Accelerator Physics

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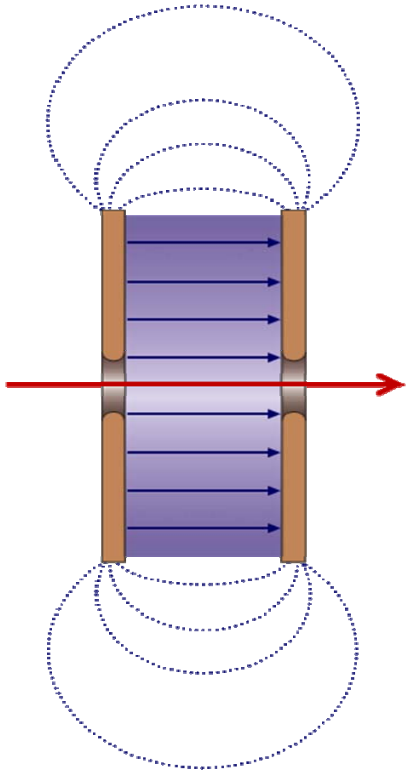
a 45 minutes crash course:

Contents:

1. Introduction

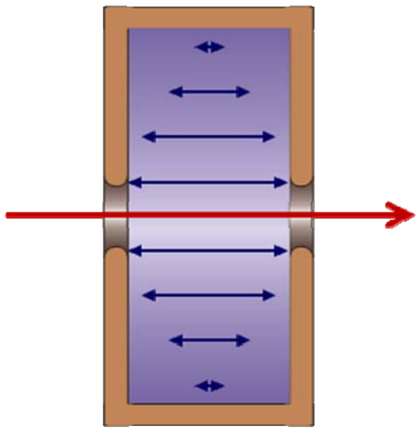
Electrostatic acceleration:



$\oint \vec{E} \cdot d\vec{s} = 0$ due to fringe fields!

RF-based acceleration:

No fringe fields!



no continuous flow of particles

"bunched particle beam"

synchronization required

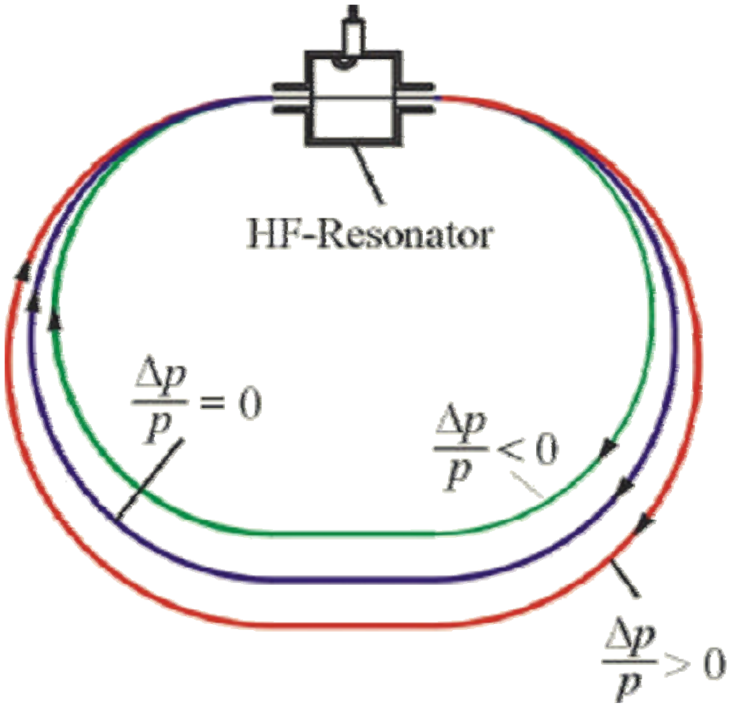
Electrical breakdown limits maximum surface fields to $E \approx U/R \approx$ 10 MV/m

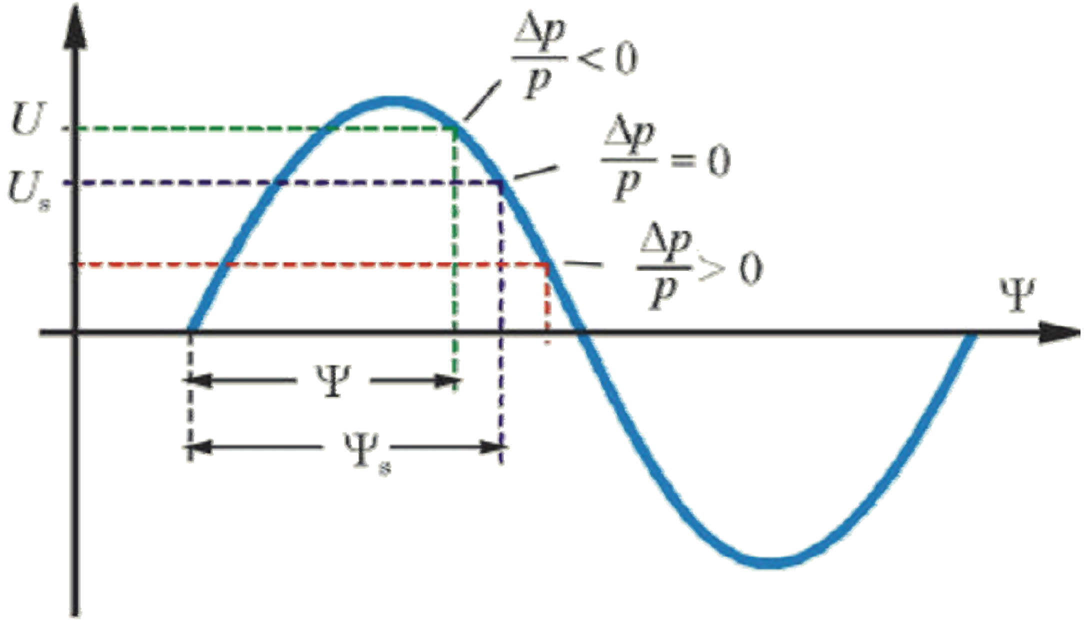
2. Beam Acceleration

2.1. Synchronization

Reference particle: $T_{\text{rev}} = h \cdot T_{\text{RF}}$ or $\omega_{\text{RF}} = h \cdot \omega_{\text{rev}}$

2.2. Longitudinal Focusing

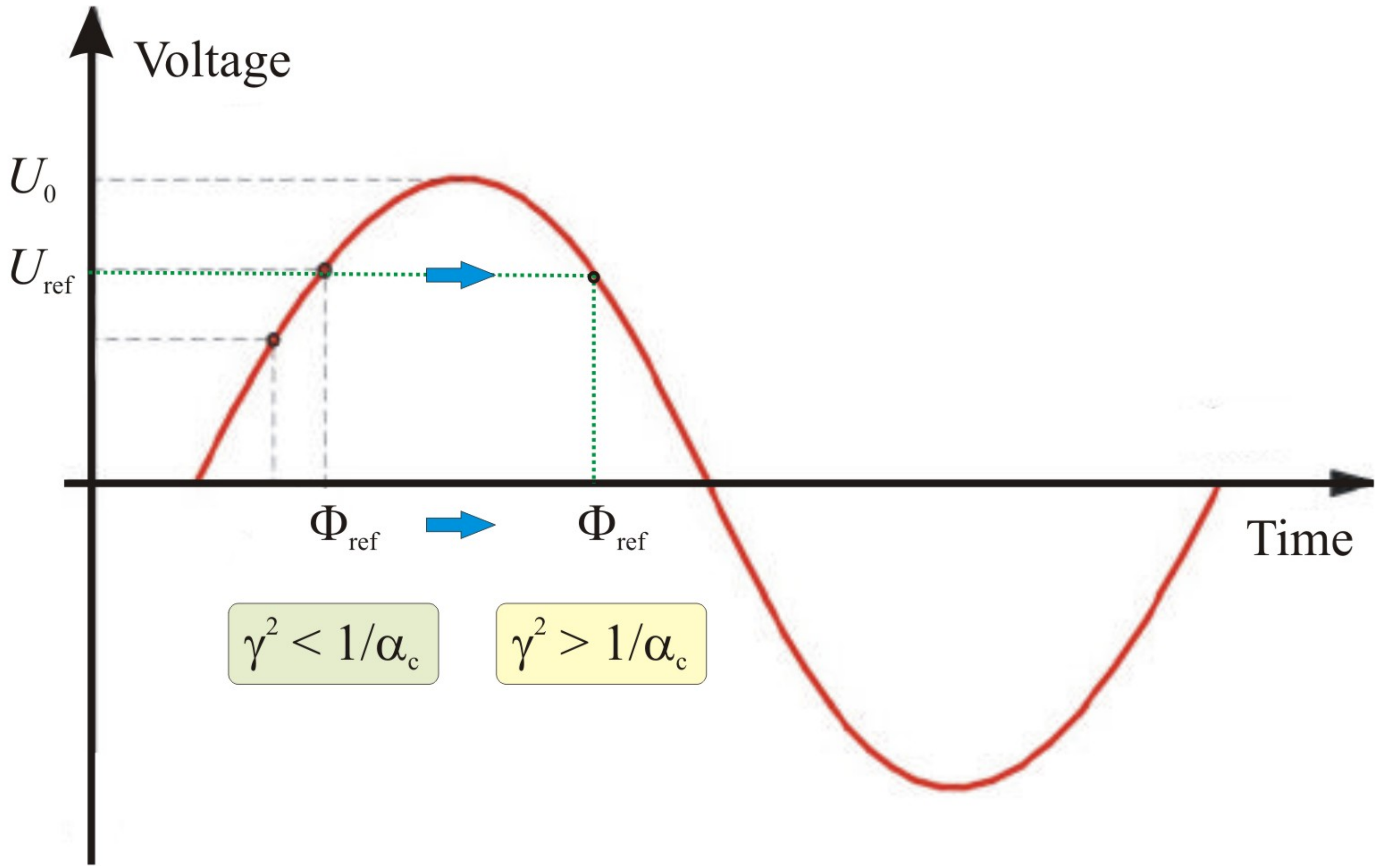




Travel time:

$$\Delta \ln T = \frac{\Delta T}{T} = \frac{\Delta L}{L} - \frac{\Delta \beta}{\beta} = \left(\alpha_c - \frac{1}{\gamma^2}\right) \cdot \frac{\Delta p}{p} = -\eta \cdot \frac{\Delta p}{p}$$

γ-Transition at proton machines:



2.3. Longitudinal Oscillations

$$\frac{d}{dt}\Delta\varphi = 2\pi h\frac{\Delta T}{T_0} = \frac{(\Delta\varphi)_{\text{rev}}}{T_0} = -\frac{2\pi h}{T_0}\cdot\eta\cdot\frac{\Delta p}{p_0} = -\frac{2\pi h\eta}{\beta^2 T_0}\cdot\frac{\Delta E}{E_0}$$

Energy gain / loss $(\Delta E)_{rev}$ per turn

$$(\Delta E)_{\text{rev}} = eU(\varphi) - W(E) = eU_0\sin\varphi - W(E) = T_0\cdot\frac{d}{dt}\Delta E$$

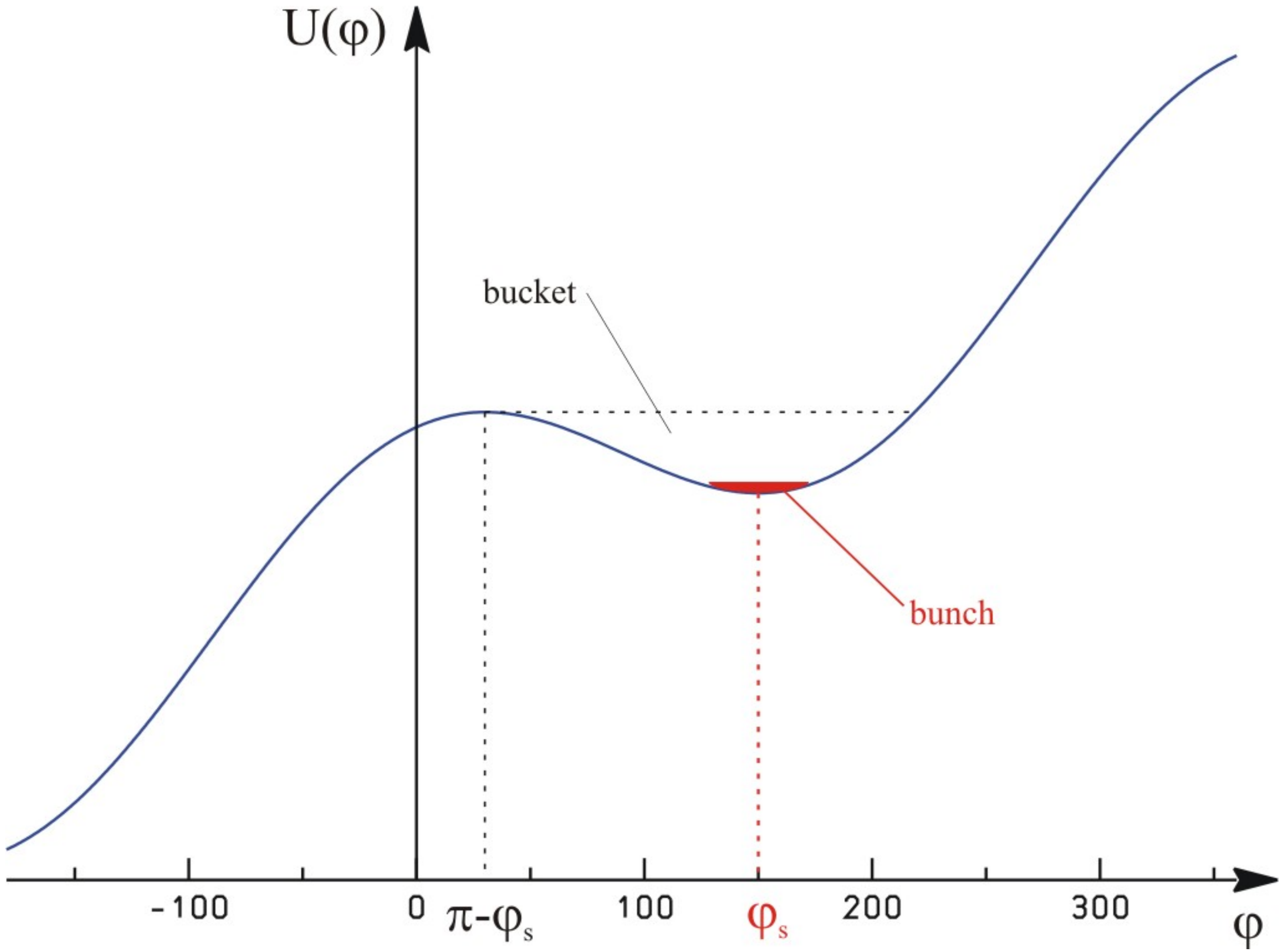
Small scale oscillations:

$$\frac{d^2\Delta\varphi}{dt^2} + 2\cdot\underbrace{\left(\frac{1}{2T_0}\cdot\frac{dW(E_0)}{dE}\right)}_{=\alpha_S}\cdot\frac{d\Delta\varphi}{dt} + \underbrace{\left(\frac{2\pi h\eta e}{\beta^2 T_0^2 E_0}\cdot U_0\cos\varphi_0\right)}_{=\Omega_S^2}\cdot\Delta\varphi = 0$$

Large scale oscillations (damping neglected):

$$\underbrace{\frac{\dot{\varphi}^2}{2}}_{\text{kinetic energy}} + \underbrace{\left\{-\frac{\Omega_S^2}{\cos\varphi_0}\left[\cos\varphi + \varphi\sin\varphi_0\right]\right\}}_{\text{potential energy}} = \text{const.}$$

Potential energy:



Stationary bunch, centre at φ_S , but particles oscillate inside!

2.4. Synchrotron Radiation

Radiated power:

$$P_\perp = \frac{e^2 c \beta^4}{6\pi\varepsilon_0 (m_0 c^2)^4} \cdot \frac{E^4}{R^2}$$

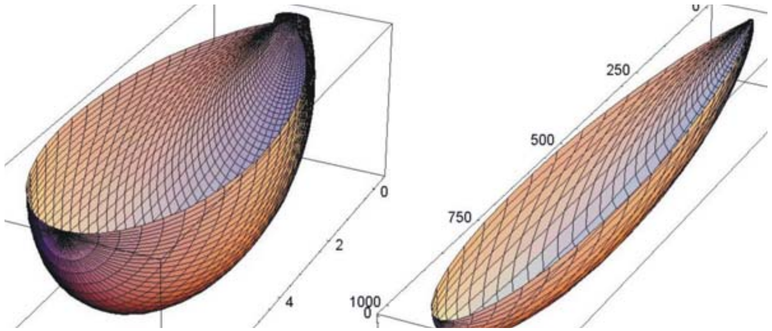
Effect on electrons / protons: $\frac{P_e}{P_p} = \left(\frac{m_p}{m_e}\right)^4 \approx 10^{13}$!!!

Circumference voltage:

$$\Delta E = \frac{e^2 \beta^3}{3\varepsilon_0 (m_0 c^2)^4} \cdot \frac{E^4}{R} \Rightarrow \Delta E[\text{keV}] \approx 88.5 \cdot \frac{(E[\text{GeV}])^4}{R[\text{m}]}$$

Radiation emitted into a cone

with angular width $\Theta \approx \frac{1}{\gamma}$



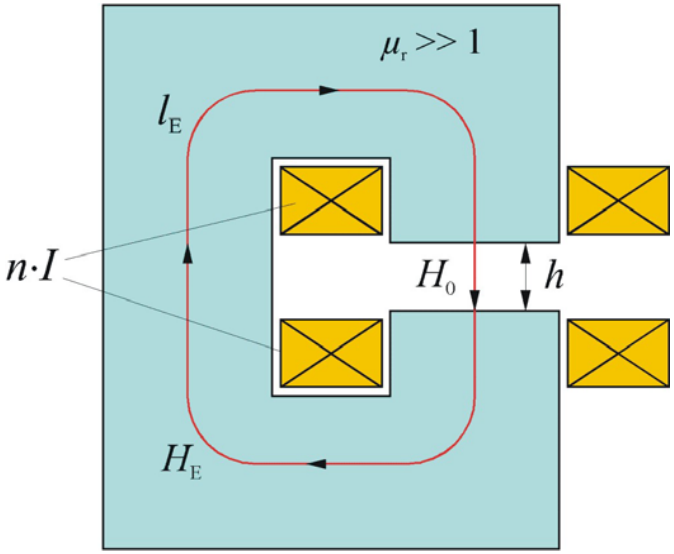
What we have learned so far for circular accelerators:

Synchronization: Path length = multiple of RF wavelength!

Electrons:	**Protons / Ions:**
Fixed revolution frequency	Revolution frequency increasing with *E*
→ **Fixed RF frequency**	→ **RF frequency has to be increased**
Power loss due to synchrotron radiation	Synchrotron radiation negligible
→ **"high" acceleration voltage**	→ **"low" acceleration voltage**
Ultrarelativistic beam: $\gamma >> 100$	Relativistic beam: $\gamma \approx 1 - 2$
→ **Reference phase $\phi_0 > 90°$**	→ **Reference phase $\phi_0 < 90°$**
no γ- Transition	**and possibly γ- Transition**
bunched beam	**bunched beam, but coasting beam possible**

3. Beam Guidance and Focusing

3.1. Guidance – Dipole Magnets



Magnetic field inside the gap:

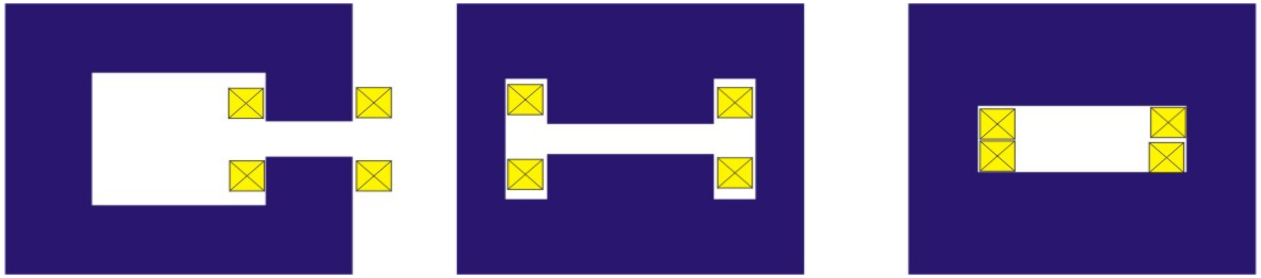
$$\int_{gap} \vec{B} \cdot d\vec{s} = \mu_0 \cdot n \cdot I$$

Dipole strength from curvature:

$$\kappa = \frac{1}{R} = \frac{e}{p} B_0 = \frac{e\,\mu_0}{p} \frac{n \cdot I}{h}$$

normalized to momentum and charge!

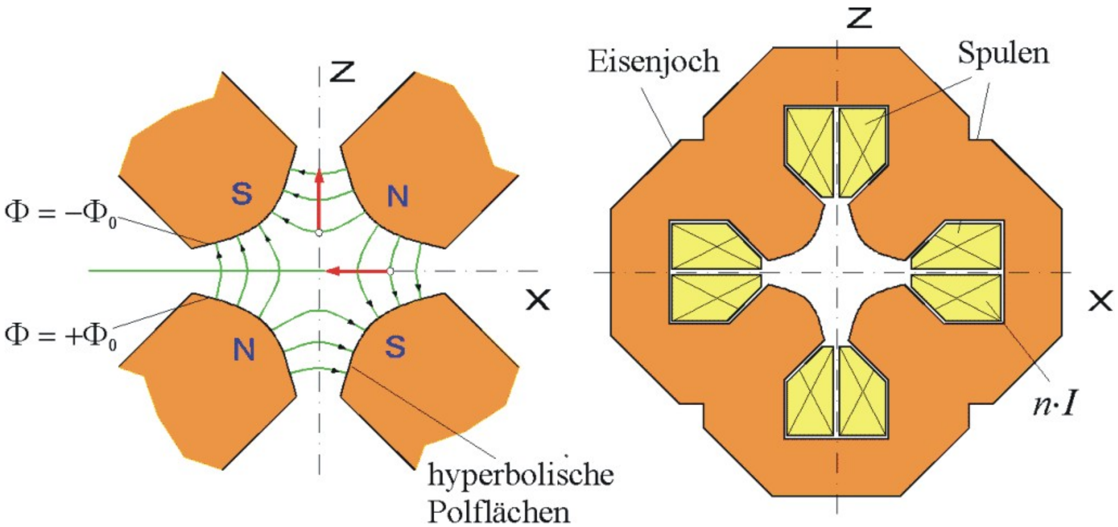
Different types:



C-Magnet H-Magnet Window-Frame Magnet

3.2. Focusing – Quadrupole Magnets

Restoring force, linear increasing with distance: $g = \partial B_z / \partial x$



Profile of the poles:

$$z(x) = \pm \frac{\Phi_0}{g \cdot x} = \pm \frac{a^2}{2x}$$

Quadrupole strength:

$$k = \frac{e}{p} g = \frac{2e\mu_0}{p} \frac{n \cdot I}{a^2}$$

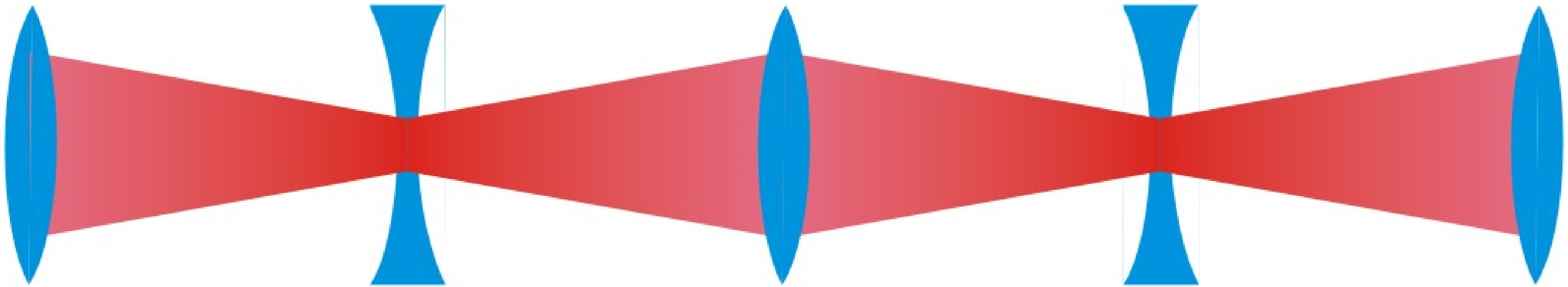
"Restoring" force: $\vec{F} = e \cdot \left(\vec{v} \times \vec{B} \right) = e\,\mathrm{v}g \cdot \left(x\hat{e}_x - z\hat{e}_z \right)$

A quadrupole magnet is therefore focusing only in one plane and defocusing in the other; depending on the sign of *g*.

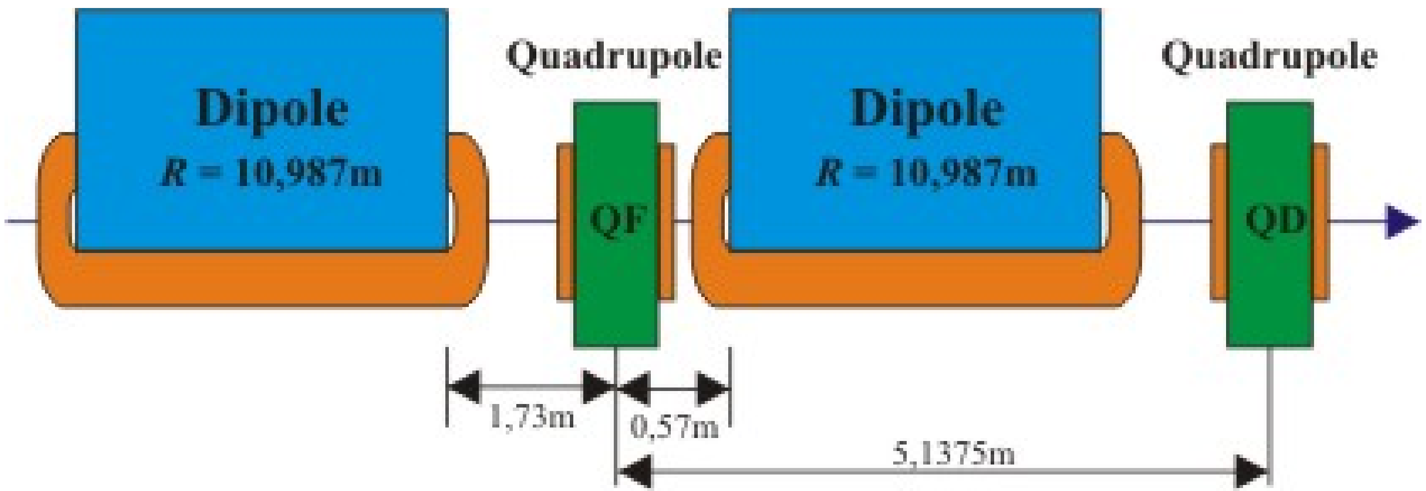
Thin lens approximation: **Focal length** from $1/f = k \cdot L$

Strong Focusing:

Light optics:



Magnet optics:



Strong focusing

or

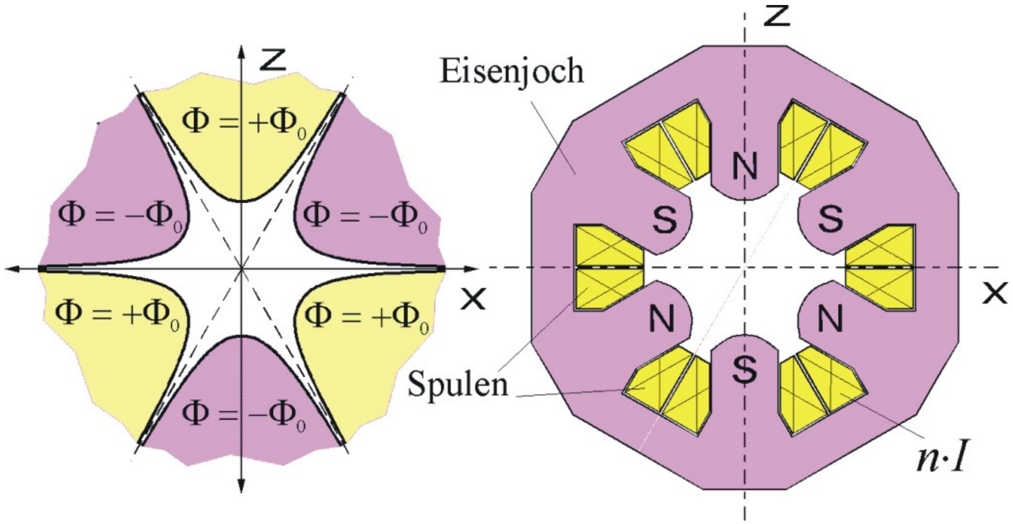
AG focusing

Simplest way:

FODO lattice

3.3. Correction of Chromatic Errors – Sextupole Magnets

Correcting force, increasing quadratically with distance: $g' = \partial^2 B_z / \partial x^2$



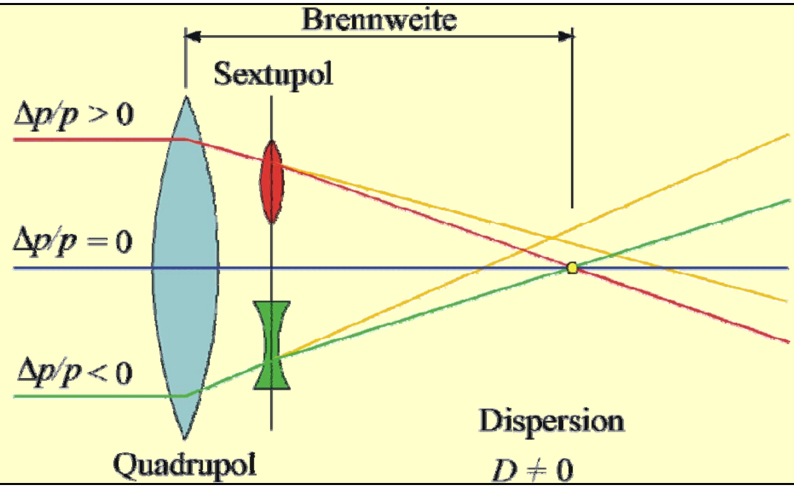
Profile of the poles:

$$x(z) = \pm\sqrt{\frac{z^2}{3} \pm \frac{2\Phi_0}{g' z}} = \pm\sqrt{\frac{z^2}{3} \pm \frac{a^3}{3z}}$$

Sextupole strength:

$$m = \frac{e}{p} g' = \frac{6e\mu_0}{p} \frac{nI}{a^3}$$

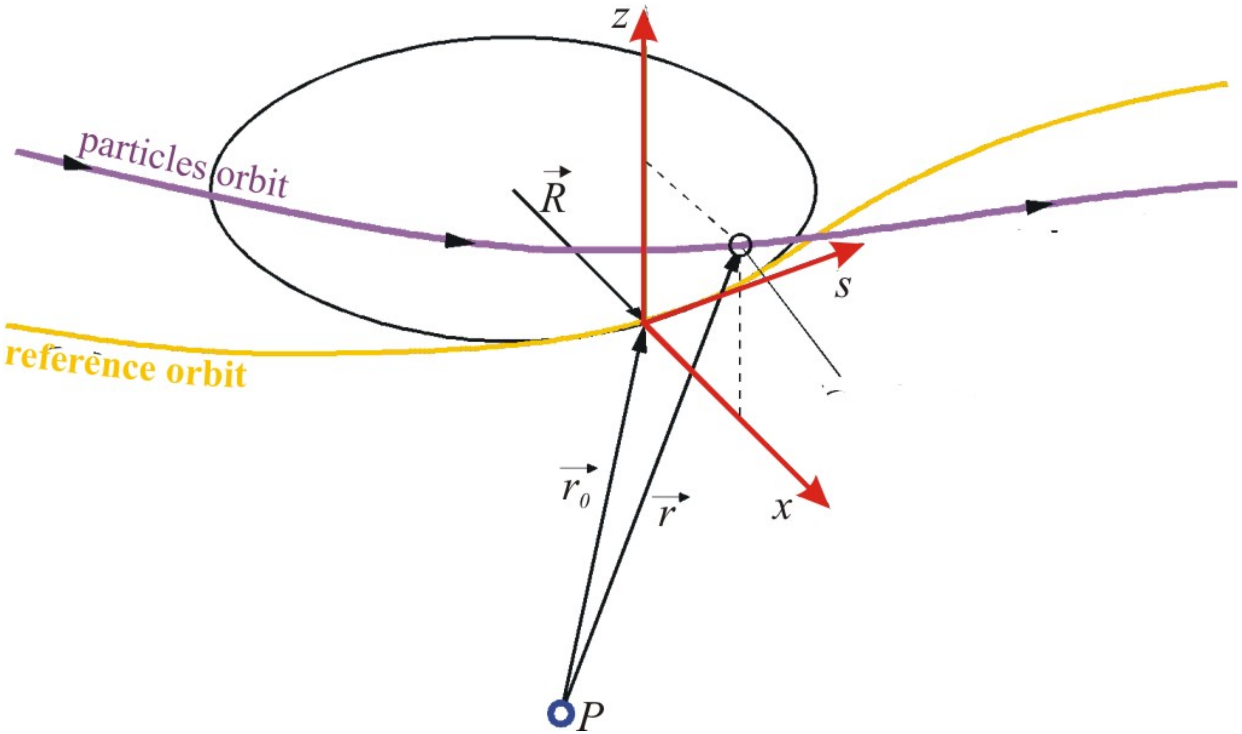
Correction of chromatic errors requires a momentum dependent beam position (dispersion):



4. Linear Beam Optics

4.1. Equations of Motion

Moving orthogonal, right-handed coordinate system (x,s,z), Frenet-Serret system:



$$\vec{r} = (R + x) \cdot \hat{e}_x + z \cdot \hat{e}_z$$

Neglecting all nonlinear terms in $x, z, and\ \Delta p/p_0$, we obtain the Hill equations:

$$x''(s) + \left(\frac{1}{R^2(s)} - k(s)\right) \cdot x(s) = \frac{1}{R(s)} \frac{\Delta p}{p}$$

$$z''(s) + k(s) \cdot z(s) = 0$$

- Particles with nominal momentum ($\Delta p/p = 0$):

  quasi harmonic oscillation (**betatron oscillation**):

$$x(s) = \sqrt{\varepsilon} \cdot \sqrt{\beta(s)} \cdot \cos\left(\phi(s) + \varphi_0\right)$$

$$\phi(s) = \int_0^s \frac{d\tilde{s}}{\beta(\tilde{s})}$$

***β*(*s*)** : position dependent oscillation amplitude function, amplitude $= \sqrt{\varepsilon \cdot \beta(s)}$

***ϕ*(*s*)** : position dependent oscillation phase advance

- Particles with **momentum deviation** ($\Delta p / p \neq 0$):

  betatron oscillations around **dispersion orbit *D(s)***, defined by a particular solution of the Hill equation when setting $\Delta p / p = 1$:

$$D''(s) + \left(\frac{1}{R^2(s)} - k(s) \right) \cdot D(s) = \frac{1}{R(s)}$$

General solution:

$$x(s) = \sqrt{\varepsilon} \cdot \sqrt{\beta(s)} \cdot \cos\left(\phi(s) + \varphi_0\right) + D(s) \cdot \frac{\Delta p}{p}$$

Betatron and dispersion functions cannot be derived analytically for real situations!!!

In case of no correlation between β and D, the 1σ (2σ) beam size is given by

$$\sigma_{rms} = \sqrt{\varepsilon \cdot \beta(s) + \left[D(s) \cdot \frac{\Delta p}{p} \right]^2}$$

4.2. Matrix Formalism

We will characterize a particles state by a vector built from its relative coordinates:

$$\vec{X} = \begin{pmatrix} x \\ x' \\ z \\ z' \\ s \\ \delta \end{pmatrix} = \begin{pmatrix} \text{radial displacement} \\ \text{radial angular displacement} \\ \text{axial displacement} \\ \text{axial angular displacement} \\ \text{longitudinal displacement} \\ \text{relative momentum deviation} \end{pmatrix}$$

Matrix formalism to describe particles trajectories: $\vec{X} = \mathbf{M} \cdot \vec{X}_0$. Upright magnets:

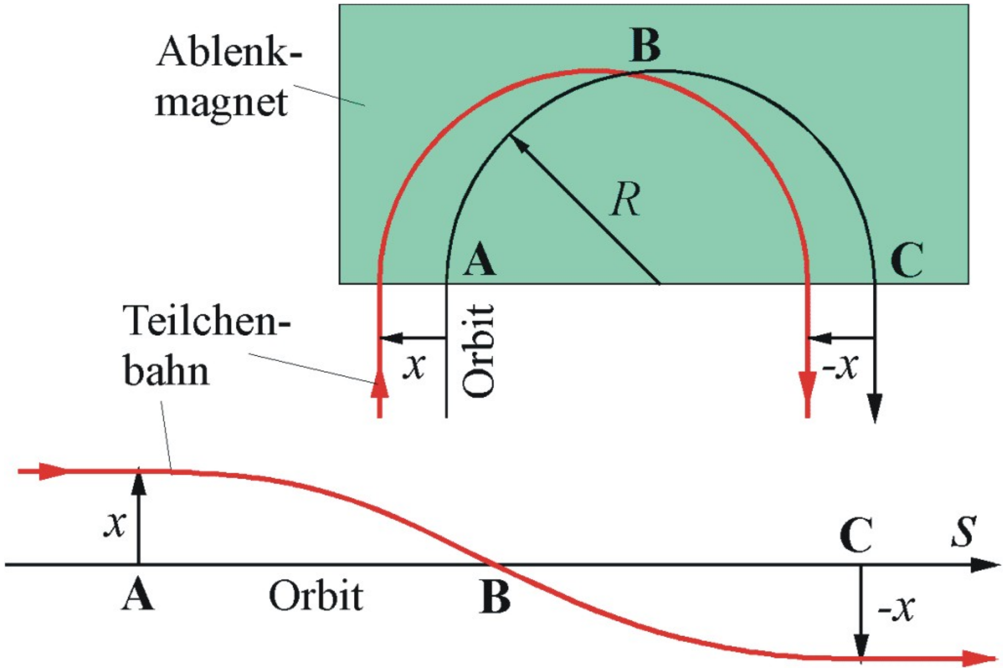
$$\mathbf{M} = \begin{pmatrix} r_{11} & r_{12} & 0 & 0 & 0 & r_{16} \\ r_{21} & r_{22} & 0 & 0 & 0 & r_{26} \\ 0 & 0 & r_{33} & r_{34} & 0 & 0 \\ 0 & 0 & r_{43} & r_{44} & 0 & 0 \\ r_{51} & r_{52} & 0 & 0 & 1 & r_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \langle x|x\rangle & \langle x|x'\rangle & 0 & 0 & 0 & \langle x|\delta\rangle \\ \langle x'|x\rangle & \langle x'|x'\rangle & 0 & 0 & 0 & \langle x'|\delta\rangle \\ 0 & 0 & \langle z|z\rangle & \langle z|z'\rangle & 0 & 0 \\ 0 & 0 & \langle z'|z\rangle & \langle z'|z'\rangle & 0 & 0 \\ \langle s|x\rangle & \langle s|x'\rangle & 0 & 0 & \langle s|s\rangle & \langle s|\delta\rangle \\ 0 & 0 & 0 & 0 & 0 & \langle \delta|\delta\rangle \end{pmatrix}$$

$$\mathbf{M}_{drift} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}_{dipole} = \begin{pmatrix} \cos\varphi & R\sin\varphi & 0 & 0 & 0 & R(1-\cos\varphi) \\ -1/R\cdot\sin\varphi & \cos\varphi & 0 & 0 & 0 & \sin\varphi \\ 0 & 0 & 1 & R\varphi & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\varphi & -R(1-\cos\varphi) & 0 & 0 & 1 & R\varphi/\gamma^2 - R(\varphi-\sin\varphi) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

A sector magnet is therefore focusing in the horizontal plane.

This effect is purely geometric in nature:



Edge-focusing in rectangular dipole magnets:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ +\dfrac{\tan\psi}{R} & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \qquad \begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{\tan\psi}{R} & 1 \end{pmatrix} \cdot \begin{pmatrix} z_0 \\ z_0' \end{pmatrix}$$

$$\mathbf{M}_{rect} = \mathbf{M}_{\psi} \cdot \mathbf{M}_{dipole} \cdot \mathbf{M}_{\psi}$$

$$\mathbf{M}_{rect} = \begin{pmatrix} \begin{matrix} 1 & R\sin\varphi \\ 0 & 1 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & R(1-\cos\varphi) \\ 0 & \sin\varphi \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 1-\frac{R\varphi}{f} & R\varphi \\ \frac{R\varphi}{f^2}-\frac{2}{f} & 1-\frac{R\varphi}{f} \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} -\sin\varphi & -R(1-\cos\varphi) \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 1 & R\varphi/\gamma^2 - R(\varphi-\sin\varphi) \\ 0 & 1 \end{matrix} \end{pmatrix}$$

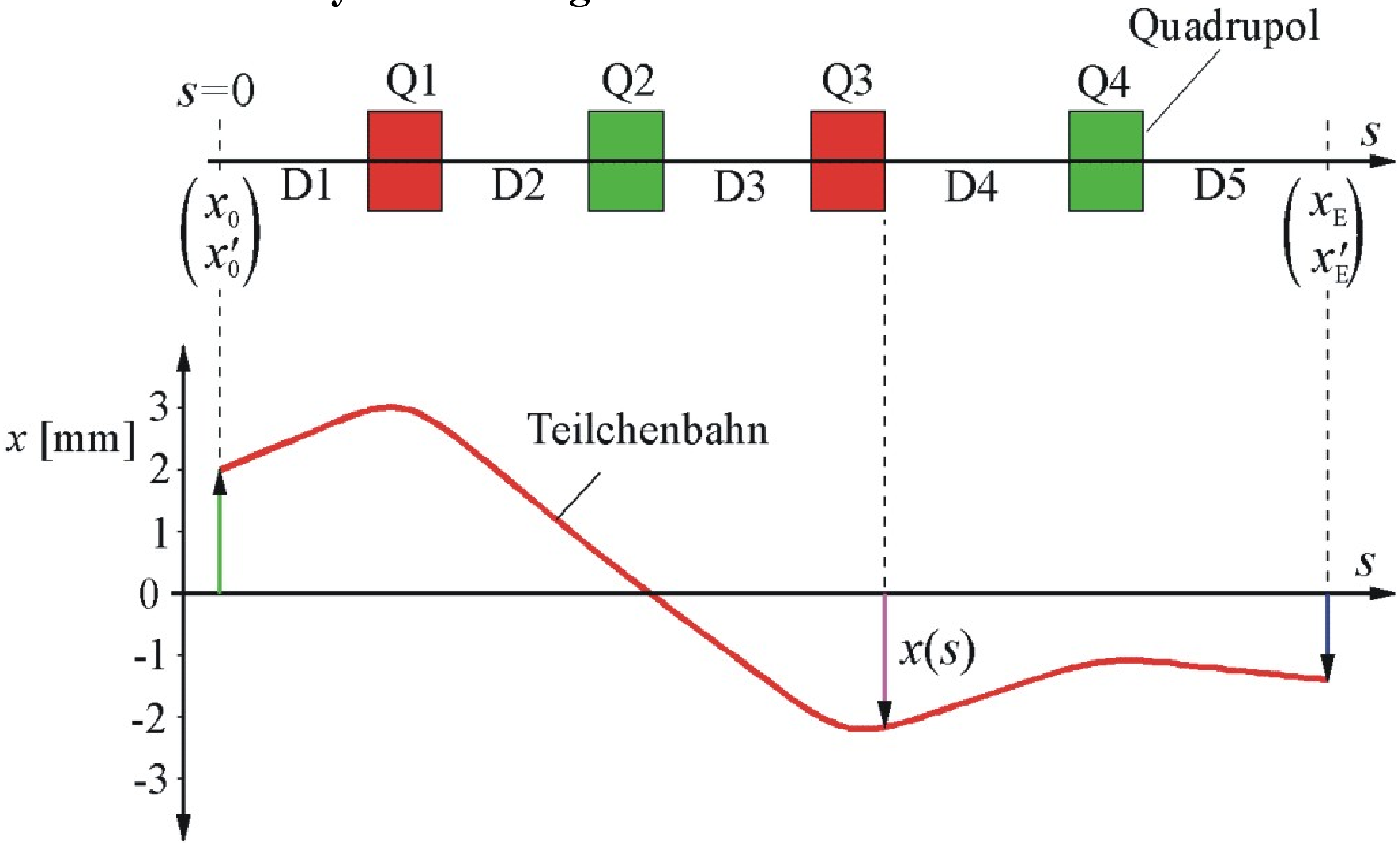
$$\frac{1}{f} \approx \frac{1}{R}\tan\psi$$

A rectangular dipole magnet is therefore focusing in the vertical plane.
It acts like a drift space in the horizontal plane!

$$\mathbf{M}_{FQ} = \begin{pmatrix} \begin{matrix} \cos\Omega & \frac{1}{\sqrt{k}}\sin\Omega \\ -\sqrt{k}\sin\Omega & \cos\Omega \end{matrix} & \cdots & 0 \\ \vdots & \begin{matrix} \cosh\Omega & \frac{1}{\sqrt{k}}\sinh\Omega \\ \sqrt{k}\sinh\Omega & \cosh\Omega \end{matrix} & \vdots \\ 0 & \cdots & \begin{matrix} 1 & L/\gamma^2 \\ 0 & 1 \end{matrix} \end{pmatrix} \quad (k<0)$$

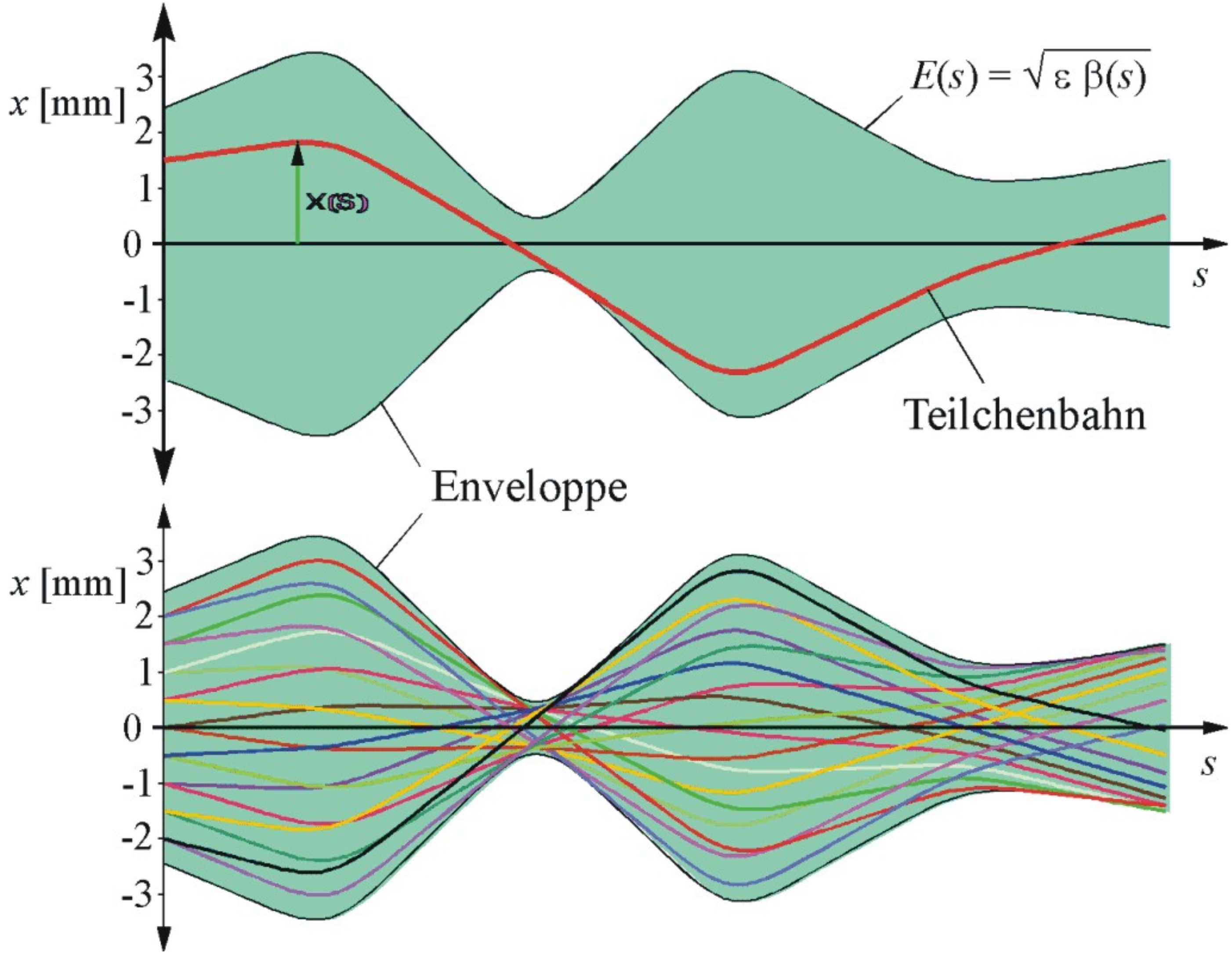
$$\mathbf{M}_{DQ} = \begin{pmatrix} \begin{matrix} \cosh\Omega & \frac{1}{\sqrt{k}}\sinh\Omega \\ \sqrt{k}\sinh\Omega & \cosh\Omega \end{matrix} & \cdots & 0 \\ \vdots & \begin{matrix} \cos\Omega & \frac{1}{\sqrt{k}}\sin\Omega \\ -\sqrt{k}\sin\Omega & \cos\Omega \end{matrix} & \vdots \\ 0 & \cdots & \begin{matrix} 1 & L/\gamma^2 \\ 0 & 1 \end{matrix} \end{pmatrix} \quad (k>0)$$

Particle Orbits in a System of Magnets:



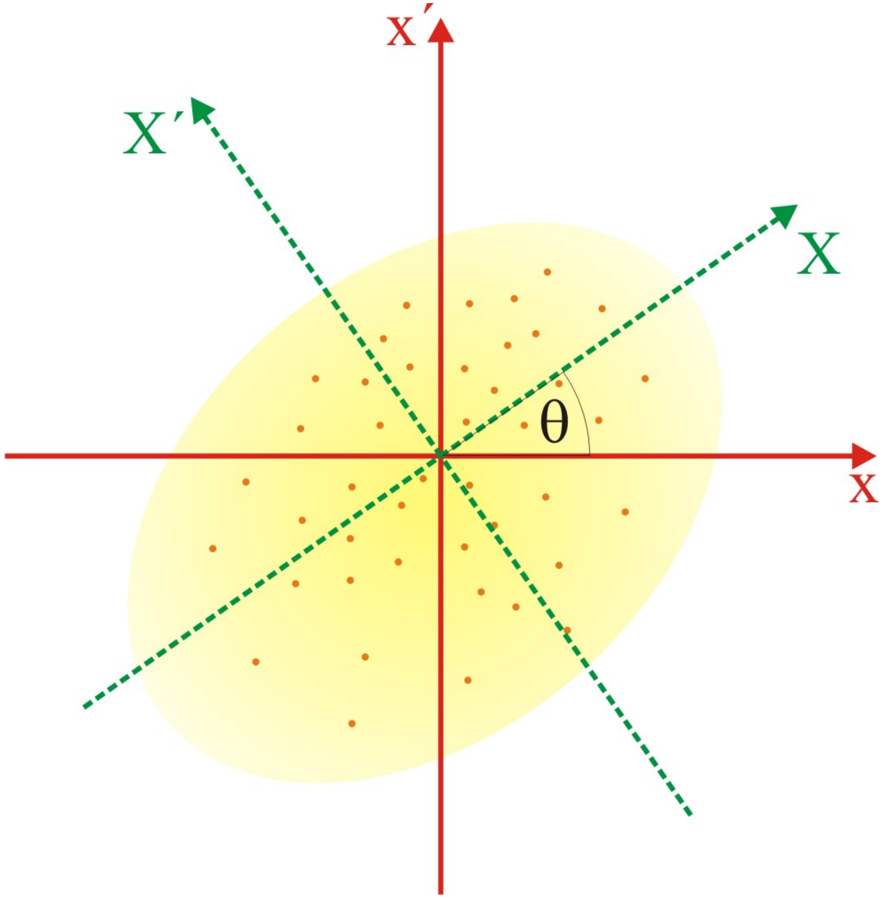
$$\vec{X}_E = \mathbf{M}_{D5} \cdot \mathbf{M}_{Q4} \cdot \mathbf{M}_{D4} \cdot \mathbf{M}_{Q3} \cdot \mathbf{M}_{D3} \cdot \mathbf{M}_{Q2} \cdot \mathbf{M}_{D2} \cdot \mathbf{M}_{Q1} \cdot \mathbf{M}_{D1} \cdot \vec{X}_0$$

The envelope cannot be derived from a single particle trajectory!



4.3. Beam Emittance and Phase Space

Beam as statistical set of points in phase space. Concentrate on 2-dim space (x, x'):



Choose origin that:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = 0 = \bar{x'} = \frac{1}{N}\sum_{i=1}^{N} x_i'$$

Choose rotated axes

$$X_i = x_i \cdot \cos\theta + x_i' \cdot \sin\theta$$
$$X_i' = -x_i \cdot \sin\theta + x_i' \cdot \cos\theta$$

that

$$\frac{\partial \sigma_X^2}{\partial\theta} = 0, \quad \frac{\partial \sigma_{X'}^2}{\partial\theta} = 0$$

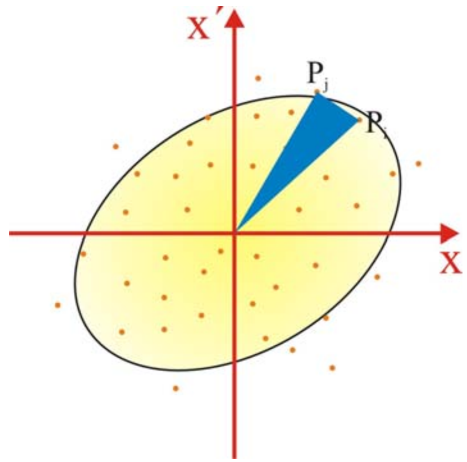
A distribution of points can be translated and rotated without changing its spread!

We will define the spread of the distribution, which is called the **emittance ε**, by

$$\varepsilon = \sigma_x \cdot \sigma_x{'} = \sqrt{\overline{x^2} \cdot \overline{x'^2} - \overline{x\,x'}^2}$$

It is important to note that this definition usually is used for electron beams, the emittance of proton beams is typically defined as 4ε !

The emittance can be considered as a statistical mean area where A_{ij} is the area of the triangle $0P_iP_j$ and ε is a measure of the spread of the points around their barycentre.



The area of the envelope-ellipse is

$$A = \pi\, a\, b = \pi\, \sigma_x\, \sigma_{x'} = \pi\, \varepsilon$$

and its equation is

$$\frac{X^2}{\sigma_x{}^2} + \frac{X'^2}{\sigma_{x'}{}^2} = \frac{X^2}{a^2} + \frac{X'^2}{b^2} = 1$$

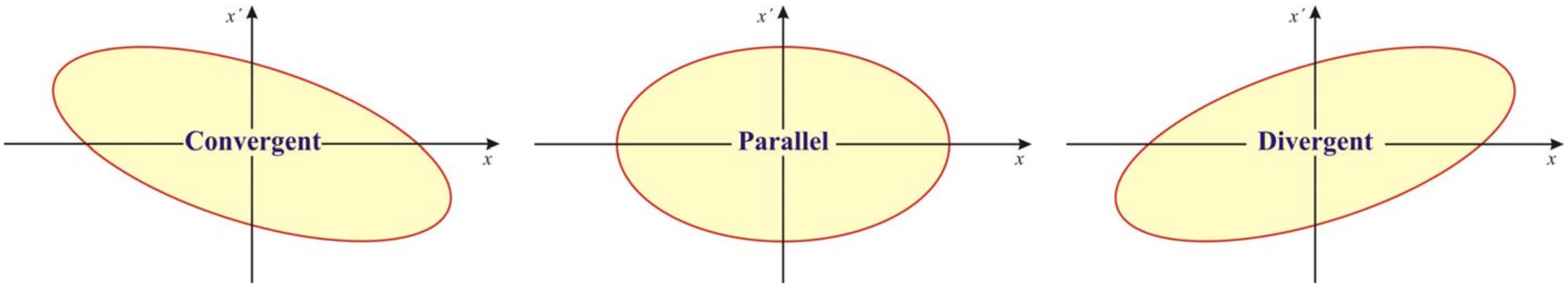
Inverse rotation of $-\theta$ gives: $\varepsilon^2 = x^2 \cdot \sigma_{x'}^2 - 2\,x\,x' \cdot \overline{x\,x'} + x'^2 \cdot \sigma_x^2$

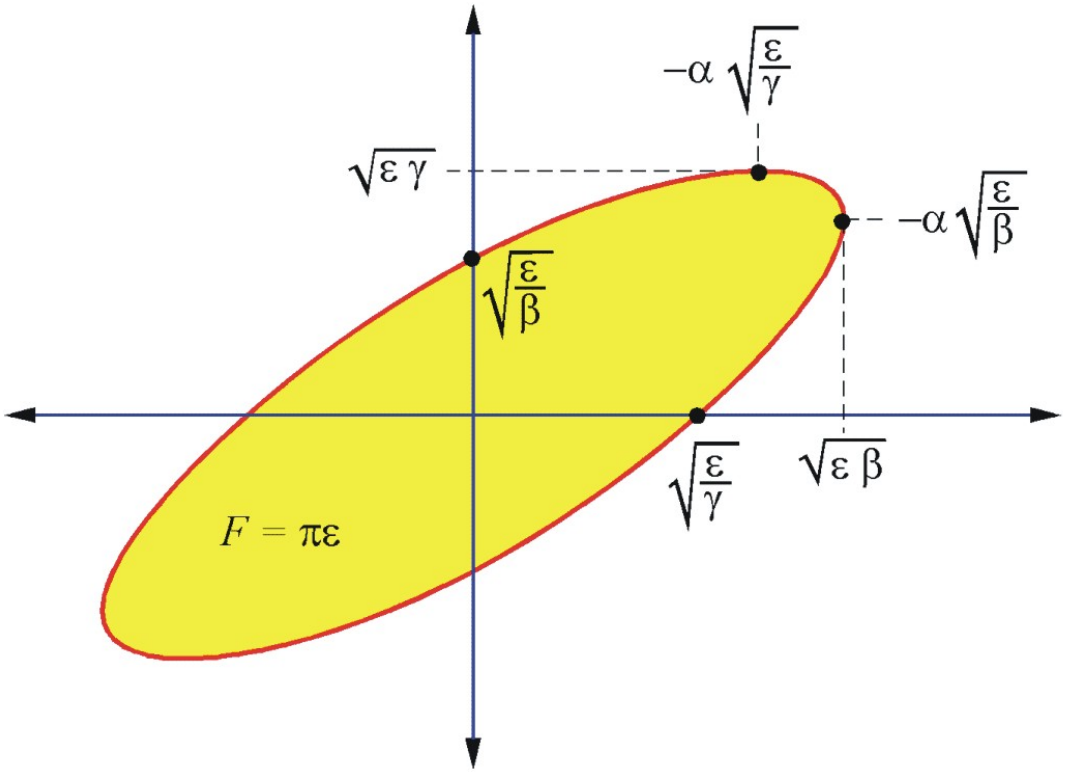
We may define the so called **Twiss-parameters α, β, and γ** such that

- r.m.s. beam envelope: $\sigma_x = \sqrt{\overline{x^2}} = \sqrt{\beta\,\varepsilon}\,,$
- r.m.s. beam divergence: $\sigma_{x'} = \sqrt{\overline{x'^2}} = \sqrt{\gamma\,\varepsilon}\,,$
- correlation r between x and x': $r\,\sigma_x\,\sigma_{x'} = \overline{x\,x'} = -\alpha\,\varepsilon\,,$

$$\gamma\,x^2 + 2\alpha\,x\,x' + \beta\,x'^2 = \varepsilon$$

Beam properties:





We can transform the solution of the Hill equation to an "ellipse equation", using

$$x = \sqrt{\varepsilon\beta}\cdot\cos\Psi, \qquad \Psi = \int_0^s \frac{d\tilde{s}}{\beta(\tilde{s})} + \phi_0, \qquad x' = -\sqrt{\frac{\varepsilon}{\beta}}\cdot(\alpha\cos\Psi + \sin\Psi)$$

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon, \qquad \text{where} \quad \alpha = -\frac{\beta'}{2} \quad \text{and} \quad \gamma = \frac{1+\alpha^2}{\beta}$$

Transformation of Twiss parameter with matrix formalism using the Beta-matrix **B**:

$$\mathbf{B} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}, \qquad |\mathbf{B}| = \beta\gamma - \alpha^2 = 1, \qquad \mathbf{B}^{-1} = \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}$$

the equation of the envelope-ellipse can be transformed to:

$$\varepsilon = {}^T\vec{X}_0 \cdot \mathbf{B}_0^{-1} \cdot \vec{X}_0 = {}^T\vec{X}_1 \cdot \mathbf{B}_1^{-1} \cdot \vec{X}_1$$

and displacement-vector $\vec{X}$ transforms according to

$$\vec{X}_1 = \mathbf{M} \cdot \vec{X}_0, \qquad {}^T\vec{X}_1 = {}^T\left(\mathbf{M} \cdot \vec{X}_0\right) = {}^T\vec{X}_0 \cdot {}^T\mathbf{M}$$

By inserting $\mathbf{1} = \mathbf{M}^{-1} \cdot \mathbf{M}$, we obtain:

$$\varepsilon = {}^T\vec{X}_1 \cdot \left(\mathbf{M} \cdot \mathbf{B}_0 \cdot {}^T\mathbf{M}\right)^{-1} \cdot \vec{X}_1$$

and we can read off the transformation of the Beta-matrix:

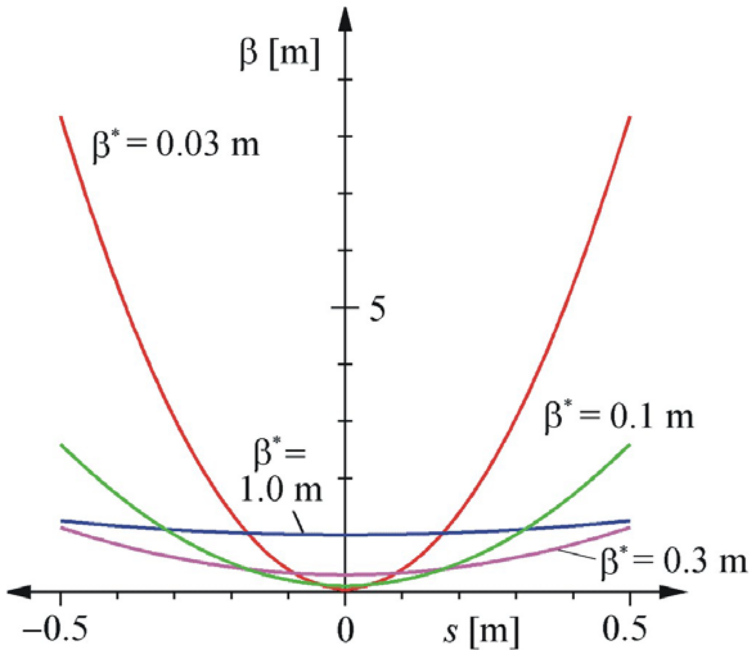
$$\mathbf{B}_1 = \mathbf{M} \cdot \mathbf{B}_0 \cdot {}^T\mathbf{M}$$

Example: Beta-function around a symmetry-point ($\alpha = 0$) of a transfer-line:

$$\mathbf{B}_1(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_{\text{sym}} & 0 \\ 0 & 1/\beta_{\text{sym}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}$$

This gives the relations for the Twiss-parameters around a symmetry-point:

$$\beta = \beta_{\text{sym}} + \frac{s^2}{\beta_{\text{sym}}}, \qquad \alpha = -\frac{s}{\beta_{\text{sym}}}$$



The corresponding beam size scales with

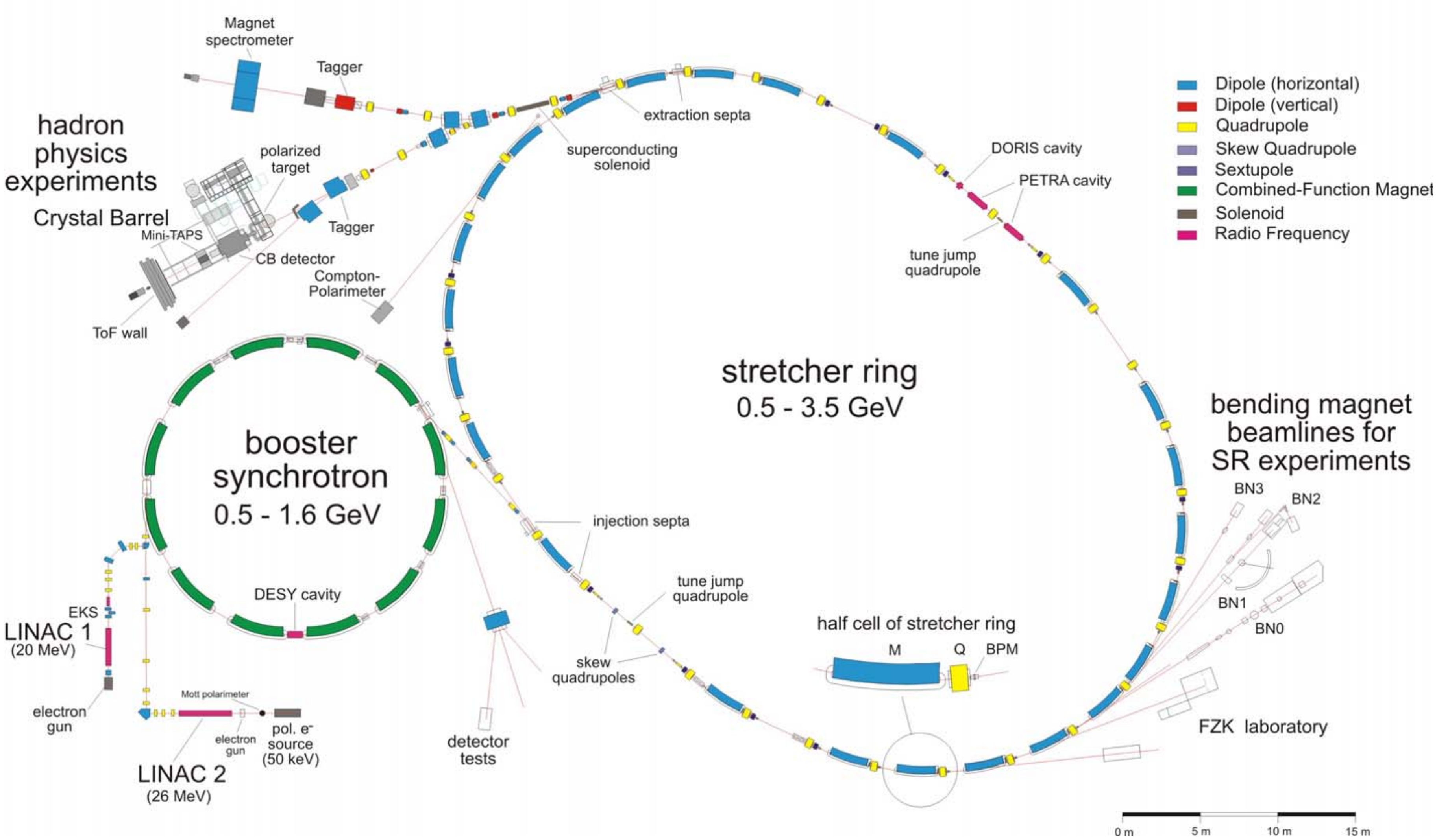
$$\sigma_x = \sqrt{\varepsilon \cdot \beta(s)} = \sigma_{sym} \cdot \sqrt{1 + \left(\frac{s}{\sigma_{sym}/\varepsilon} \right)^2}$$

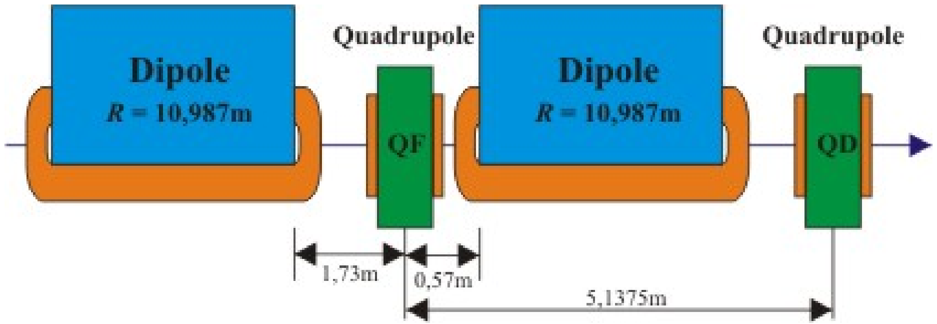
Comparing this with the formulas for light optics, $w(s) = w_0 \cdot \sqrt{1 + \left(\frac{s}{z_R} \right)^2}, \quad z_R = \frac{\pi \cdot w_0^2}{\lambda}$

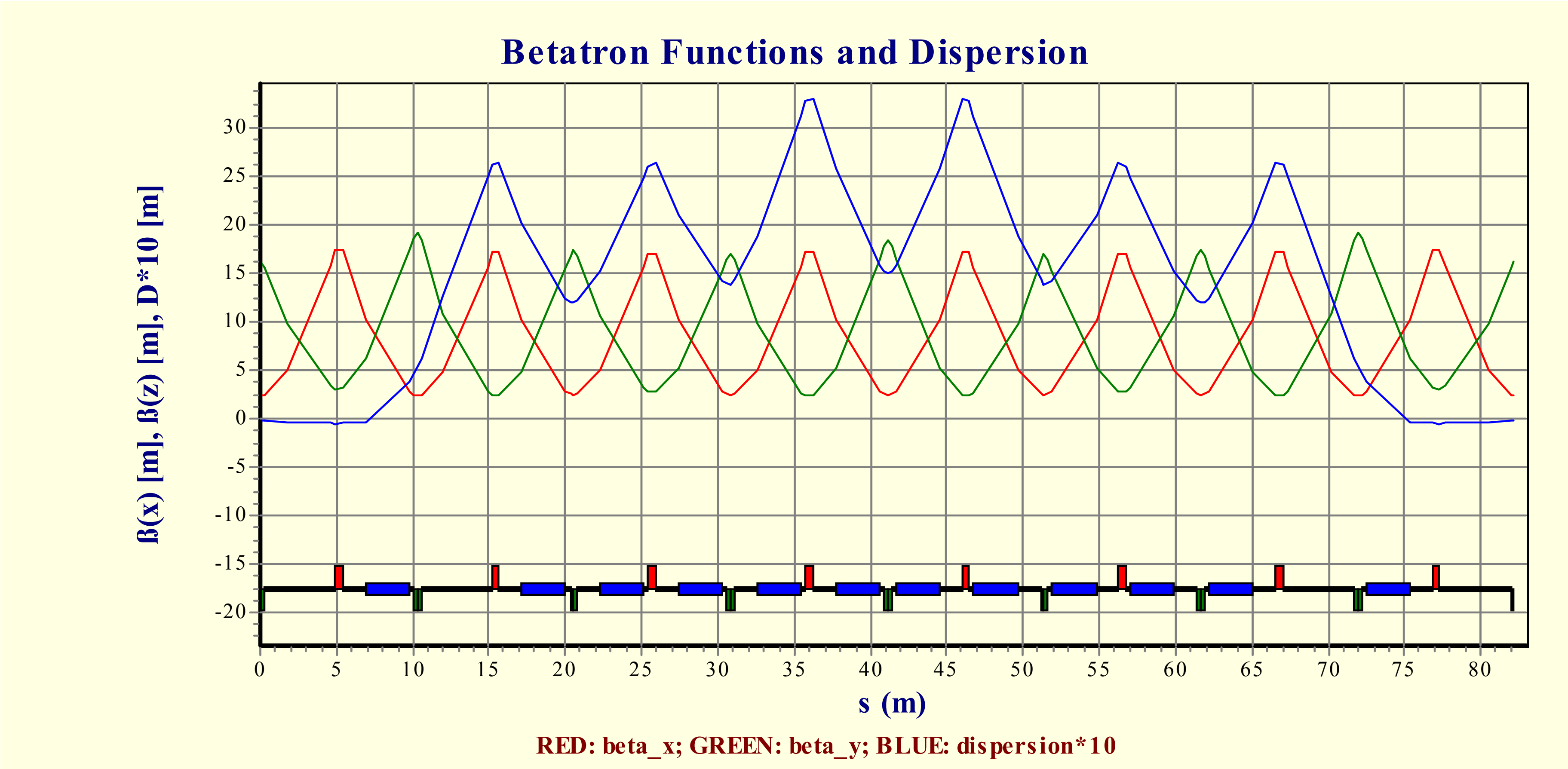
we obtain the import relation $\pi \cdot \varepsilon \triangleq \lambda$

5. Circular Accelerators

Example: **El**ectron **S**tretcher **A**ccelerator **ELSA**




Dipole
R = 10,987m
Quadrupole
QF
Dipole
R = 10,987m
Quadrupole
QD
1,73m
0,57m
5,1375m

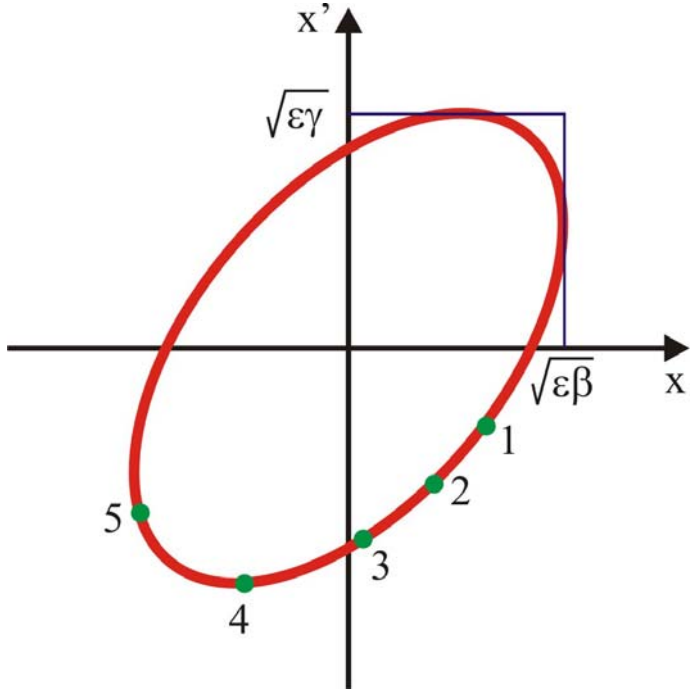

Betatron Functions and Dispersion
β(x) [m], β(z) [m], D*10 [m]
s (m)
RED: beta_x; GREEN: beta_y; BLUE: dispersion*10

5.1. Betatron Tune

The betatron tune Q is defined as the number of oscillations per revolution:

$$Q_{x,z} = \frac{1}{2\pi} \cdot \oint \frac{ds}{\beta_{x,z}(s)}$$

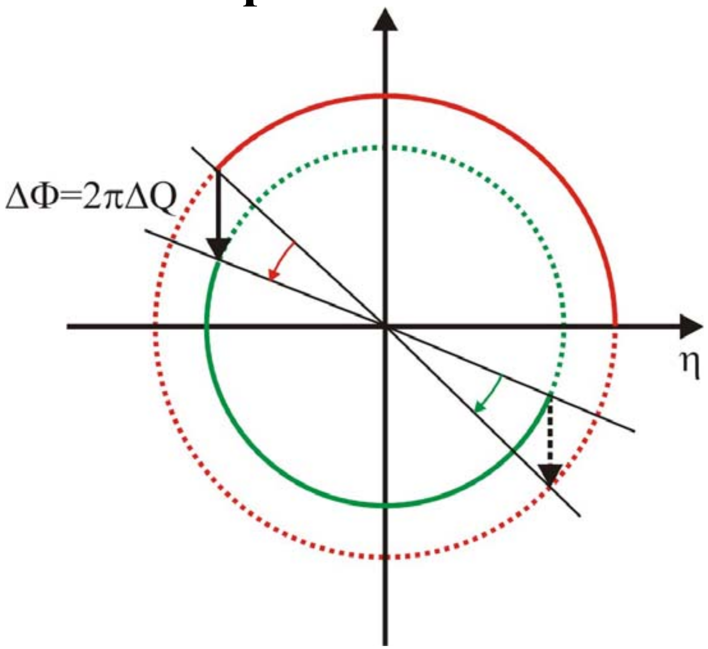
If one regards the phase space at an arbitrarily chosen point, a single particle moves on its phase space ellipse ,where the points represents the parameters after 1,2, … 5 revolutions.



5.2. Optical Resonances

Descriptive discussion in **normalized phase-space** (ellipse → circle):

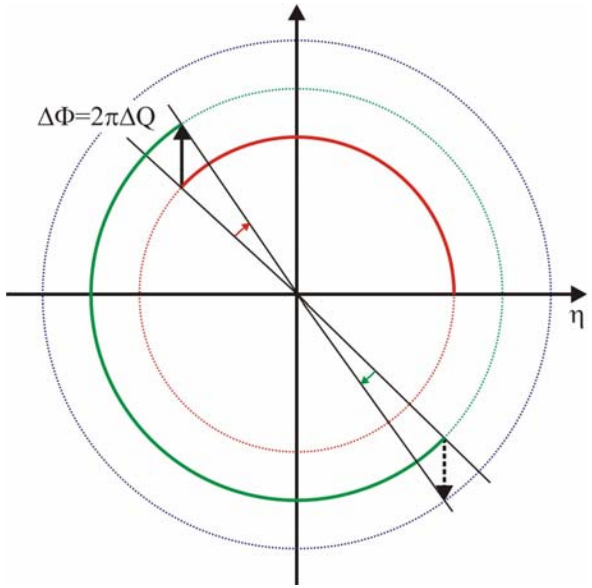
Dipole Errors:



No average tune shift $\Delta Q = 0$

Tune modulation amplitude dQ

Gradient Errors:



Average tune shift $\Delta Q = \frac{1}{4\pi} \beta \delta(kl)$

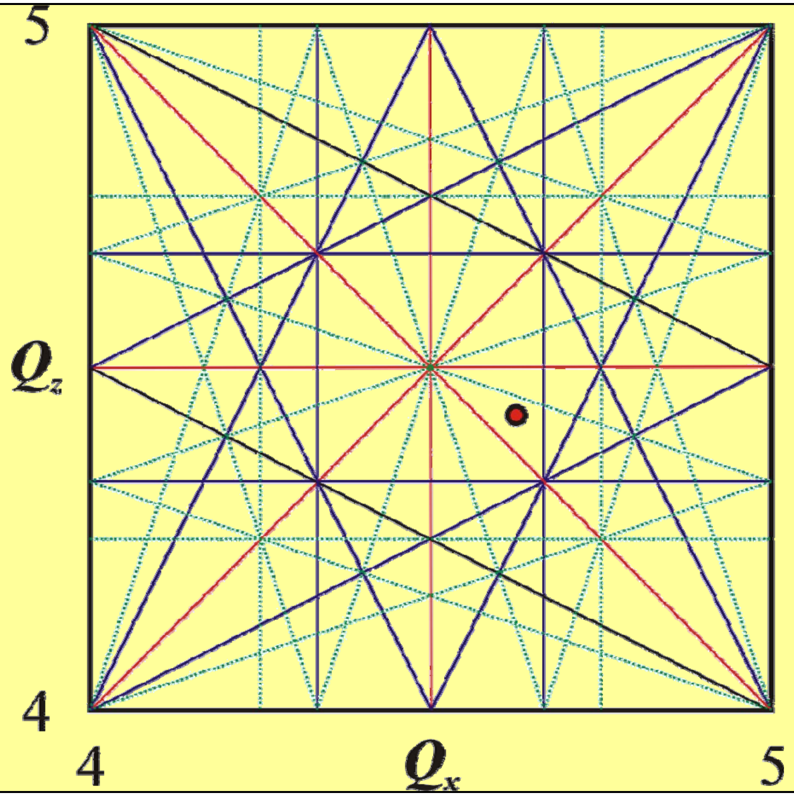
Tune modulation amplitude $dQ = \Delta Q$

Any particle whose unperturbed Q lies in the stop band width *dQ* will lock into resonance and is lost.

We may generalize and give a list of resonances and their driving multipoles:

resonance type	**driving multipole**
integer resonance: $Q = n$	dipole errors
half-integer resonance $2 \cdot Q = n$	quadrupole errors
third-integer resonance $3 \cdot Q = n$	sextupole errors

...



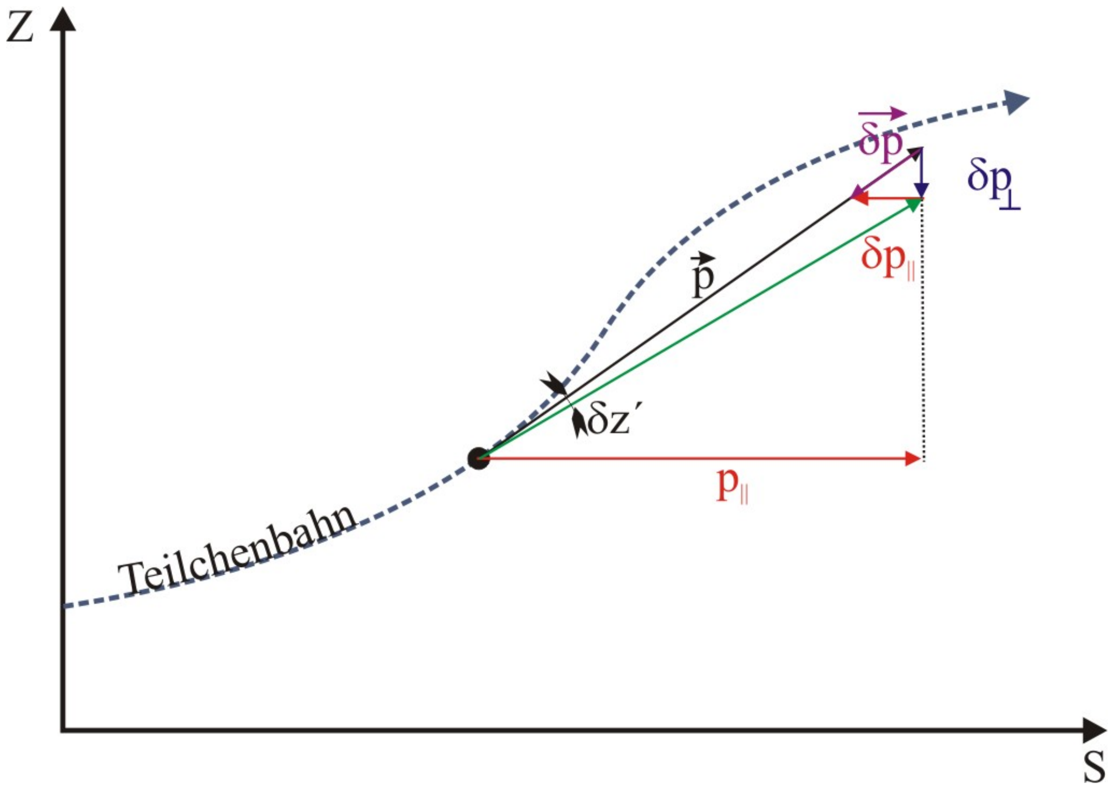
Due to betatron coupling, perturbations may depend on the betatron amplitude in both planes. These coupling terms lead to the generalized resonance condition

$$j \cdot Q_x + k \cdot Q_z = N$$

where $j+k$ indicates the order of the resonance.

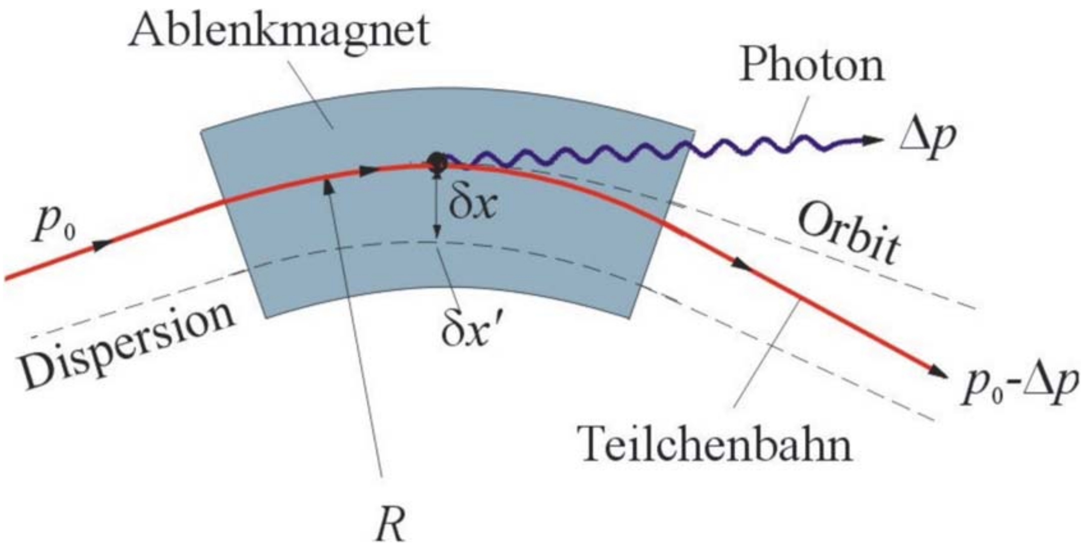
5.3. Radiation Damping – Natural Emittance

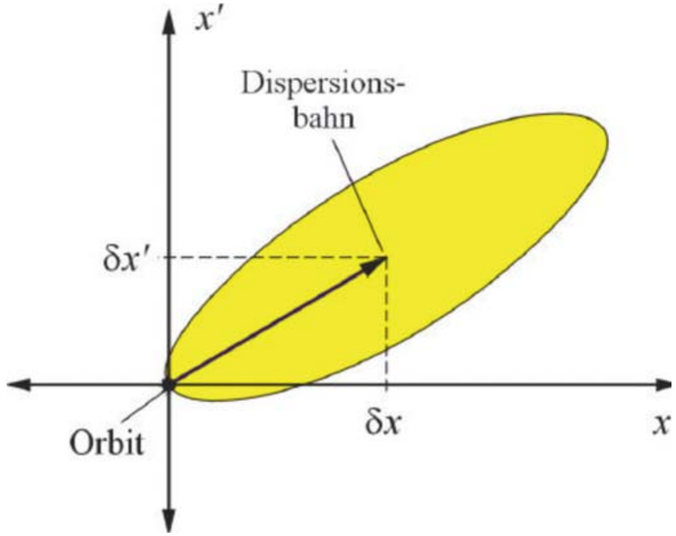
Damping of betatron oscillations:



$$\frac{d^2\Delta\varphi}{dt^2} + 2\alpha_S \cdot \frac{d\Delta\varphi}{dt} + \Omega_S^2 \cdot \Delta\varphi = 0, \qquad \text{damping time} \quad \tau = \frac{1}{\alpha_S} \sim \frac{T_0}{R \cdot E^3}$$

Excitation of betatron oscillations:





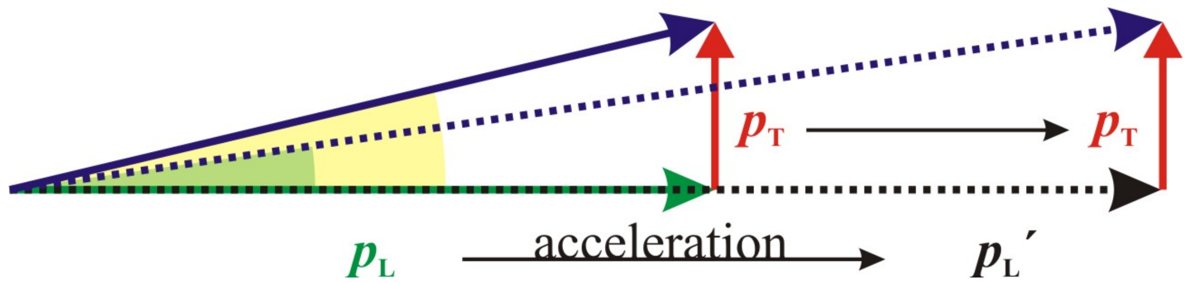
natural emittance: $\varepsilon_{\text{nat}} \sim \frac{E^2}{R \cdot l_{\text{m}}} \cdot \int_0^{l_{\text{m}}} \mathcal{H}$

natural energy spread: $\frac{\sigma_E}{E_0} \sim \frac{E}{\sqrt{R}}$

Proton Beams: negligible influence of synchrotron radiation!

"A proton machine remembers everything done to the beam like an elephant!"

What remains is adiabatic damping:



beam emittance $\varepsilon \sim \frac{1}{E}$

Further ways out (additional cooling):

- electron cooling
- stochastic cooling

6. Summary

I have tried to discuss the following topics:

- RF-based acceleration, longitudinal focusing and synchronization
- Beam guidance and transverse focusing (AG focusing)
- Linear beam optics, matrix formalism
- Phase space and emittance
- Optical resonances
- Beam excitation and cooling

Topics which could not have been touched within 45 minutes:

- Slow beam extraction
- Production and acceleration of polarized beams
- Intensity limitations (space charge, wake-fields, …)
- …