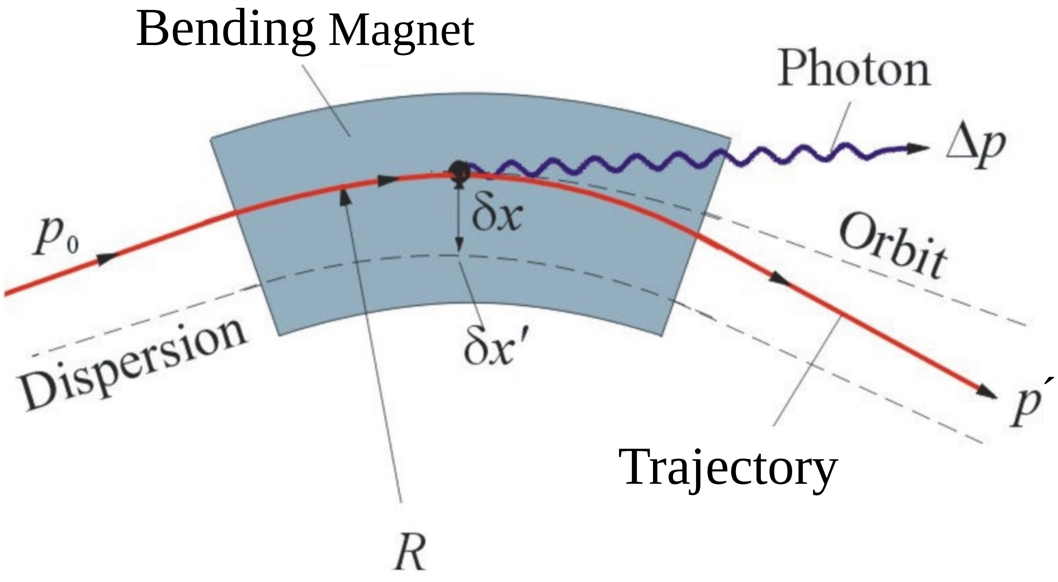


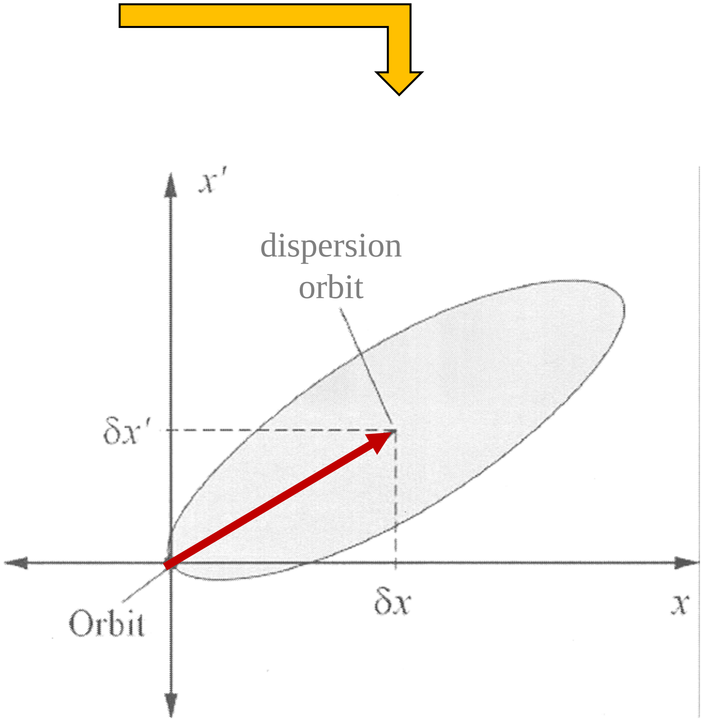


but:

Equilibrium Emittance

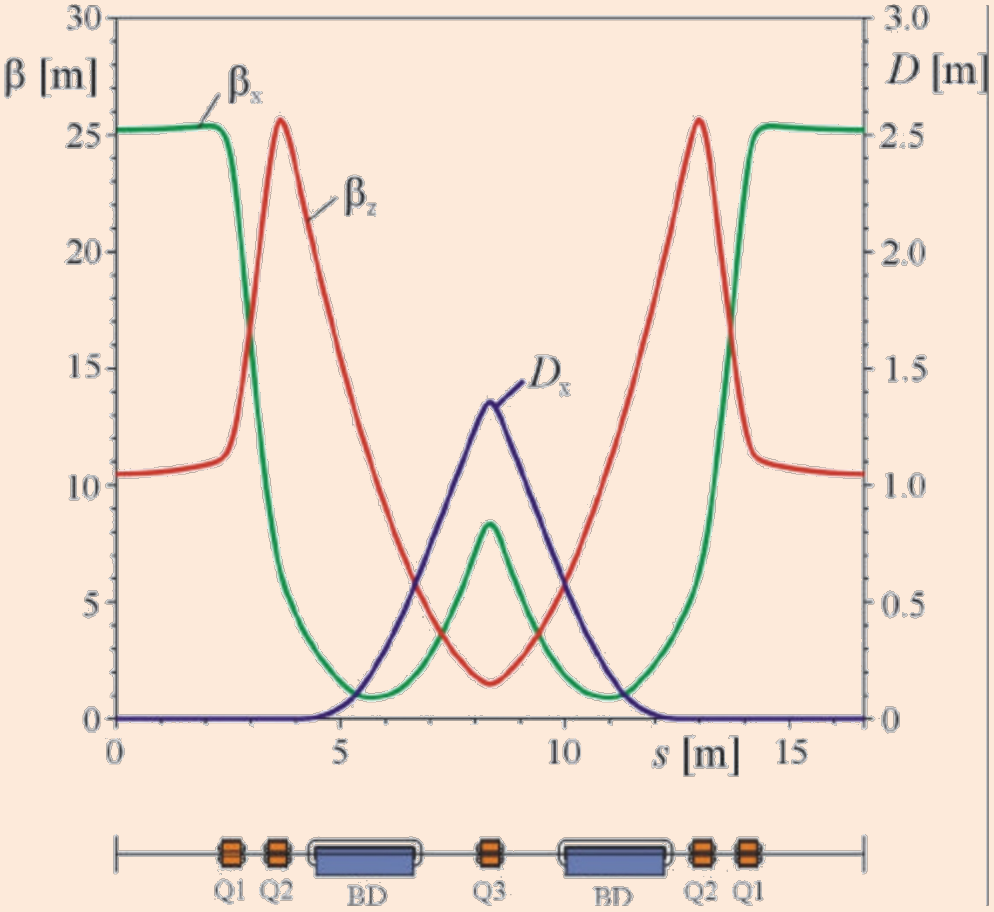
Heating in dispersive sections:





ε : Equilibrium of Heating and Cooling

Low Emittance Lattice



Ideas:
- use short dipole magnets
- suppress dispersion in straights

Simplest lattice:
DBA (e.g. Chasman-Green)

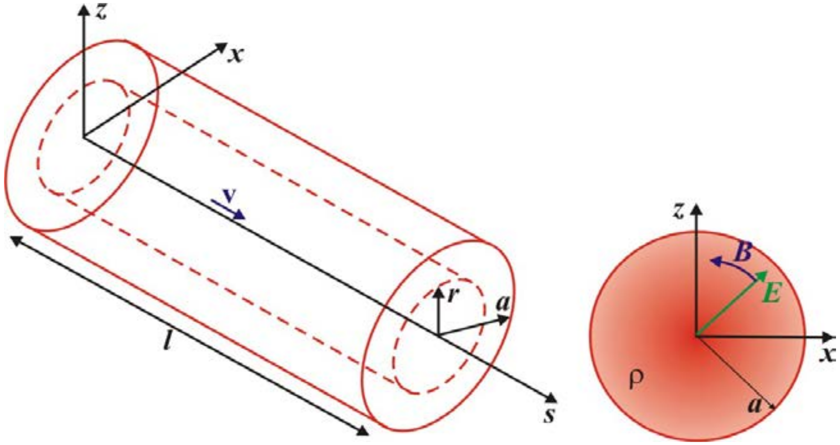
More sophisticated lattices
+ damping wigglers
used in SR-sources

Energy-scaling of emittance:

$$\varepsilon \sim E^2$$

Direct Space Charge

Coulomb Repulsion caused by Neighbors:



$$\oiint_{\partial V} \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\iiint_V \rho\cdot d^3r$$

$$\Rightarrow E_r(r) = \frac{\rho}{2\varepsilon_0}\cdot r$$

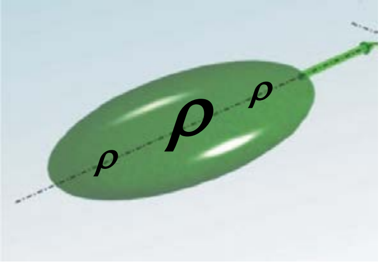
$$F^*(r) = \frac{e\rho}{2\varepsilon_0}\cdot r$$

Transformation to the Lab-Frame:

- Lorentz contraction → Factor γ
- Dilution of space charge density → Factor γ

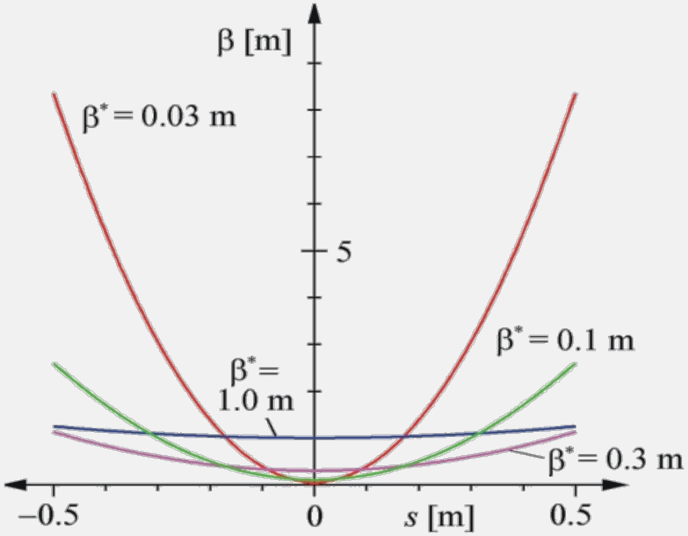
$$\gamma m_0 \ddot{x} = F_x = \frac{F_x^*}{\gamma^2}$$

$$\Delta Q_{x,z} = -\frac{e}{8\pi^2\varepsilon_0 m_0 (\beta c)^3 \gamma^3}\cdot\boxed{\frac{I}{\varepsilon_{x,z}}}\cdot L$$

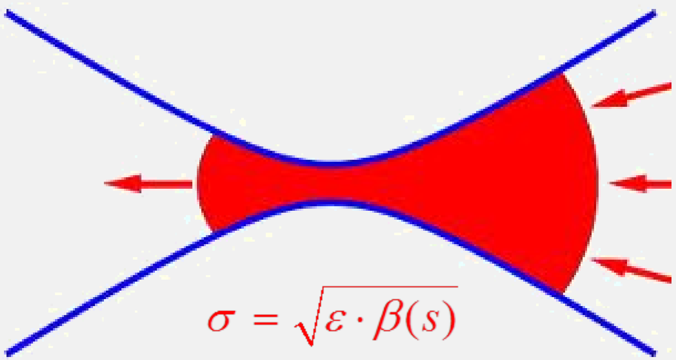


Small Beam Sizes

Beam Broadening:



Hourglass-Effect:



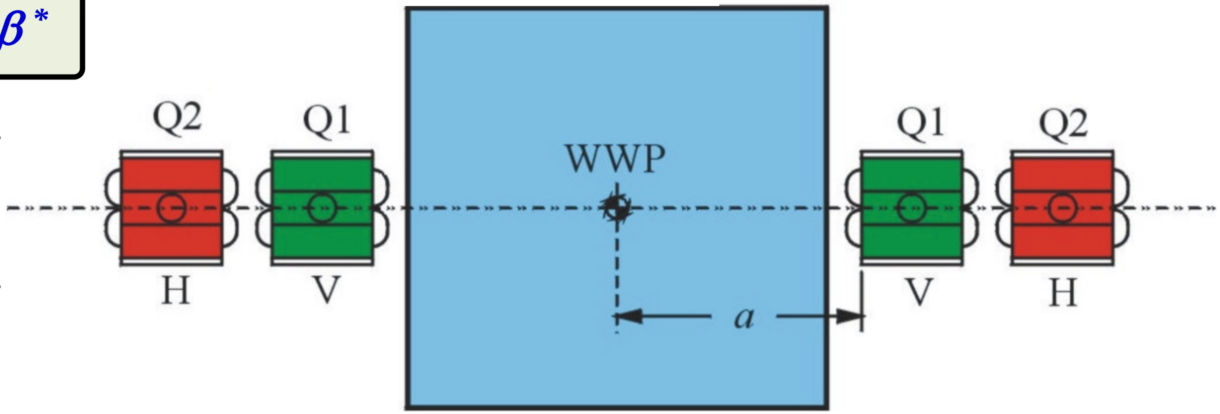
$\sigma = \sqrt{\varepsilon \cdot \beta(s)}$

short bunches!

$\sigma < 1{,}7 \cdot \beta^*$

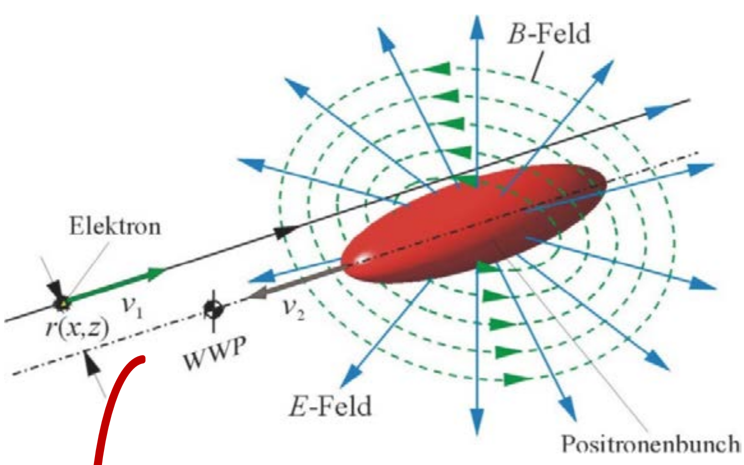
$$\mathfrak{L} = \frac{1}{2\, e^2\, \omega_0\, n} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$

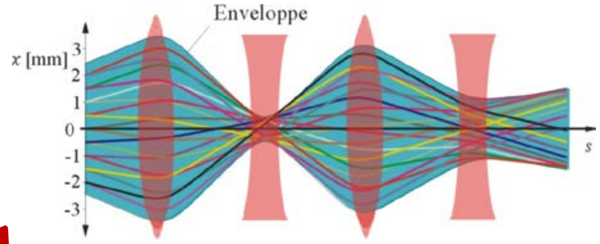
particle detector

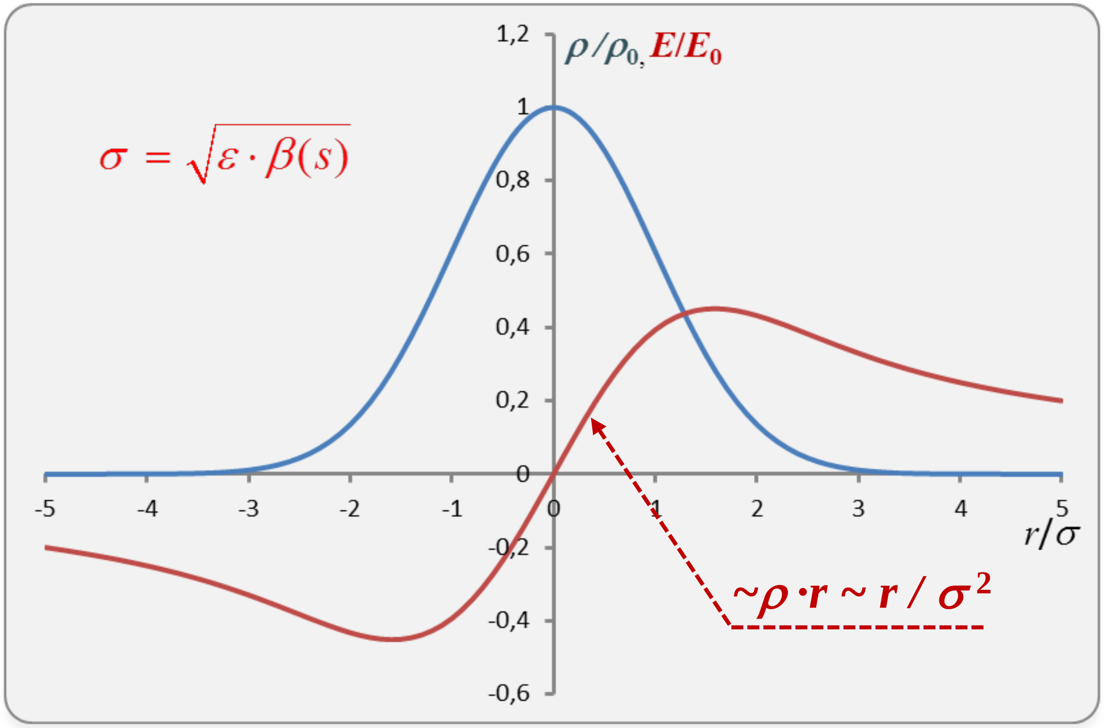


Beam-Beam Interaction

Additional focussing resp. defocussing in beam crossing region







Beam-Beam Parameters:

$$\Delta Q_x = -\frac{N r_e}{2\pi}\cdot\frac{1}{\gamma}\cdot\frac{\beta_x^*}{(\sigma_x+\sigma_z)\sigma_x}$$

$$\Delta Q_z = -\frac{N r_e}{2\pi}\cdot\frac{1}{\gamma}\cdot\frac{\beta_z^*}{(\sigma_x+\sigma_z)\sigma_z}$$

Accelerators for Hadronists

Conclusions:

GeV ↔ acceleration in RF electric fields

circular accelerators / recirculating LINACs (electrons only)

- **Magnets:** **dipoles:** deflection / **quadrupoles:** focussing
- **Beam Dynamics:** betatron oscillations, **betatron tune**
  field errors: optical resonances
- **Beam Quality:** **beam emittance = area in phase space / π**
  - adiabatic damping
  - electron cooling } (protons, ions)
  - stochastic cooling } (protons, ions)
  - equilibrium emittance (electrons)
- **Limitations:** direct space charge, beam-beam effects, …