RF Resonant Polarimetry:

a way to non-invasive fast measurement of beam polarization and spin tunes?

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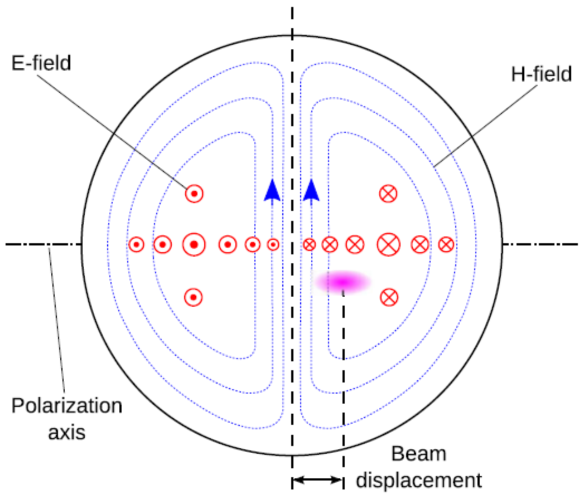
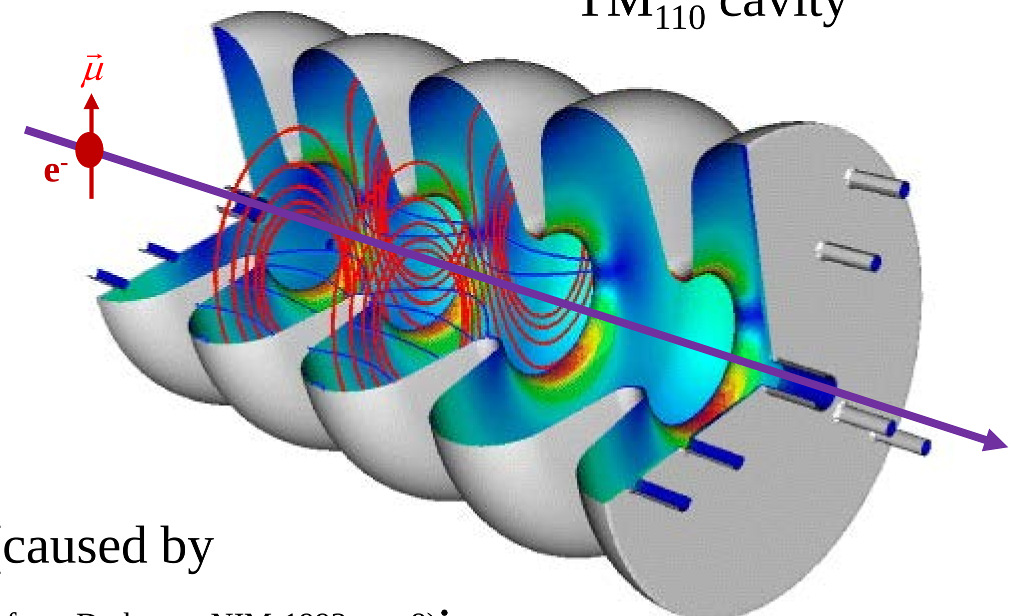
1. Functional Principle
2. Relativistic Stern-Gerlach Force
3. Cavity Modes
4. Energy Transfer per Particle Passage
5. Signal Power
6. Example: Respol with TE_{011}, TE_{111}

Resonant Polarimetry

Principle Idea (Derbenev 1993):



TM_{110} cavity

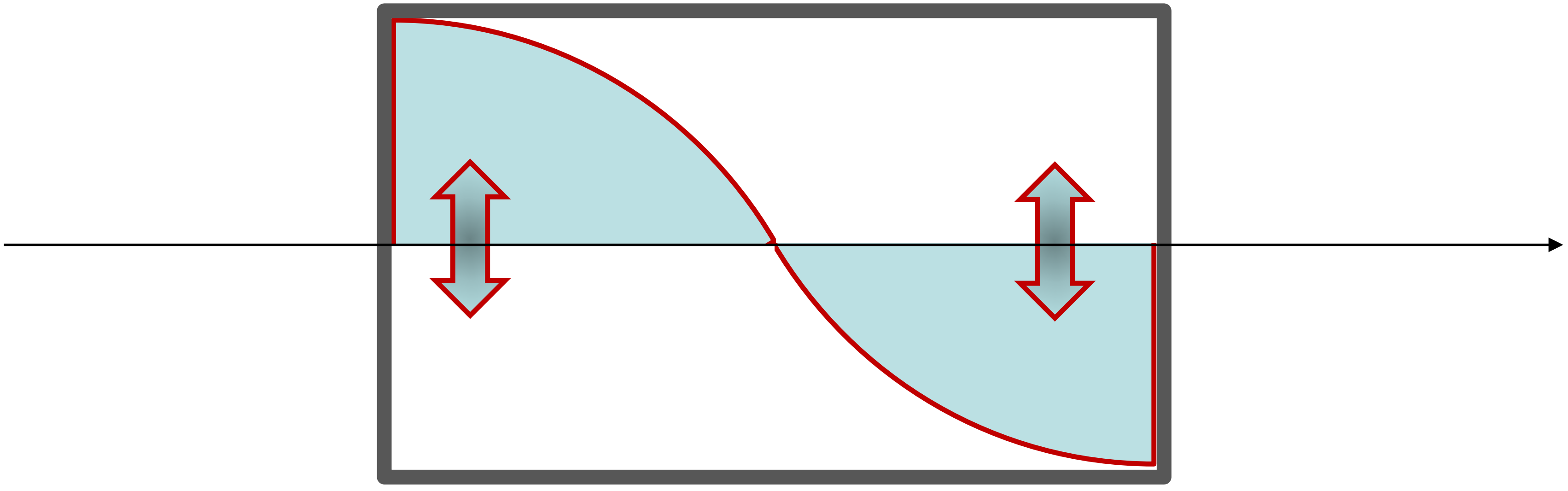


Coupling of the magnetic moment (caused by the spin) to the cavity's B-field (taken from Derbenev-NIM-1993, eq. 8):

$$W_C = \omega_c |a|^2 = \omega_c N^2 \left| \left\langle \frac{e}{2mc\sqrt{2\omega_c}} \left(\left(G + \frac{1}{\gamma} \right) B_\perp^c + \frac{1+G}{\gamma} B_\parallel^c \right) \vec{e} \cdot e^{ik\theta} \right\rangle \right|^2 \frac{\hbar^2 t^2}{4} P_e \sin^2 \alpha$$

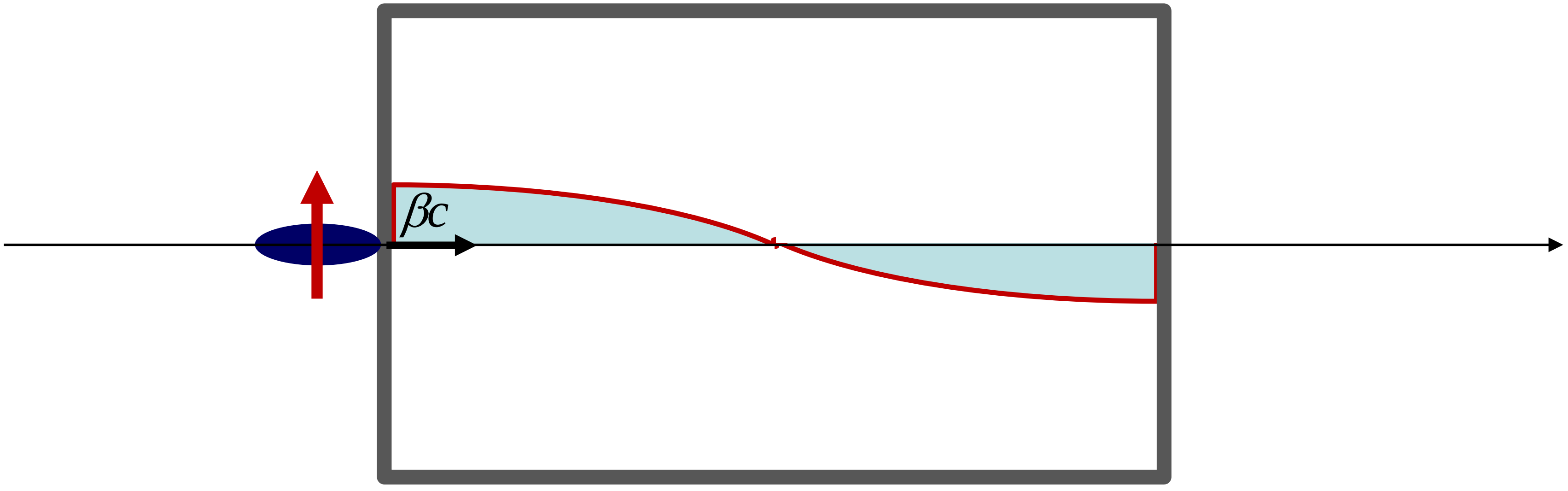
?Physical understanding?   ?γ and G scaling?

Transverse Mode



$$\Delta W = \int \frac{\partial}{\partial z}\left(\vec{\mu} \bullet \vec{B}\right) \cdot dz$$

Transverse Mode



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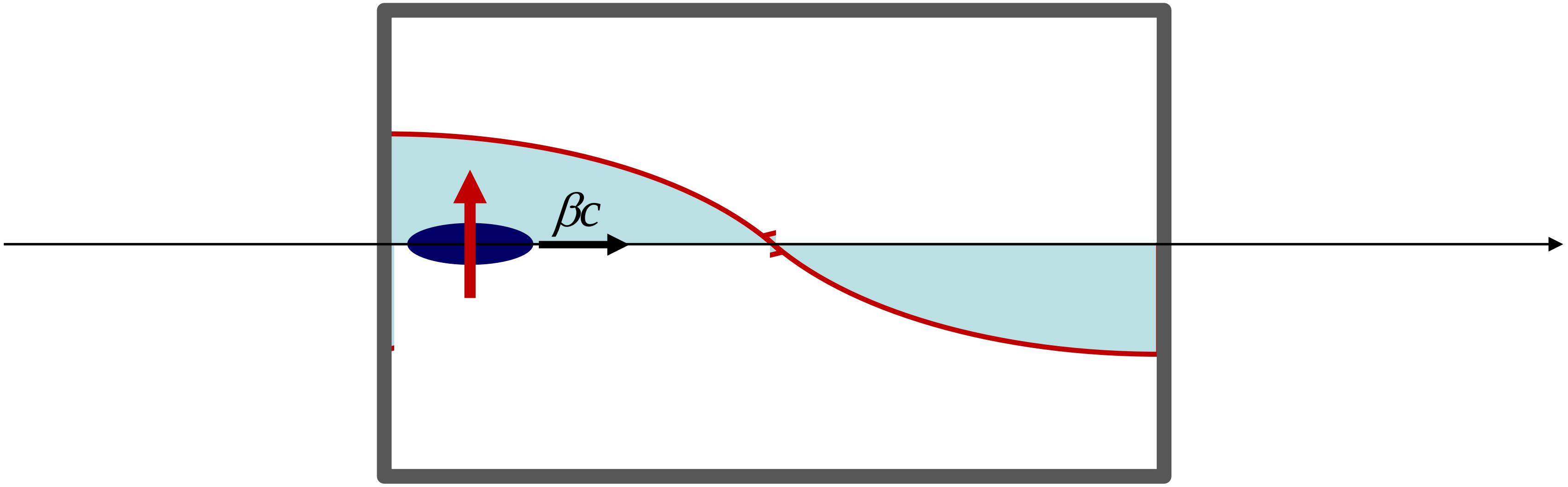

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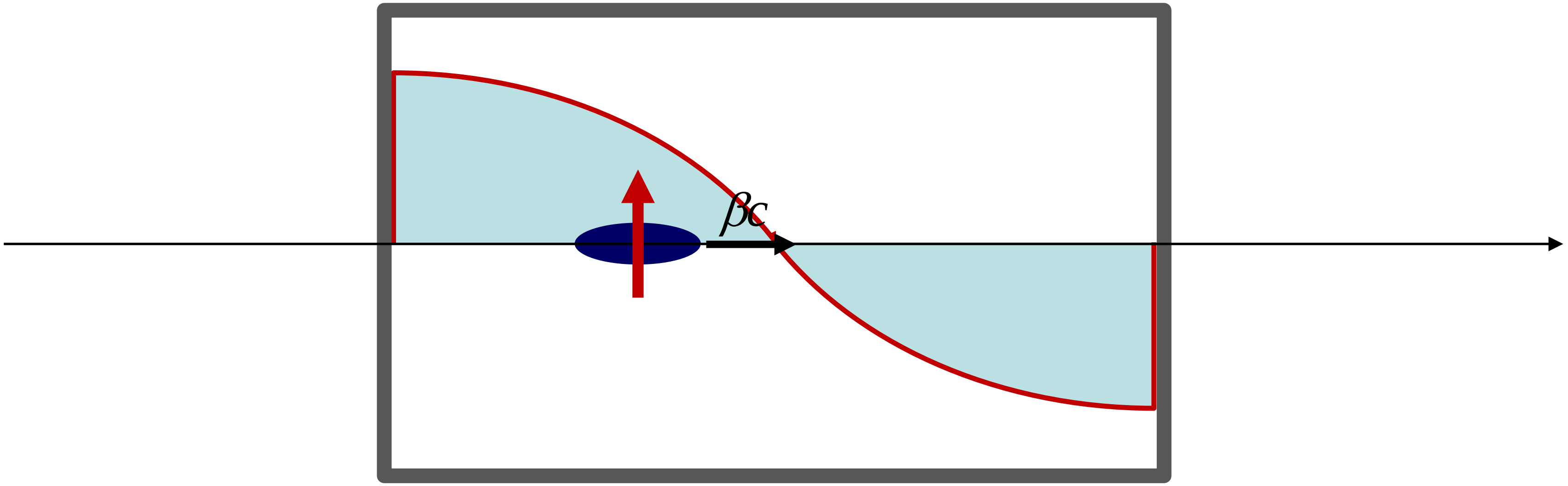

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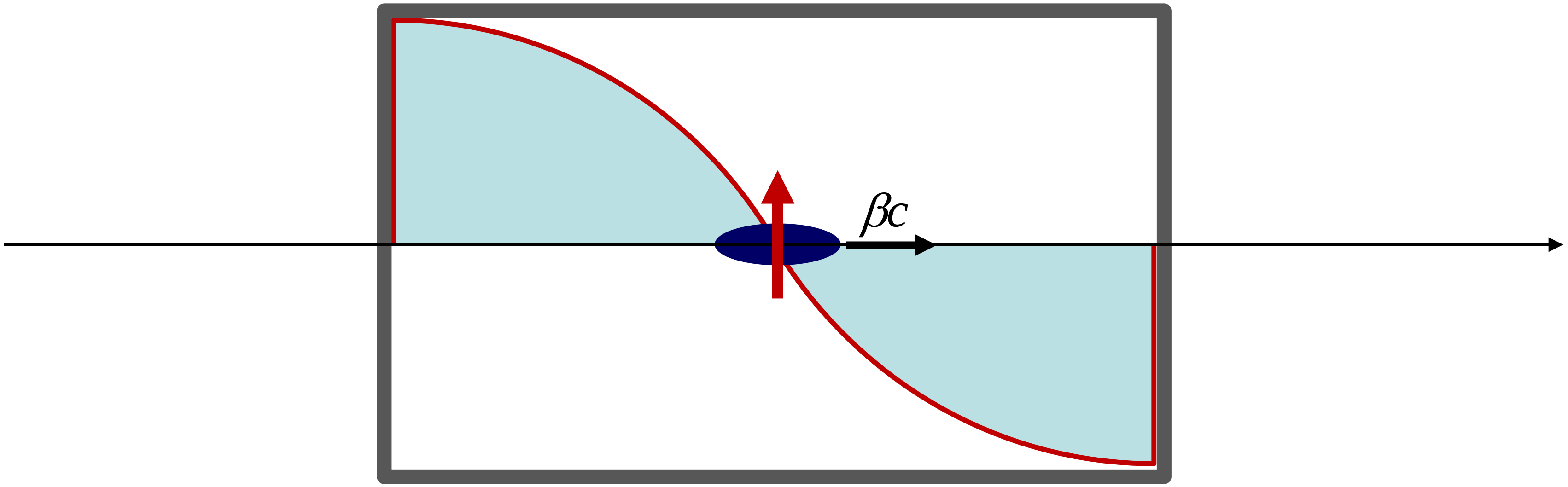

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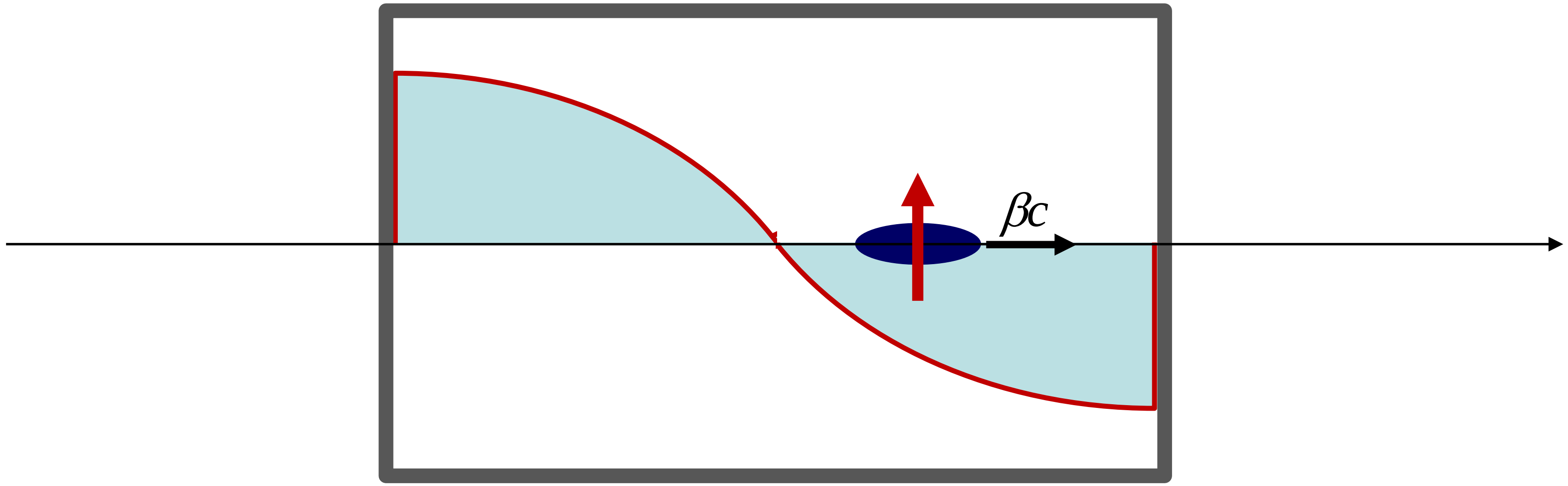

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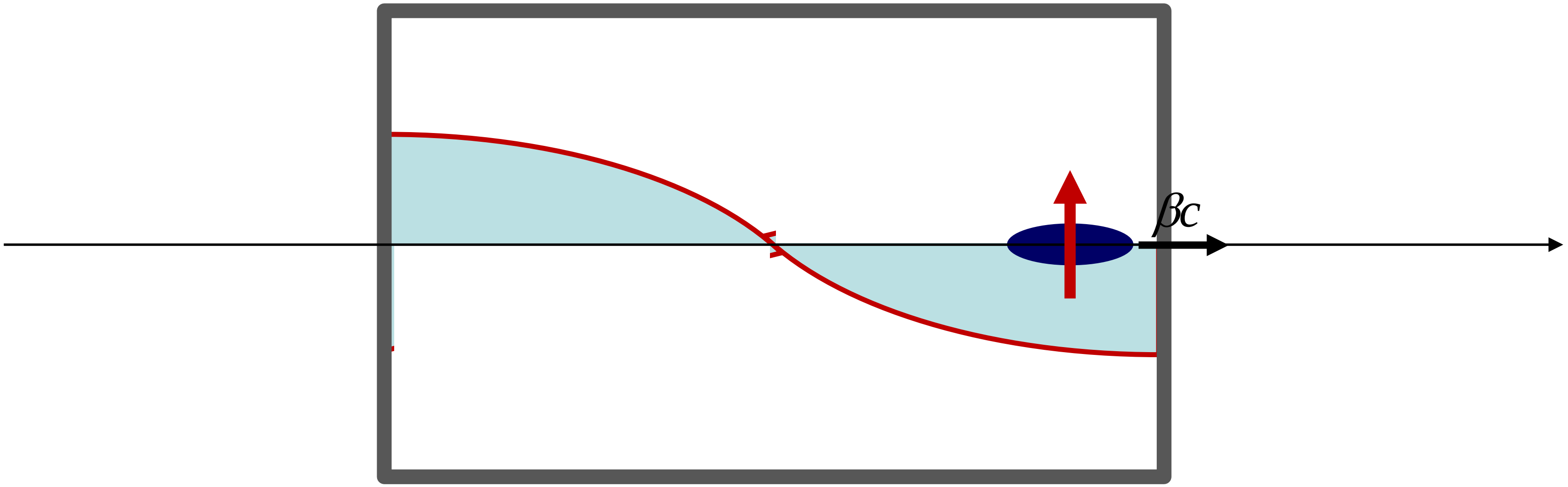

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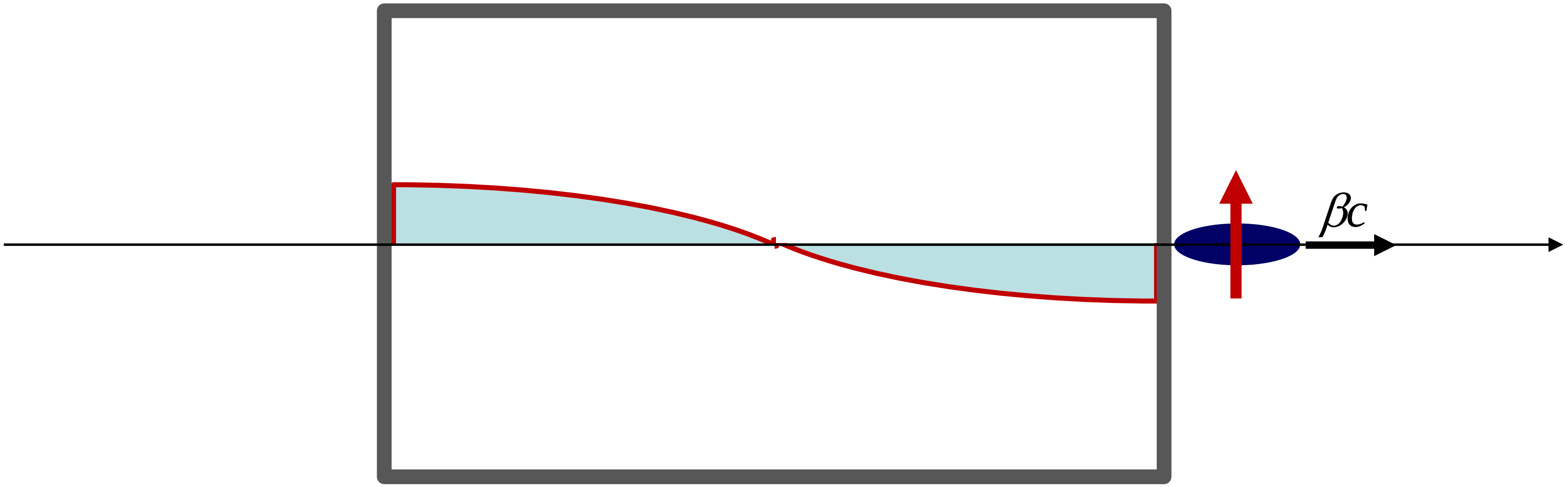

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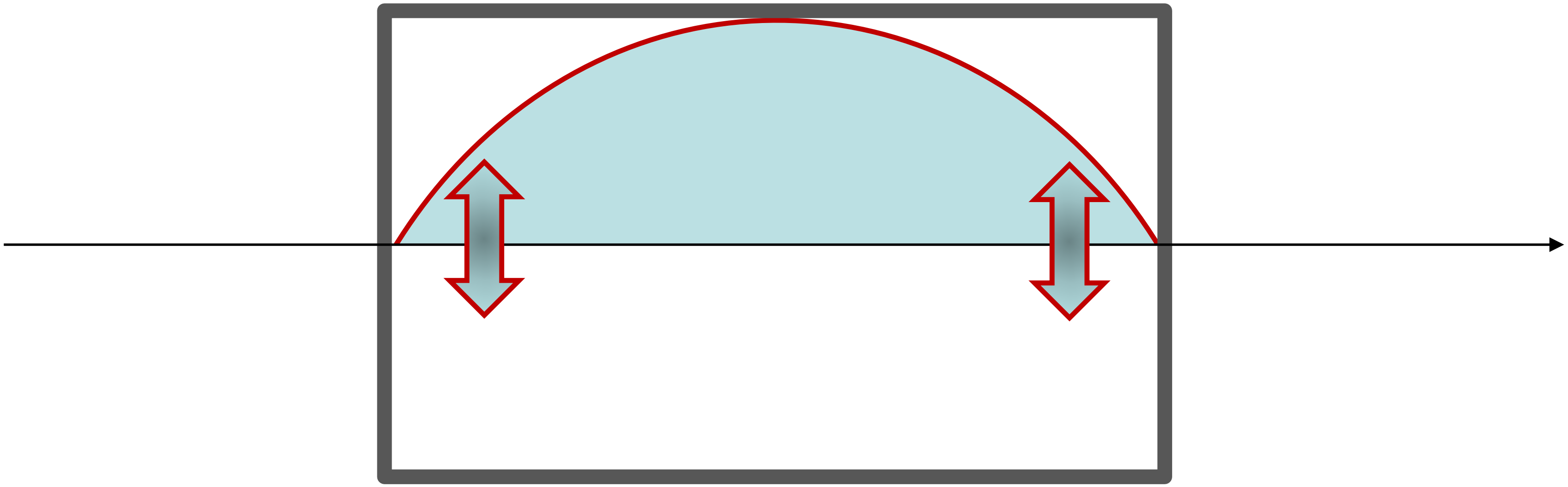
Longitudinal Mode



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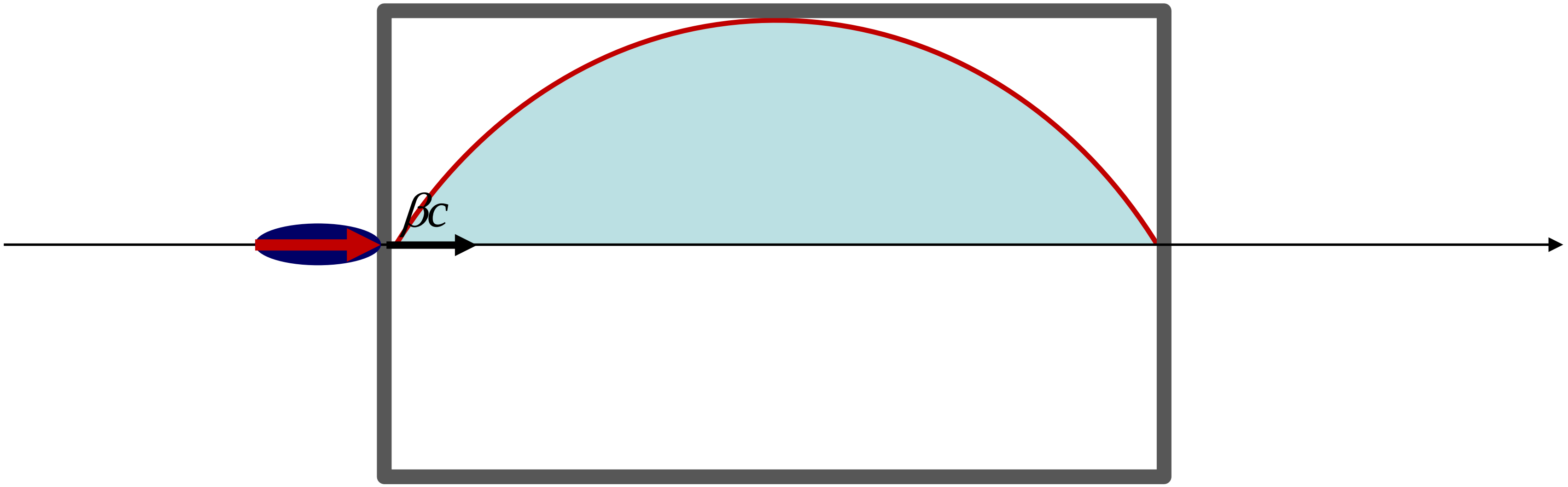

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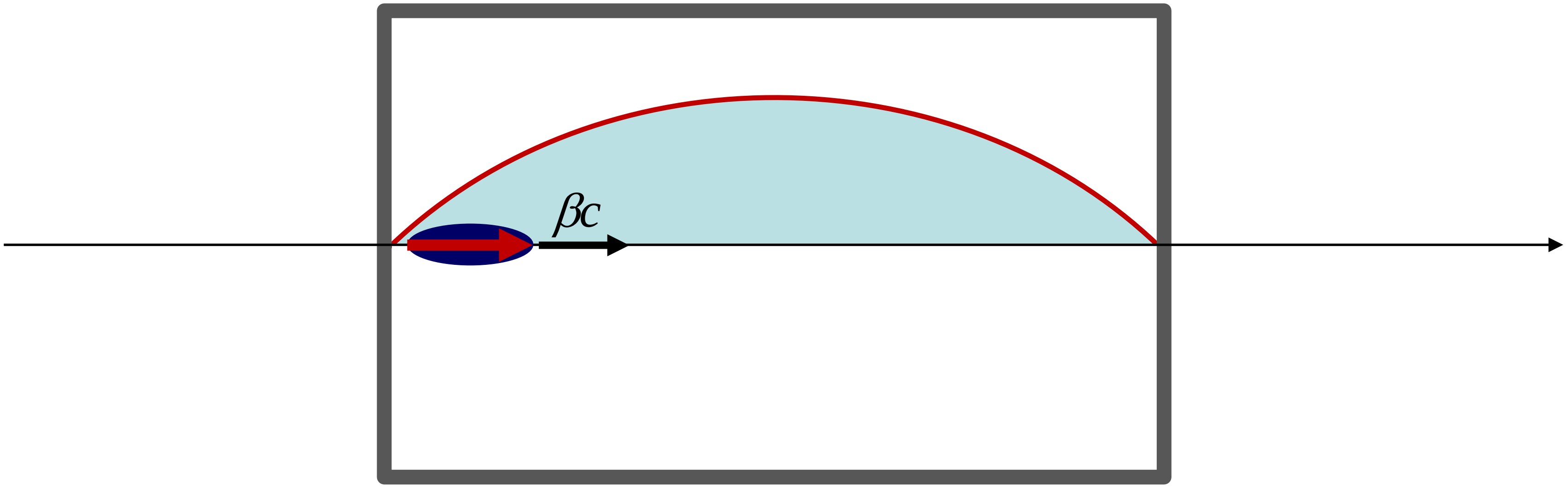

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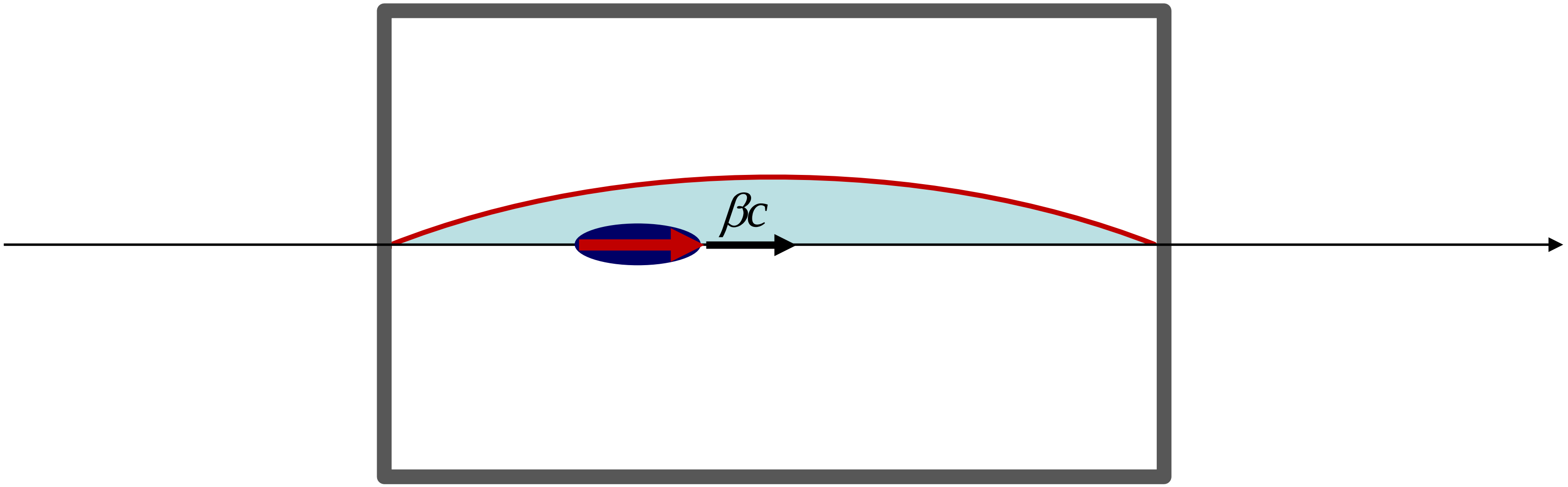

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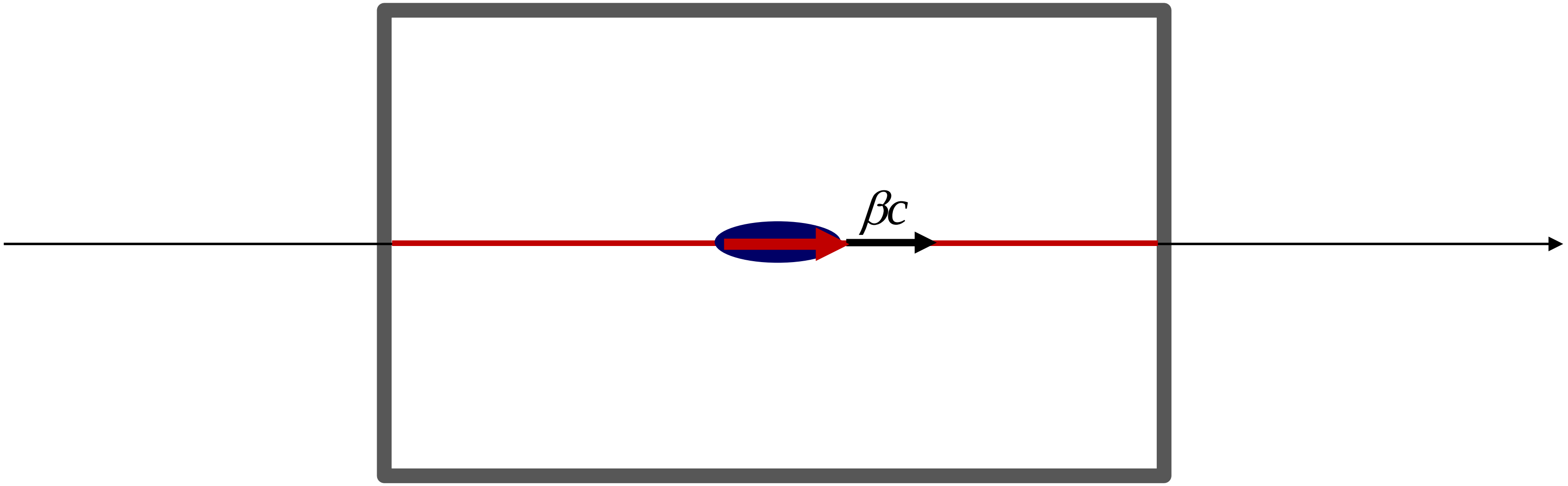

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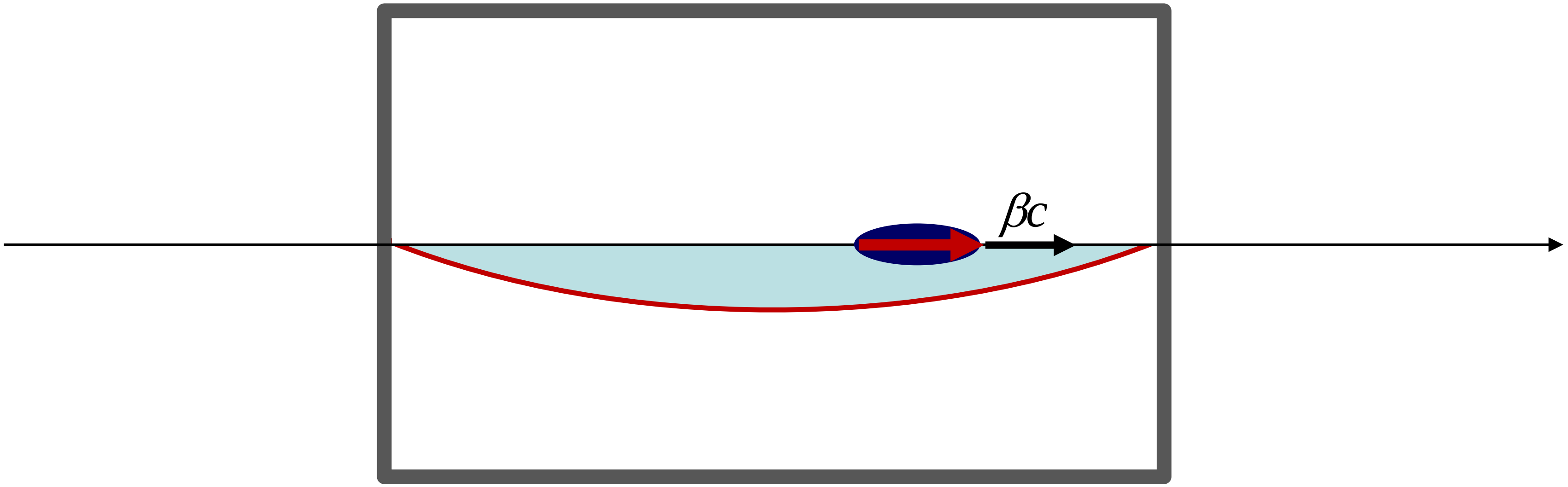

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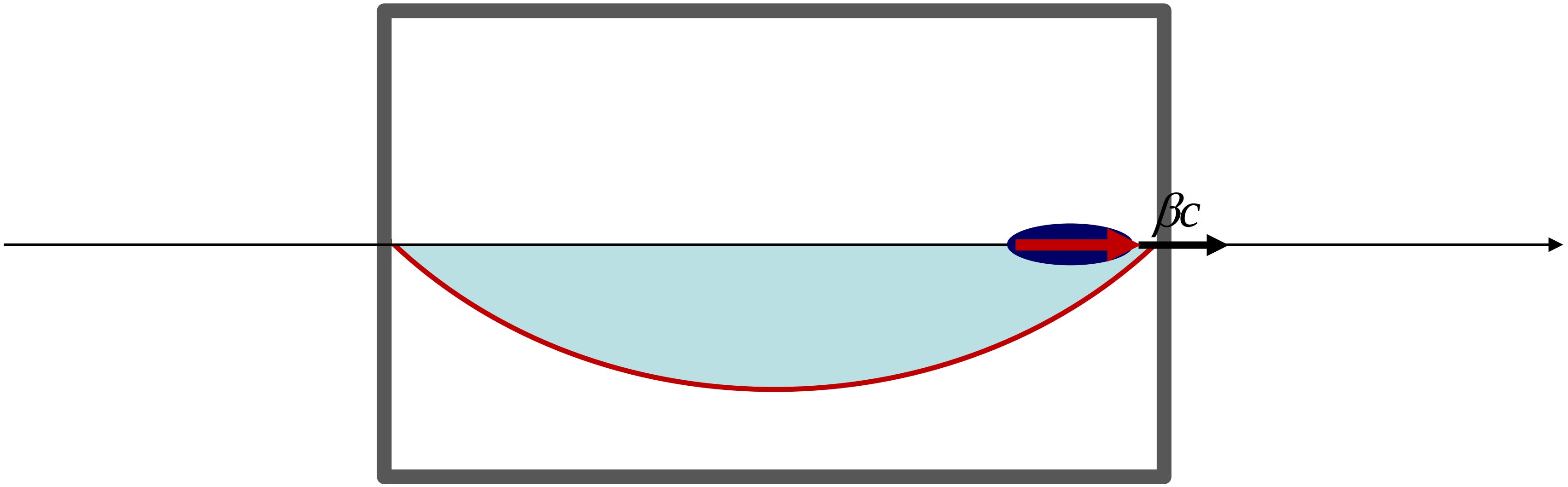

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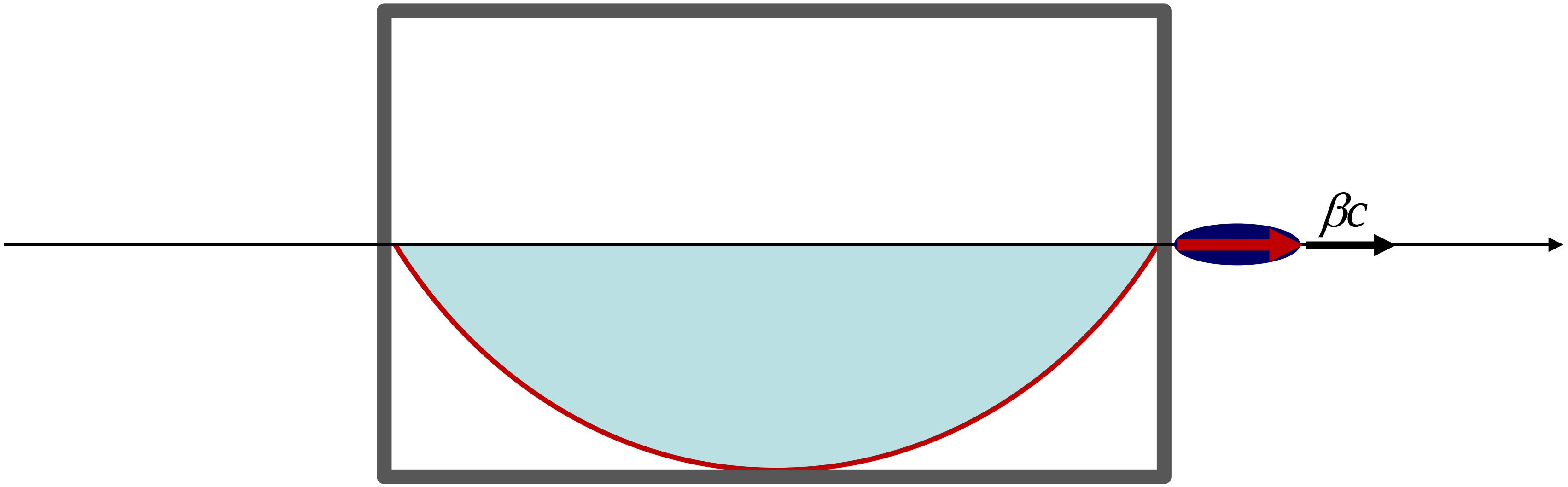

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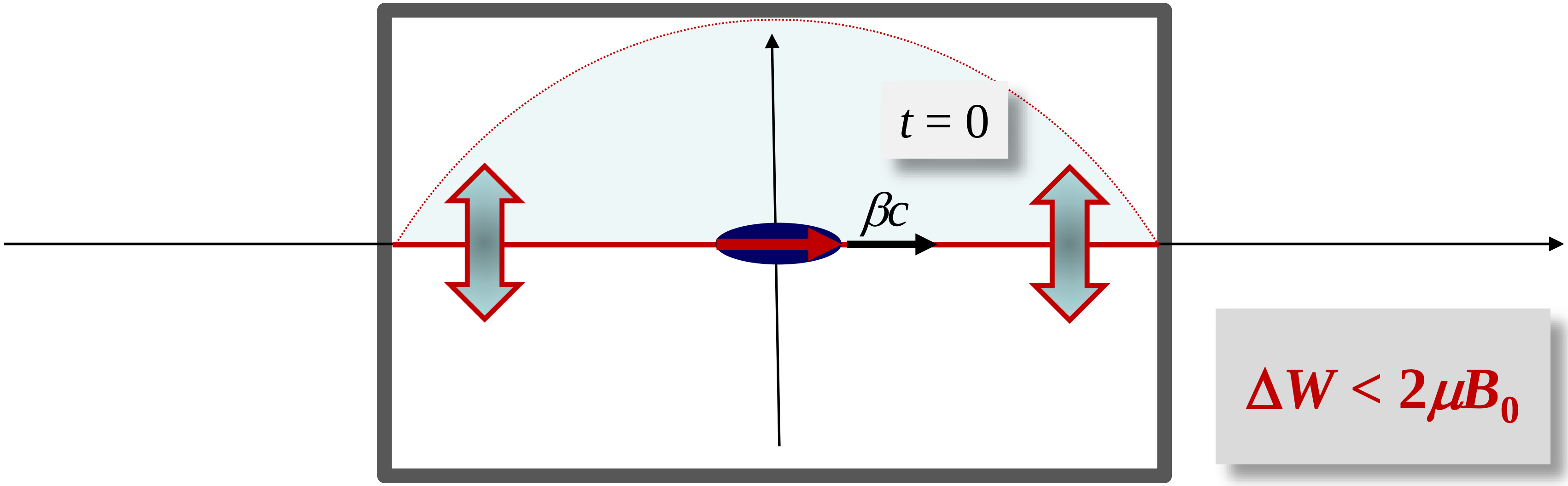


$$\Delta W < 2\mu B_0$$

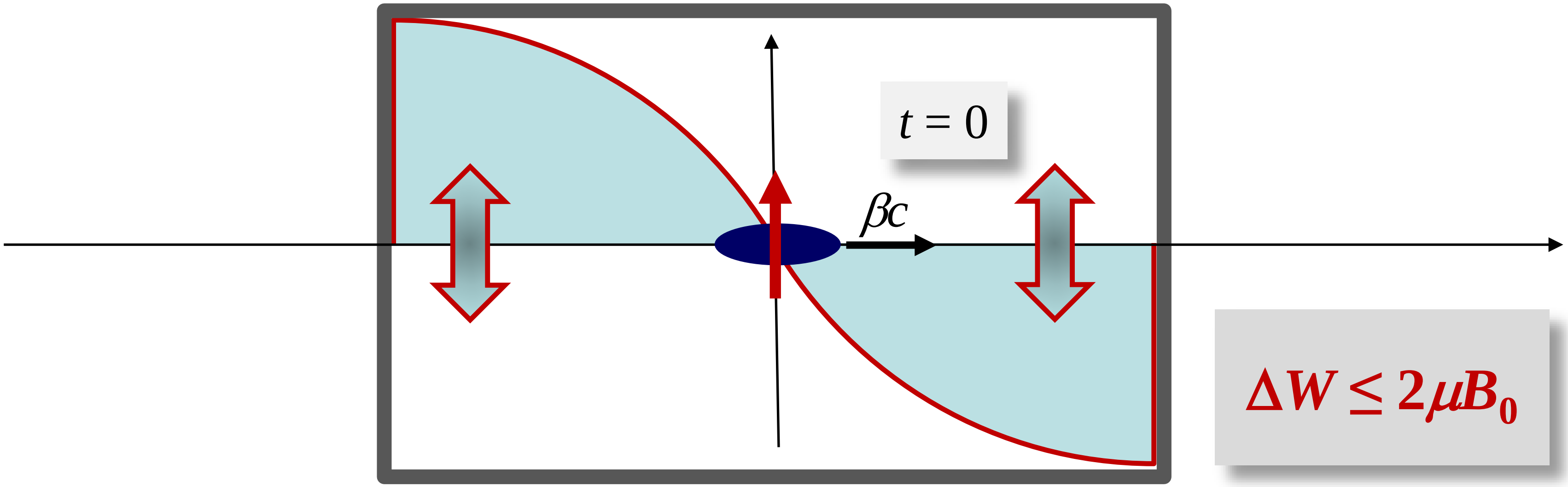
Findings:

$$B_{\perp} = B_0 \cdot \cos\left(\omega t + \phi\right) \Rightarrow \phi_{opt} = \frac{\pi}{2}, \quad \beta_{ph} \approx 1$$

?

Transverse Mode

$$\Delta W = \int \frac{\partial}{\partial z}\left(\vec{\mu}\bullet\vec{B}\right)\cdot dz$$



$$\Delta W \le 2\mu B_0$$

Findings:

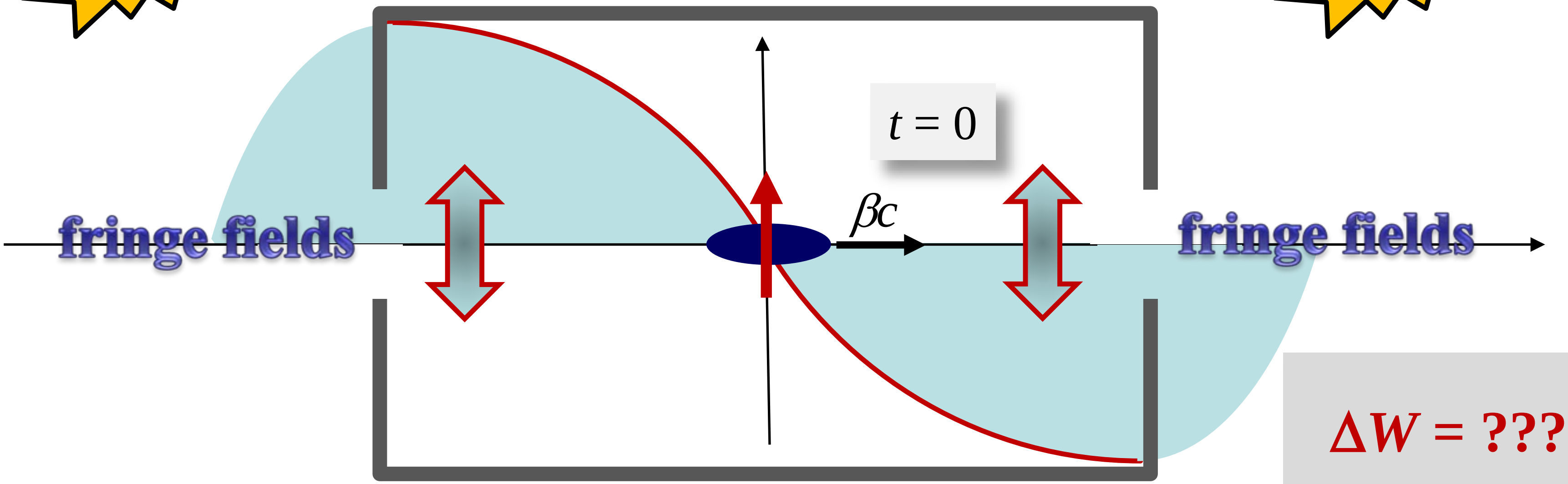
$$B_\perp = B_0 \cdot \cos\left(\omega t + \phi\right) \;\Rightarrow\; \phi_{opt} = 0, \;\; \beta_{ph} \gg 1$$

Transverse Mode

$$\Delta W = \int \frac{\partial}{\partial z}\left(\vec{\mu}\bullet\vec{B}\right)\cdot dz$$

but:

but:



$\Delta W = ???$

Findings:

$$B_\perp = B_0 \cdot \cos\left(\omega t + \phi\right) \quad \Rightarrow \quad \phi_{opt} = 0, \quad \beta_{ph} = ???$$

Some Approaches

Conte (arXiv: 0907.2161v1-2009)

Longitudinal Stern-Gerlach force:

$$F_z^{SG} = \frac{\partial}{\partial z^*}\left(\vec{\mu}^* \bullet \vec{B}^*\right) = \gamma\left(\frac{\partial}{\partial z} + \frac{\beta}{c}\frac{\partial}{\partial t}\right)\left(\vec{\mu}^* \bullet \gamma\left[\left(\vec{B} - \frac{\vec{\beta}}{c} \times \vec{E}\right) - \frac{\gamma^2}{\gamma+1}\vec{\beta}\left(\vec{\beta} \bullet \vec{B}\right)\right]\right)$$

Energy transfer to the cavity:

$$\Delta U = \int_0^L F_z^{SG} \cdot dz = \gamma^2 \cdot \int_0^L \left(\frac{\partial}{\partial z} + \frac{\beta}{c}\frac{\partial}{\partial t}\right)\vec{\mu} \bullet \left(\vec{B}_\perp - \frac{\vec{\beta}}{c} \times \vec{E}_\perp + \frac{1}{\gamma}\vec{B}_\parallel\right) \cdot dz$$

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Energy transfer to the cavity:

$$\Delta U = \int_0^L F_z^{SG}\cdot dz = \gamma^2 \cdot \int_0^L \left(\frac{\partial}{\partial z} + \frac{\beta}{c}\frac{\partial}{\partial t}\right)\vec{\mu}\bullet\left(\vec{B}_\perp - \frac{\vec{\beta}}{c}\times\vec{E}_\perp + \frac{1}{\gamma}\vec{B}_\parallel\right)\cdot dz$$

Improper procedures in Conte:

- Treatment of the fringe fields: $F \simeq \gamma^2 \frac{\partial B_y}{\partial z}$ neglecting temporal changes
- No relativistic cancellation by taking use of the total derivative
- Neglecting beam deflection and spin precession in the transverse magnetic fields in the cavity using

$$\vec{B}^* = \gamma\left(\vec{B}_\perp - \frac{\vec{\beta}}{c}\times\vec{E}\right) + \vec{B}_\parallel$$

A simple but (hopefully) correct Approach

Transformation of derivatives: $$\frac{\partial}{\partial z^*} = \gamma\left(\frac{\partial}{\partial z} + \frac{\beta}{c}\frac{\partial}{\partial t}\right) = \gamma\frac{d}{dz} - \frac{1}{\beta\gamma c}\frac{\partial}{\partial t}$$

Transformation of the fields:

$$\vec{\mu}^* \bullet \vec{B}^* = \vec{\mu} \bullet \left[\frac{\gamma}{1+G}\left\{\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp - \left(G+\frac{1}{1+\gamma}\right)\frac{\vec{\beta}}{c}\times\vec{E}\right\} + \vec{B}_\parallel\right]$$

Taking use of the relativistic compensation:

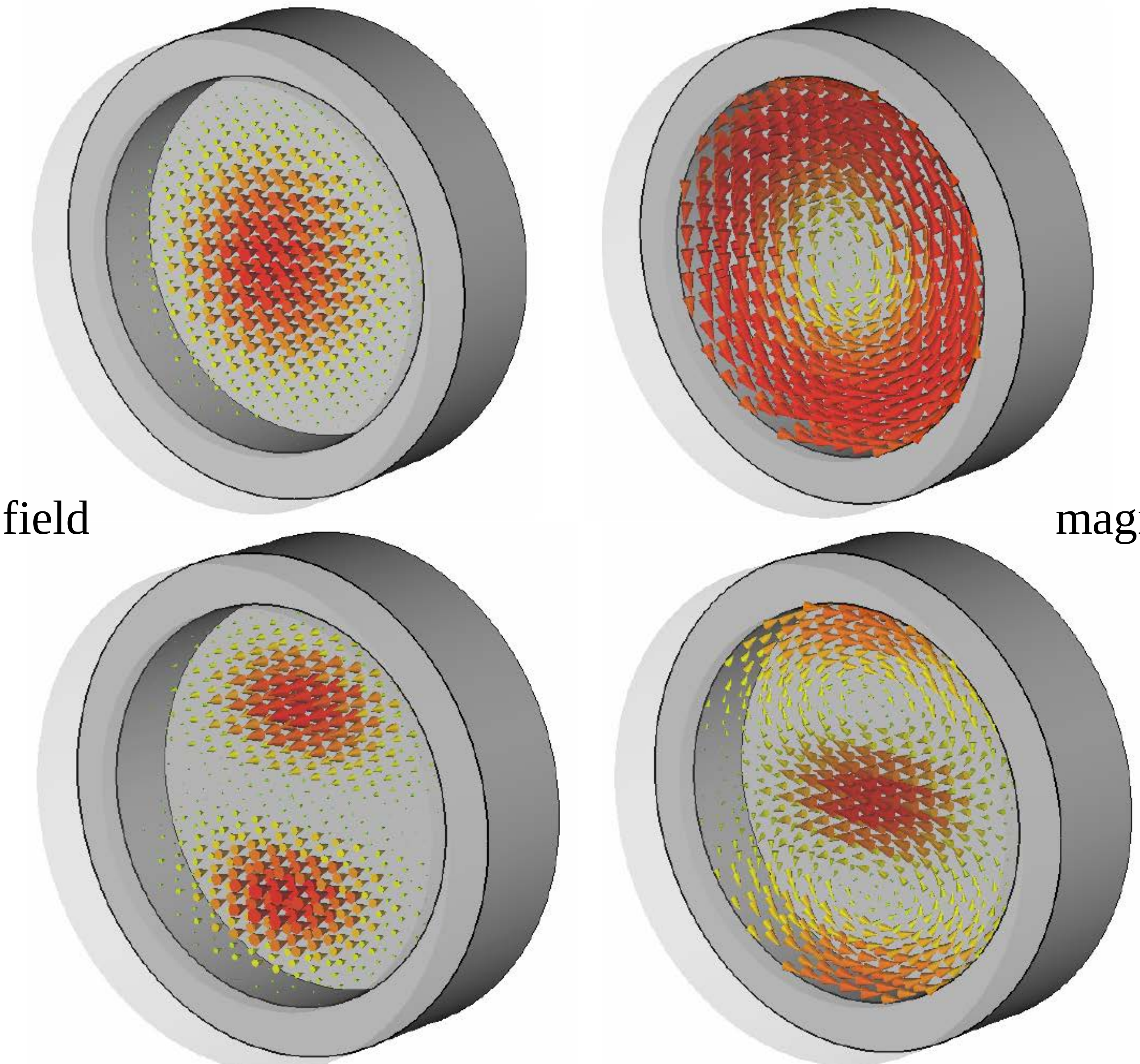
$$\Delta U = \int_0^d F_z^{SG}\cdot dz = \underbrace{\gamma\vec{\mu}^* \bullet \vec{B}^*\Big|_0^d}_{=0} - \frac{\vec{\mu}^*}{\gamma\beta c}\bullet\int_0^d \frac{\partial}{\partial t}\left[\frac{\gamma}{1+G}\left\{\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp - \left(G+\frac{1}{1+\gamma}\right)\frac{\vec{\beta}}{c}\times\vec{E}\right\} + \vec{B}_\parallel\right]dz$$

→ Energy transfer to the cavity:

$$\Delta U = \int_C F_z^{SG}\cdot dz = -\frac{\vec{\mu}}{\beta c}\bullet\frac{\partial}{\partial t}\int_C\left\{\underbrace{\frac{G+1/\gamma}{1+G}}_{=\xi_B}\vec{B}_\perp - \underbrace{\left(\frac{G}{1+G}+\frac{1}{(1+G)(1+\gamma)}\right)}_{=\xi_E}\frac{\vec{\beta}}{c}\times\vec{E} + \frac{1}{\gamma}\vec{B}_\parallel\right\}dz$$

Cavity Modes: TM

TM_{010}



electric field

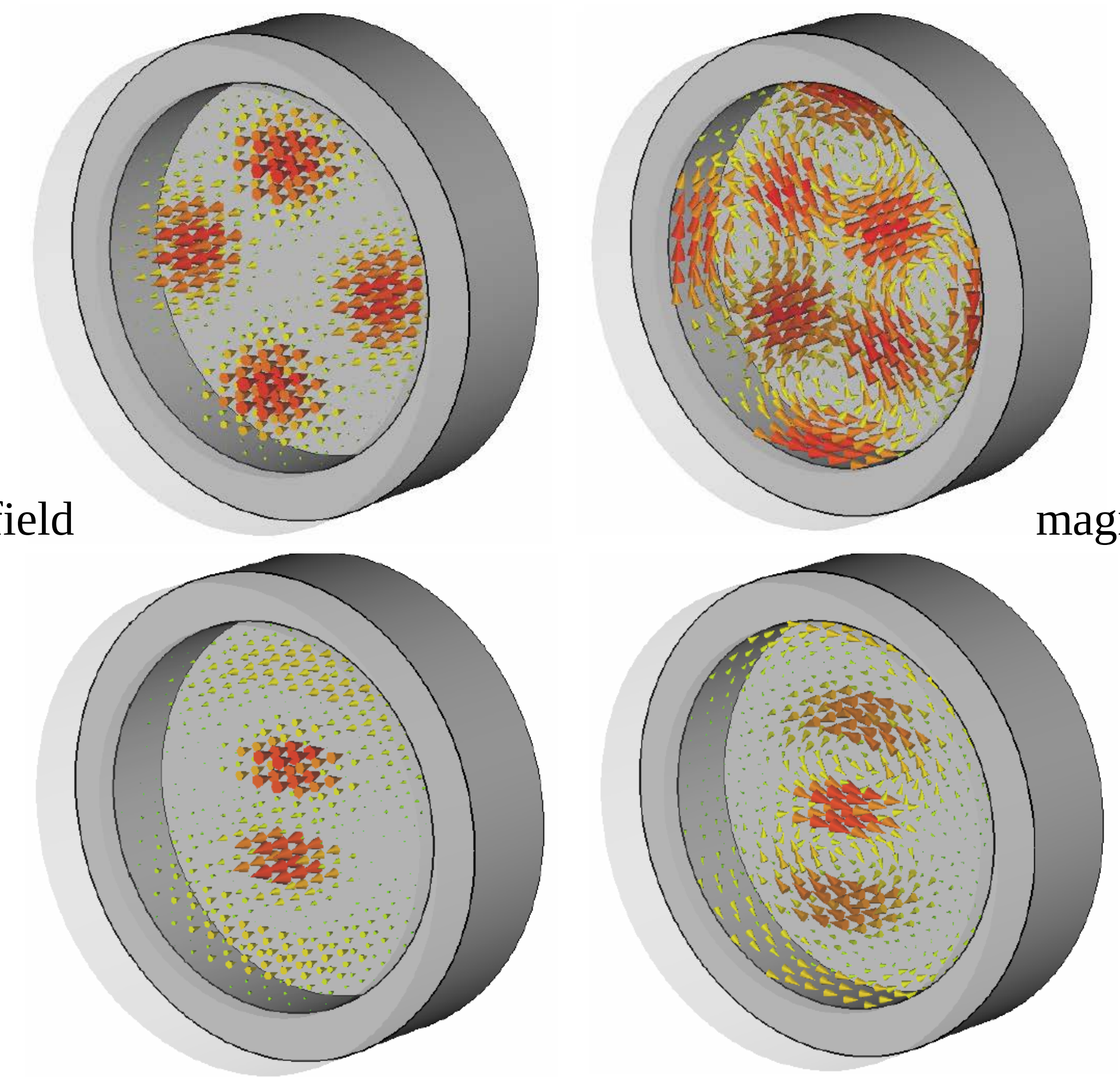
magnetic field

TM_{110}

Cavity Modes: TM

TM_{210}

TM_{120}

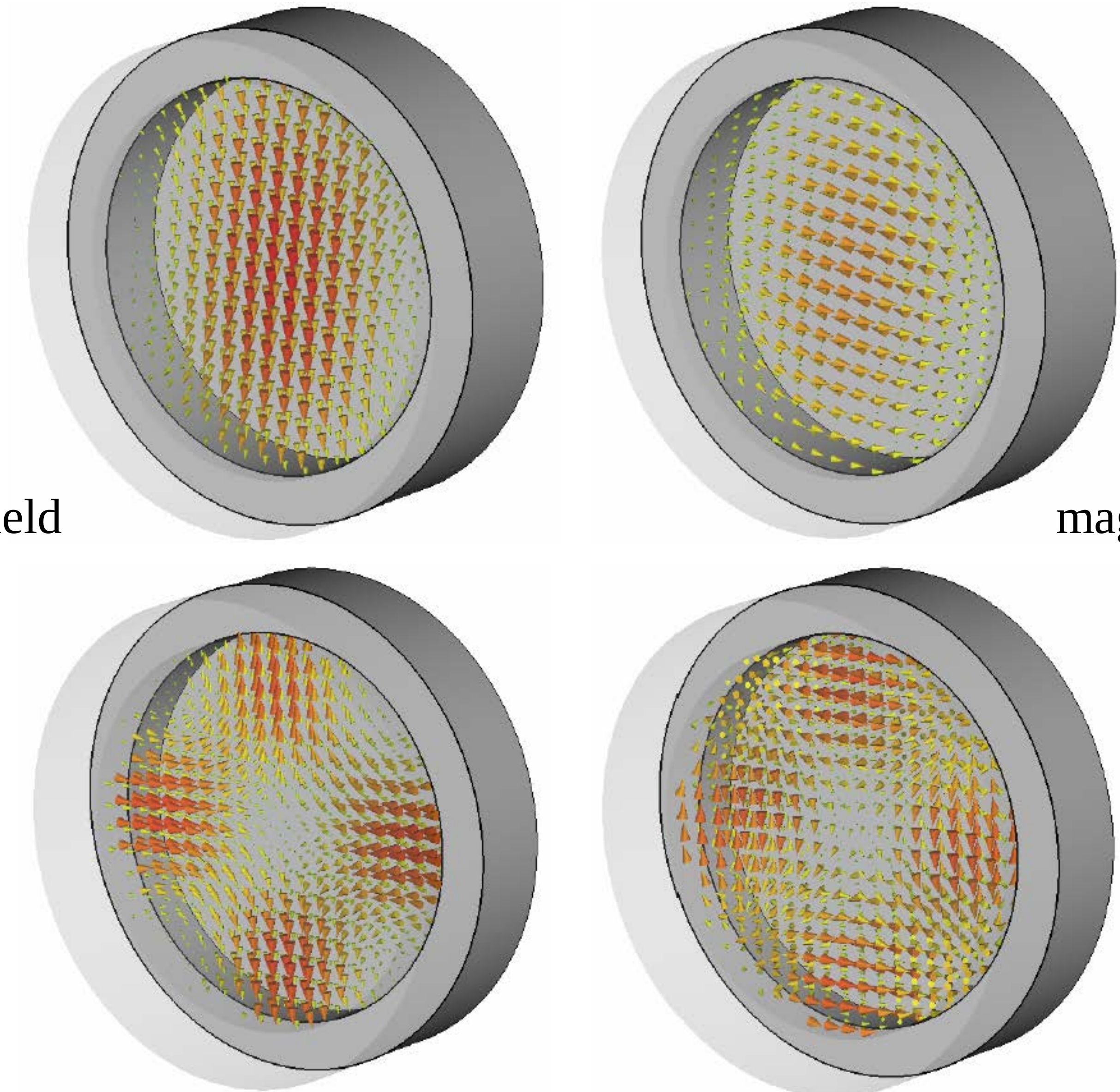


electric field

magnetic field

Cavity Modes: TE

TE_{111}



electric field

magnetic field

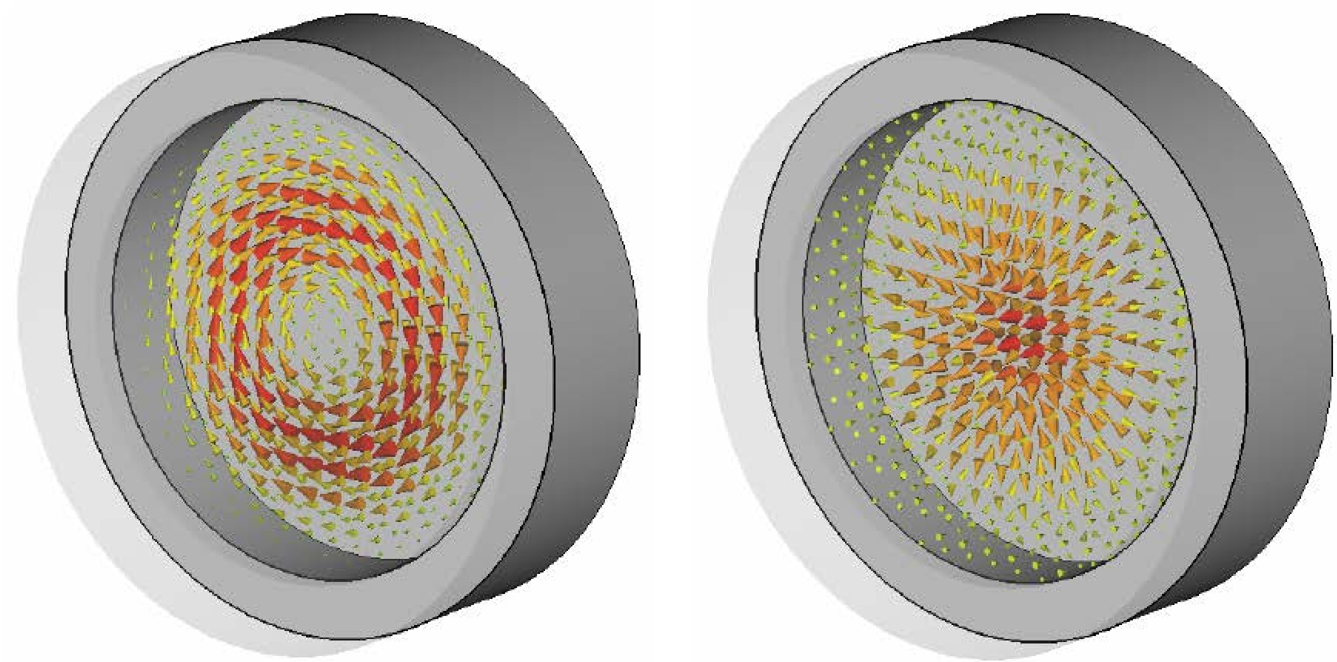
TE_{211}

Cavity Modes: TE

and longitudinal:

electric field　　　　magnetic field

TE$_{011}$



General Findings and Set-Up

TM$_{mnp}$ and TE$_{mnp}$ modes, on-axis fields = 0 for m > 1!!!

odd longitudinal *p*:	**even longitudinal *p*:**
$\vec{B}_\perp(z,t) = \vec{B}_\perp^0 \cdot \sin\left(\frac{p\pi z}{L}\right) \cdot \cos(\omega t + \phi)$	$\vec{B}_\perp(z,t) = \vec{B}_\perp^0 \cdot \cos\left(\frac{p\pi z}{L}\right) \cdot \cos(\omega t + \phi)$
$B_z(z,t) = B_z^0 \cdot \cos\left(\frac{p\pi z}{L}\right) \cdot \cos(\omega t + \phi)$	$B_z(z,t) = B_z^0 \cdot \sin\left(\frac{p\pi z}{L}\right) \cdot \cos(\omega t + \phi)$
$\vec{E}_\perp(z,t) = \vec{E}_\perp^0 \cdot \cos\left(\frac{p\pi z}{L}\right) \cdot \sin(\omega t + \phi)$	$\vec{E}_\perp(z,t) = \vec{E}_\perp^0 \cdot \sin\left(\frac{p\pi z}{L}\right) \cdot \sin(\omega t + \phi)$
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Origin of coordinate system at the center of the cavity!

Single Particle Energy Transfer

Integration of the Stern-Gerlach force:

- **odd longitudinal *p*:**

$$\Delta U_{\perp} = \frac{-2\cos\phi}{1-\left(\beta/\beta_{ph}\right)^2}\sin\left(\frac{p\pi}{2}\right)\cos\left(\frac{p\pi\beta_{ph}}{2\beta}\right)\vec{\mu}\bullet\left\{\xi_B\vec{B}_{\perp}^{0}\cdot+\xi_E\frac{\beta}{\beta_{ph}}\left(\hat{e}_z\times\frac{\beta}{c}\vec{E}_{\perp}^{0}\right)\right\}$$

$$\Delta U_{\parallel} = -\frac{2}{\gamma}\mu_z B_z^0\frac{\sin\phi}{1-\left(\beta/\beta_{ph}\right)^2}\frac{\beta}{\beta_{ph}}\sin\left(\frac{p\pi}{2}\right)\cos\left(\frac{p\pi\beta_{ph}}{2\beta}\right)$$

- **even longitudinal *p*:**

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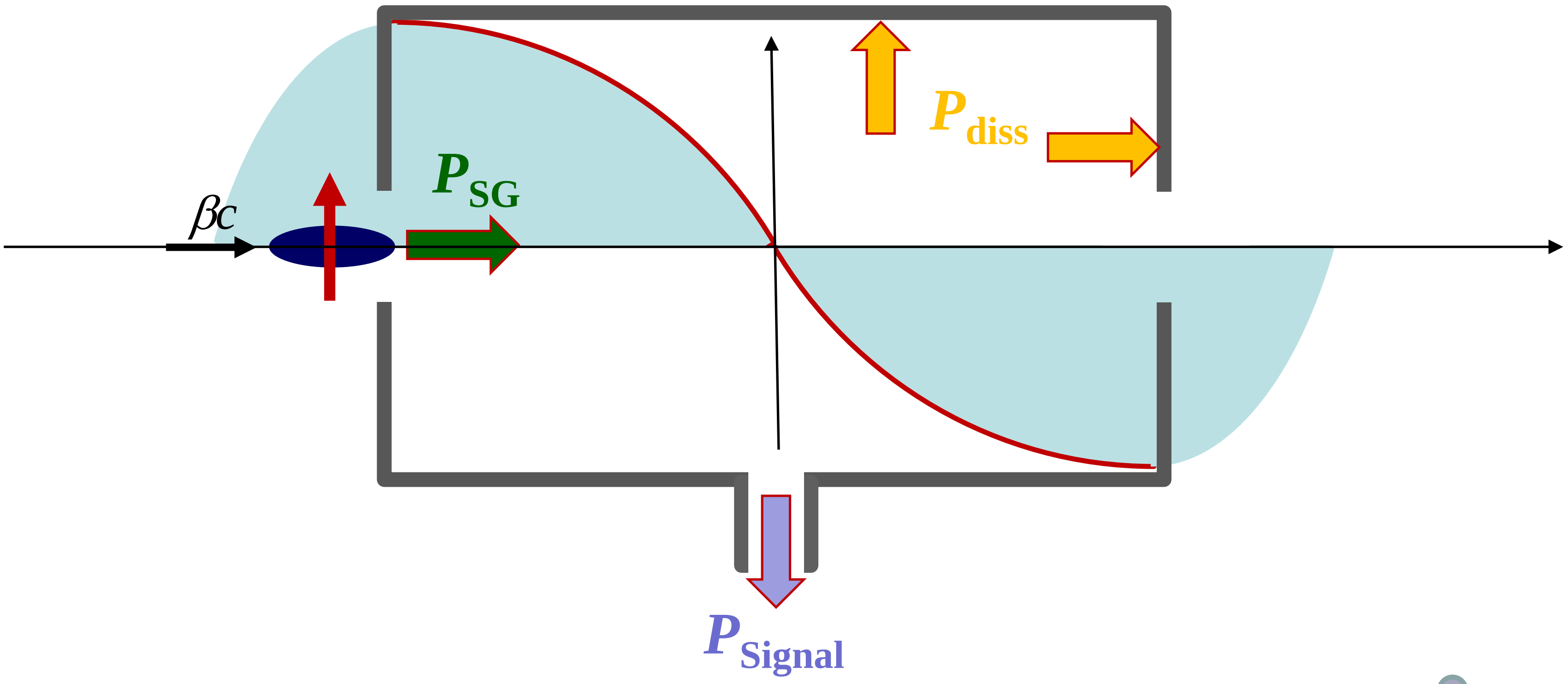
Signal Power



Steady State: $P_{SG} = P_{Signal} + P_{diss}$

Signal Power

Energy transfer: $P_+ = \frac{I}{e} \cdot \eta_b \cdot \Delta U$ **, bunch factor:** $\eta_b = \int \rho(s) \cdot \cos\left(\frac{\omega s}{\beta c}\right) \cdot ds$

Stored energy: $W_C = \frac{1}{2\mu_0} \int_V B^2 dV = \frac{1}{2\varepsilon_0} \int_V E^2 dV = \upsilon_b \cdot B_0^2 = \upsilon_e \cdot E_0^2$

→ Energy transfer: $dW_C = P_+ \cdot dt = \frac{I}{e} \cdot \eta_b \cdot \Delta U \cdot dt = \frac{I}{e} \cdot \eta_b \cdot s_\mu \cdot B_0 \cdot dt = \varsigma \cdot \sqrt{W_C} \cdot dt$

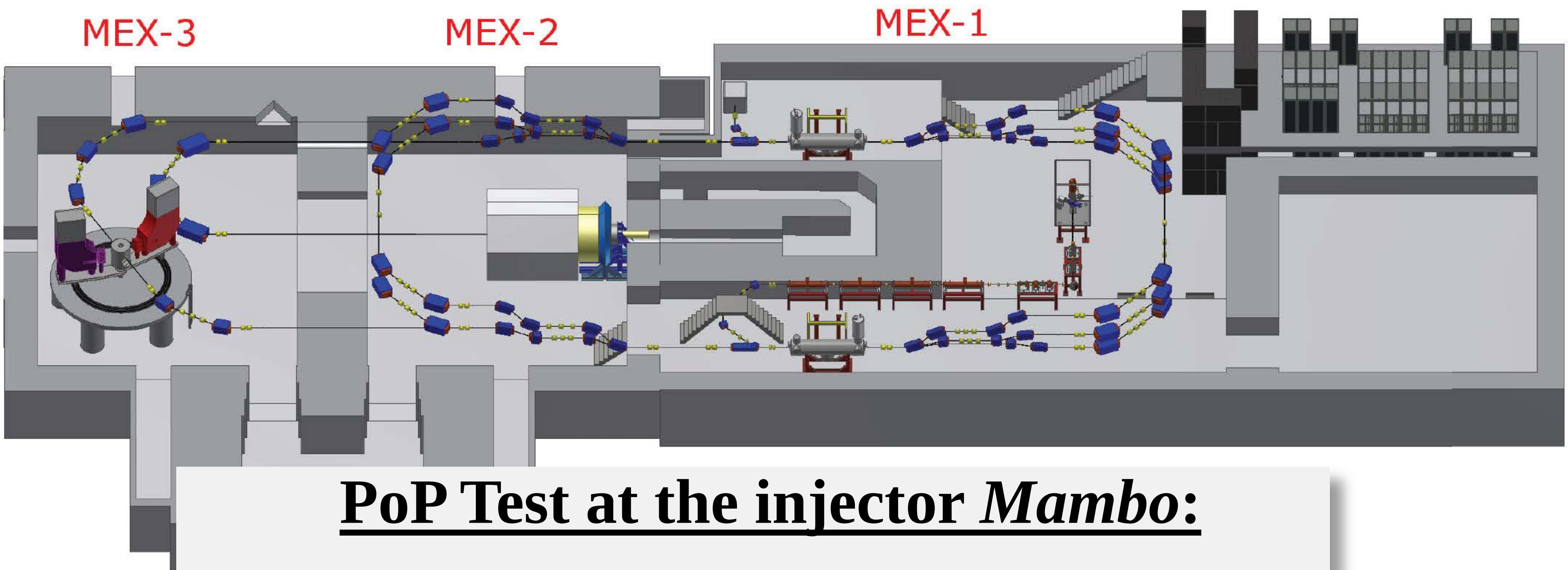
Energy dissipation: $P_- = \frac{\omega}{Q_l} \cdot W_C = \frac{1+\kappa}{Q_0} \cdot \omega \cdot W_C = \frac{1}{\tau} \cdot W_C$

Build-up of stored energy: $\frac{d}{dt} W_C = \varsigma \cdot \sqrt{W_C} - \frac{1}{\tau} \cdot W_C \quad \rightarrow \quad W_C(t) = (\varsigma\tau)^2 \cdot \left(1 - e^{-\frac{t}{2\tau}}\right)$

Steady state conditions: $W_C^\infty = (\varsigma\tau)^2 = \frac{I^2 \cdot \eta_b^2 \cdot s_\mu^2}{e^2 \cdot \upsilon} \cdot \frac{Q_0^2}{(1+\kappa)^2} \cdot \frac{1}{\omega^2}$

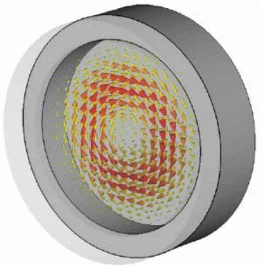
Signal Power: $P_S = \kappa \cdot P_- = \kappa \cdot \frac{\omega \cdot W_C}{Q_0} = \frac{I^2 \cdot \eta_b^2 \cdot s_\mu^2}{e^2 \cdot \upsilon} \cdot \frac{\kappa}{(1+\kappa)^2} \cdot \frac{Q_0}{\omega}$

Experiment @ *MESA*/Mainz:

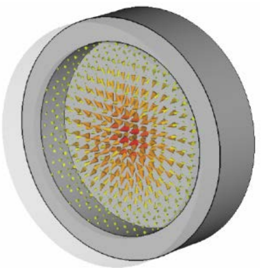


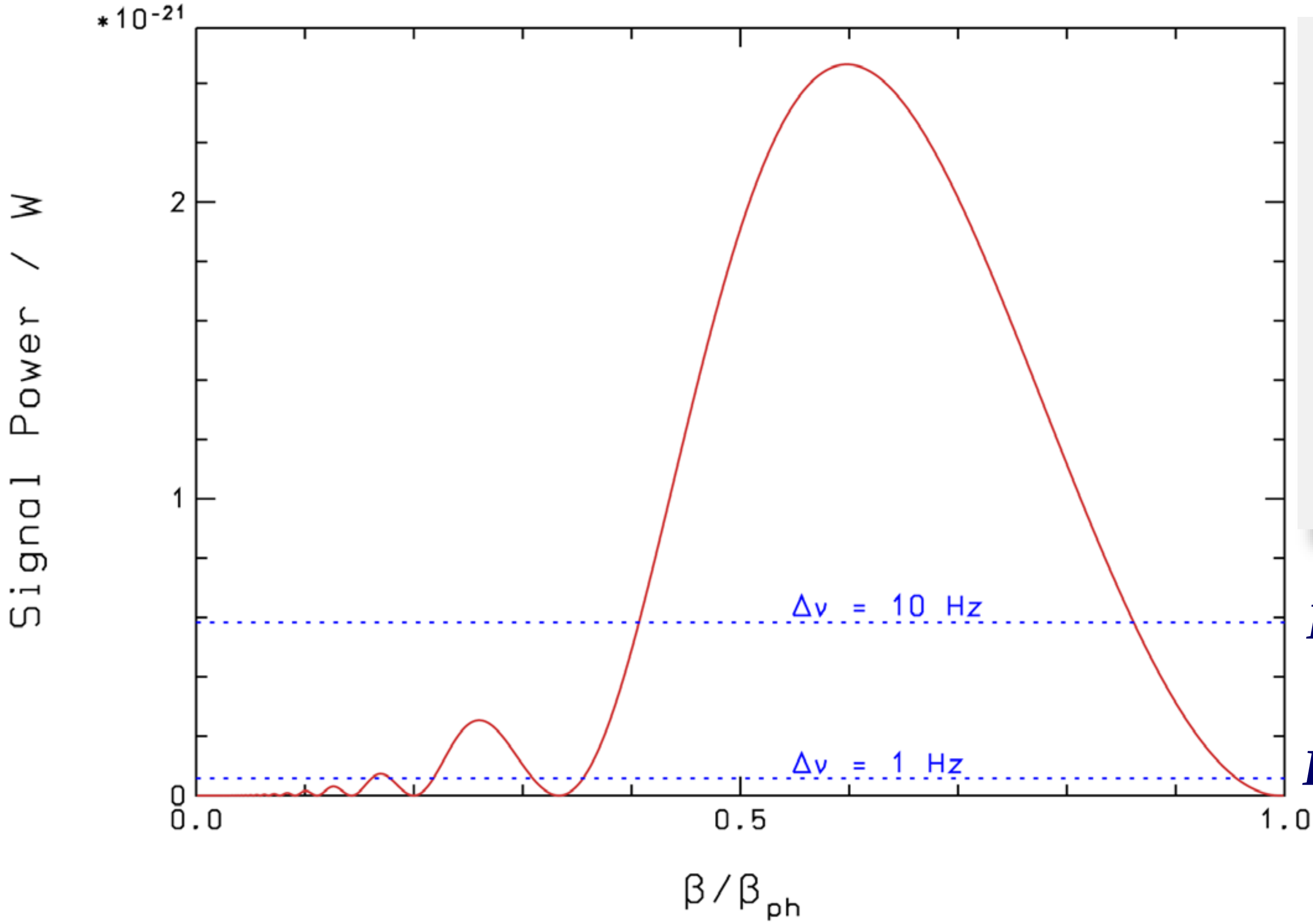
PoP Test at the injector *Mambo*:

- Longitudinal polarisation ↔ long. magn. field
- Low Lorentz gamma ($\gamma \approx 10$)
- Flip helicity every bunch (2 lasers?!)
- Tune cavity to ½ bunch repetition frequency
- Use TE mode with no long. electric fields
- Phase locking of polarimeter signal to RF



Longitudinal: TE_{011}





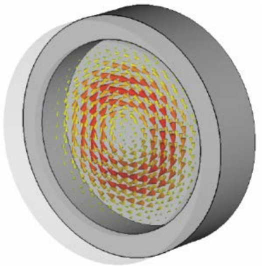
Parameters:

- I_e = 1 mA
- η_b = 1
- ν_C = 0.65 GHz
- Q_0 = 10^{10}
- κ = 10
- γ = 10

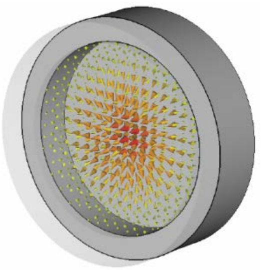
P_{noise} @ 10 Hz Bwdth

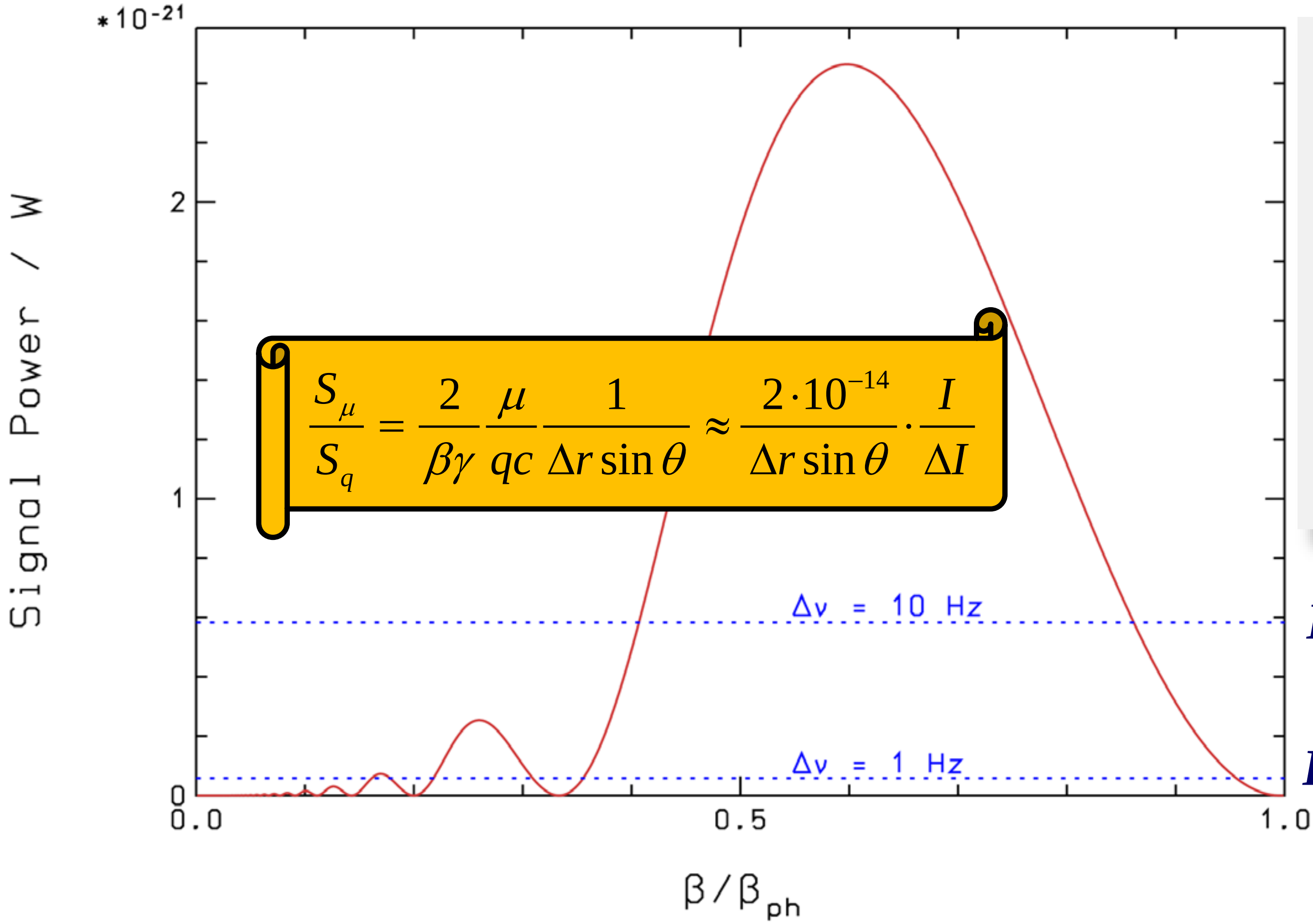
P_{noise} @ 1 Hz Bwdth

Expected Signal Power: $$P_S = \left(\frac{I \cdot \eta_b}{e}\right)^2 \cdot \frac{16\mu_0\mu_e^2}{\pi^2 c^3} \cdot \frac{f\left(\beta_{ph}\right)}{F\left(j_{11}\right)} \cdot \frac{\kappa Q_0}{\left(1+\kappa\right)^2} \cdot \left(\frac{\omega_C}{\gamma}\right)^2$$



Longitudinal: TE_{011}





$$\frac{S_\mu}{S_q} = \frac{2}{\beta\gamma}\frac{\mu}{qc}\frac{1}{\Delta r \sin\theta} \approx \frac{2\cdot 10^{-14}}{\Delta r \sin\theta}\cdot\frac{I}{\Delta I}$$

Parameters:

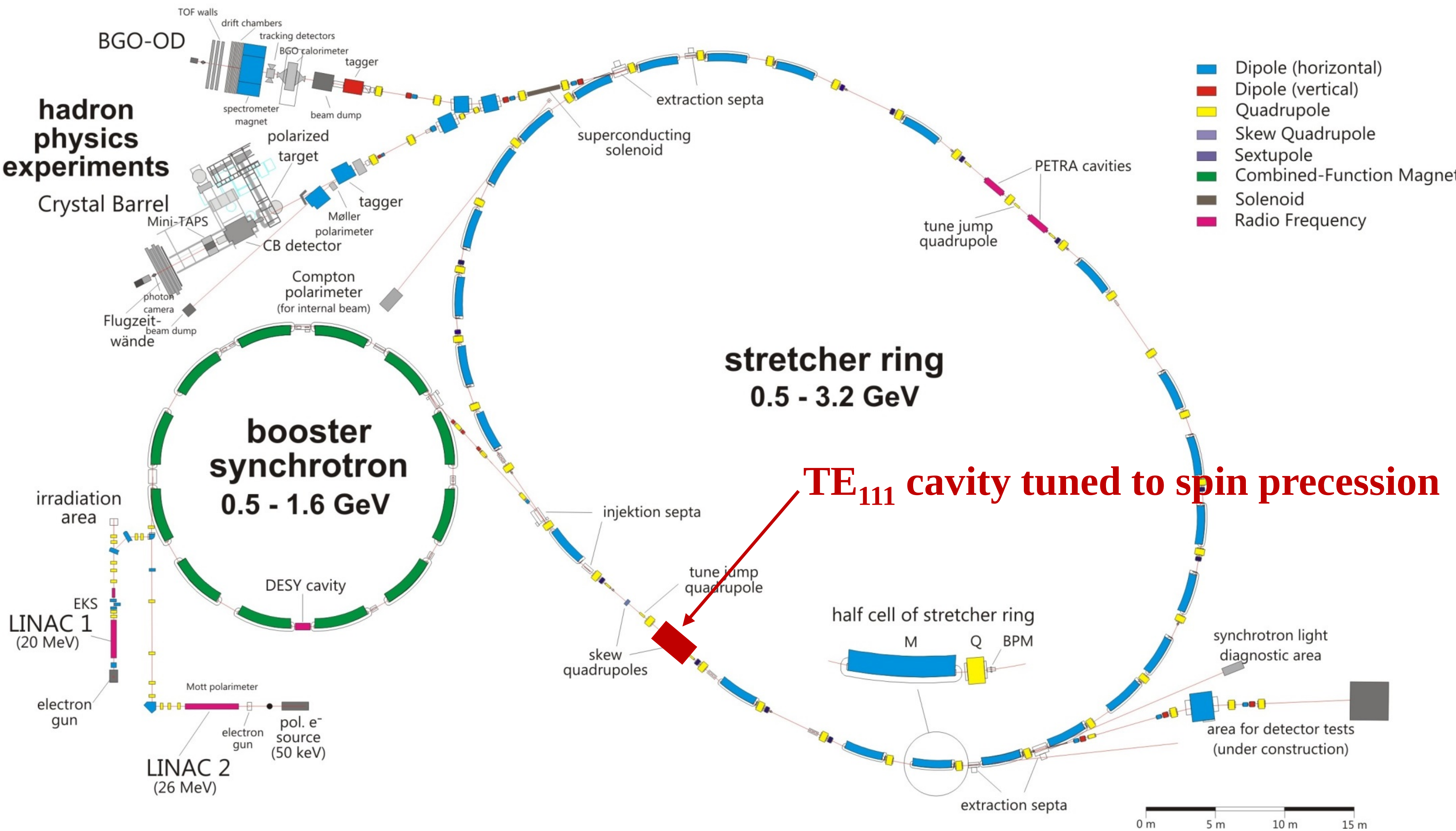
- I_e = 1 mA
- η_b = 1
- ν_C = 0.65 GHz
- Q_0 = 10^{10}
- κ = 10
- γ = 10

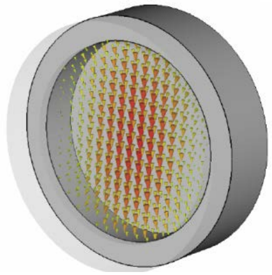
P_{noise} @ 10 Hz Bwdth

P_{noise} @ 1 Hz Bwdth

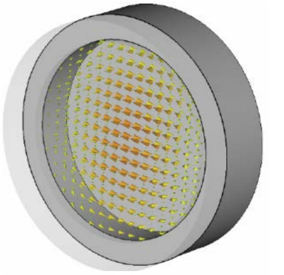
Expected Signal Power: $$P_S = \left(\frac{I\cdot\eta_b}{e}\right)^2 \cdot \frac{16\mu_0\mu_e^2}{\pi^2 c^3}\cdot\frac{f\left(\beta_{ph}\right)}{F\left(j_{11}\right)}\cdot\frac{\kappa Q_0}{(1+\kappa)^2}\cdot\left(\frac{\omega_C}{\gamma}\right)^2$$

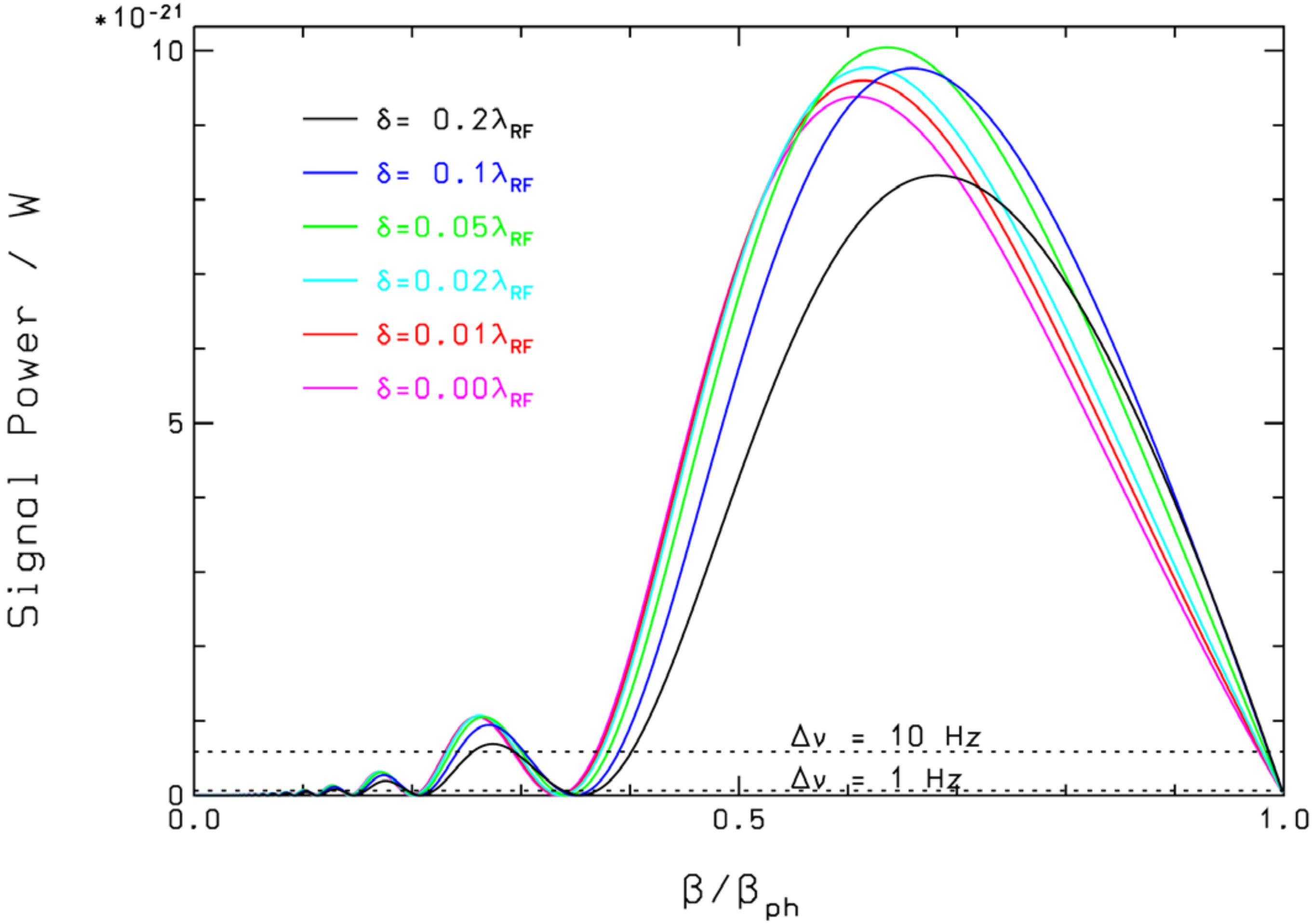
Experiment @ *ELSA*/Bonn





Transverse: TE_{111}



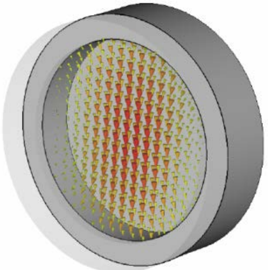


Parameters:

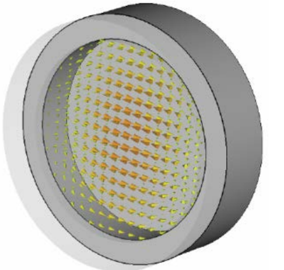
- I_e = 50 mA
- η_b = 0.9
- ν_C = 0.5 GHz
- Q_0 = 10^{10}
- κ = 10
- γ = 3018
- γG = 3.5

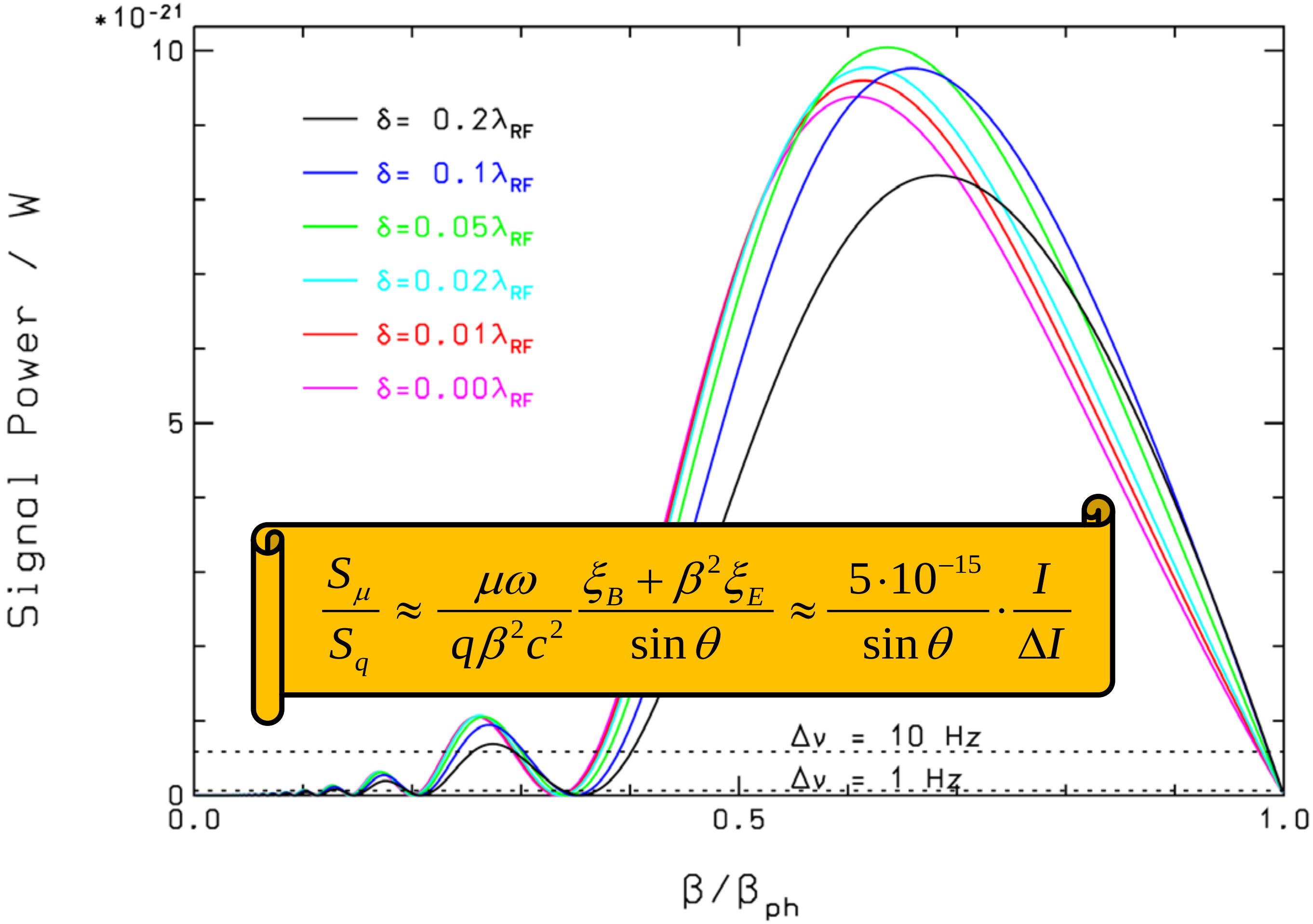
P_{noise} @ 10 Hz Bwdth

Expected Signal Power: $P_S \approx \left(\frac{I \cdot \eta_b}{e}\right)^2 \cdot \frac{32\mu_0\mu_e^2}{\pi^2 c^3} \cdot \frac{f\left(\beta_{ph}\right)}{F\left(j'_{11}\right)} \cdot \frac{\kappa Q_0}{\left(1+\kappa\right)^2} \cdot \left(G \cdot \omega_C\right)^2$



Transverse: TE_{111}





$$\frac{S_\mu}{S_q} \approx \frac{\mu\omega}{q\beta^2 c^2}\frac{\xi_B + \beta^2\xi_E}{\sin\theta} \approx \frac{5\cdot 10^{-15}}{\sin\theta}\cdot\frac{I}{\Delta I}$$

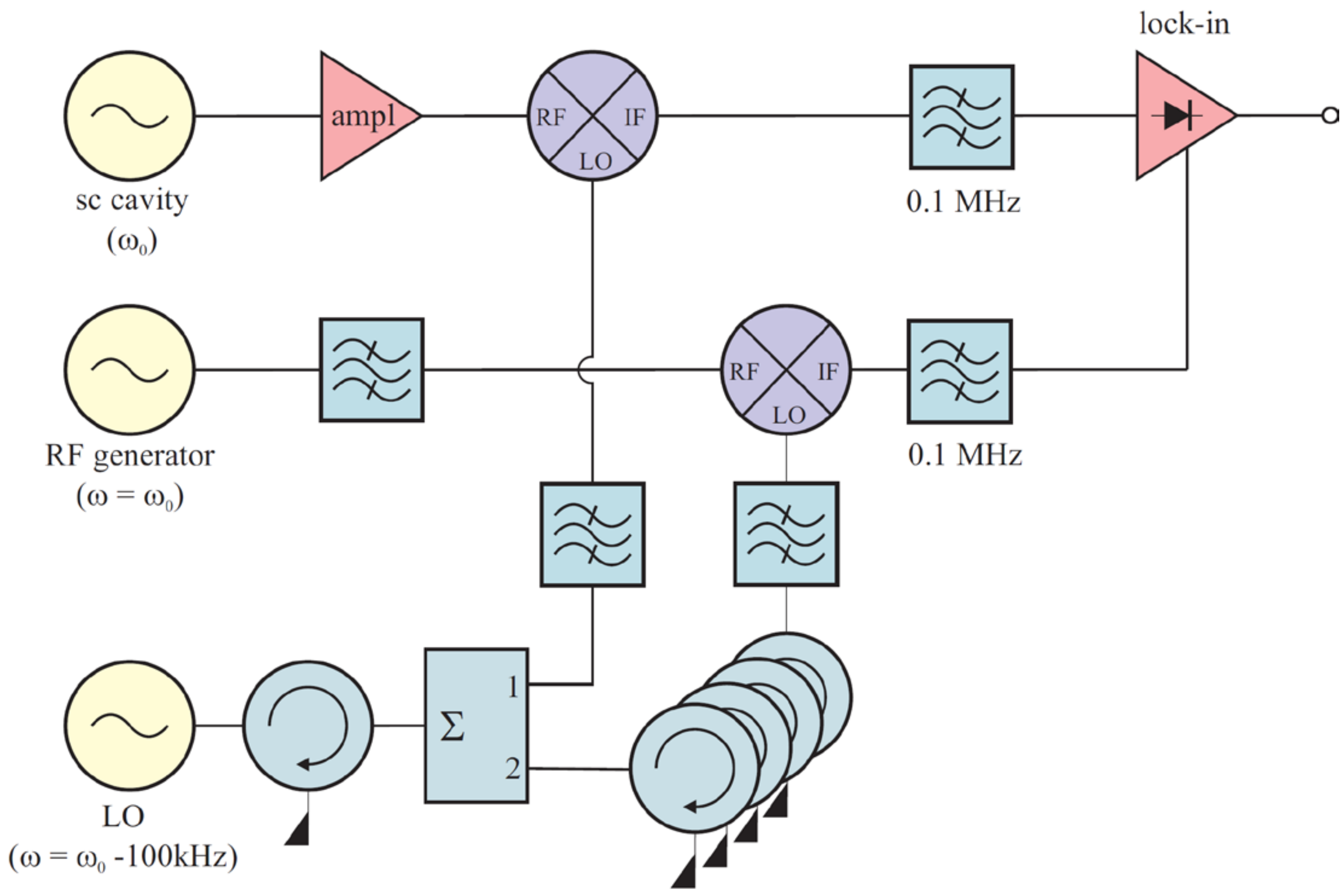
Parameters:

- I_e = 50 mA
- η_b = 0.9
- ν_C = 0.5 GHz
- Q_0 = 10^{10}
- κ = 10
- γ = 3018
- γG = 3.5

P_{noise} @ 10 Hz Bwdth

Expected Signal Power: $$P_S \approx \left(\frac{I\cdot\eta_b}{e}\right)^2 \cdot \frac{32\mu_0\mu_e^2}{\pi^2 c^3}\cdot\frac{f\left(\beta_{ph}\right)}{F\left(j'_{11}\right)}\cdot\frac{\kappa Q_0}{(1+\kappa)^2}\cdot\left(G\cdot\omega_C\right)^2$$

Low Bwdth Signal Processing



More information cf. T. Pusch et al., PRSTAB **15**, 112801 (2012)

Conclusions

- Expected signal power is extremely low!
- sc cavities ($Q_0 \approx 10^{10}$) with weak coupling essential!
- Phase-lock techniques required
- Coupling to charge is about 14 orders of magnitude greater!

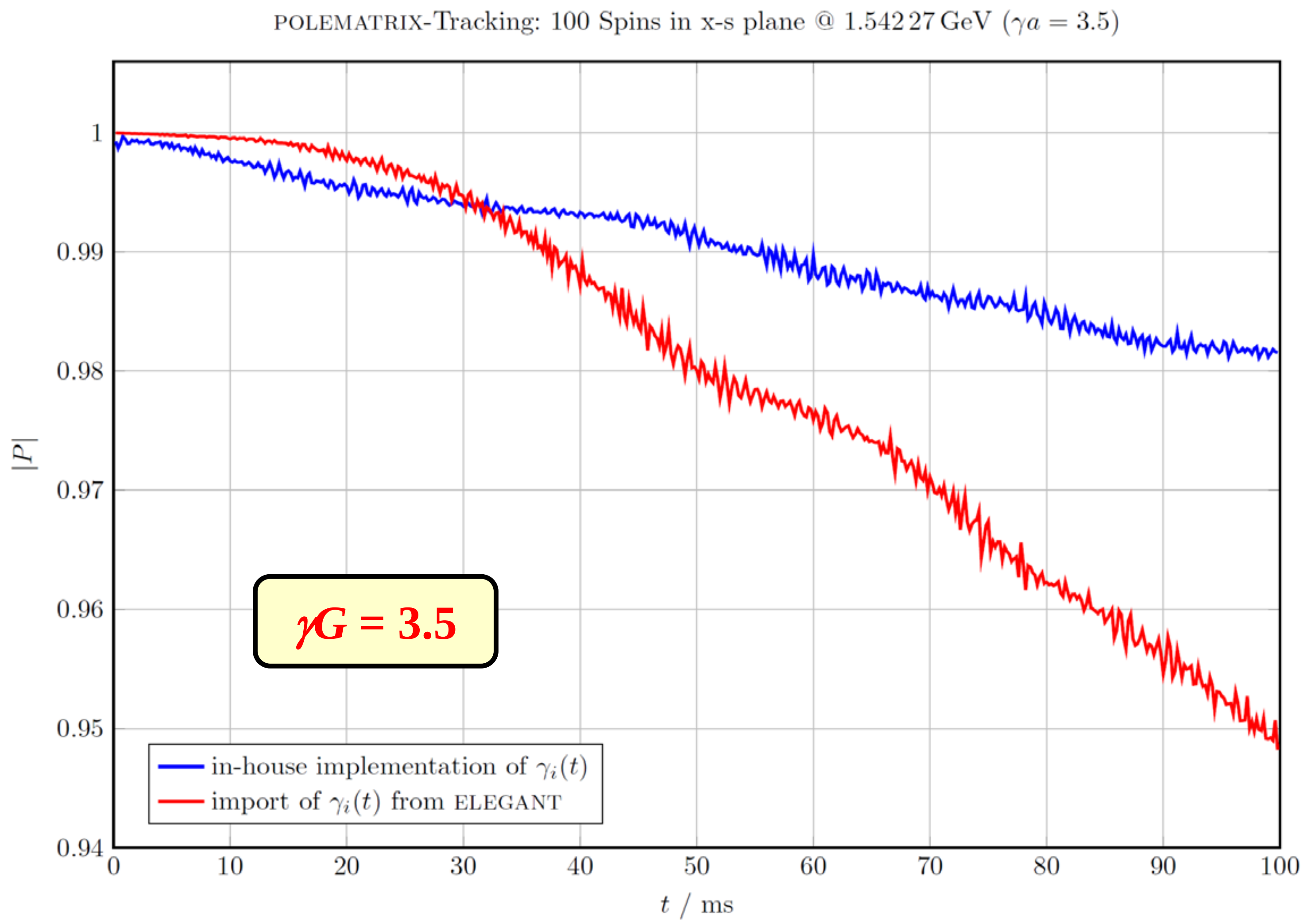
PoP will be a really hard task but doable?!

LIGO demonstrated: ultimate precision can be achieved!

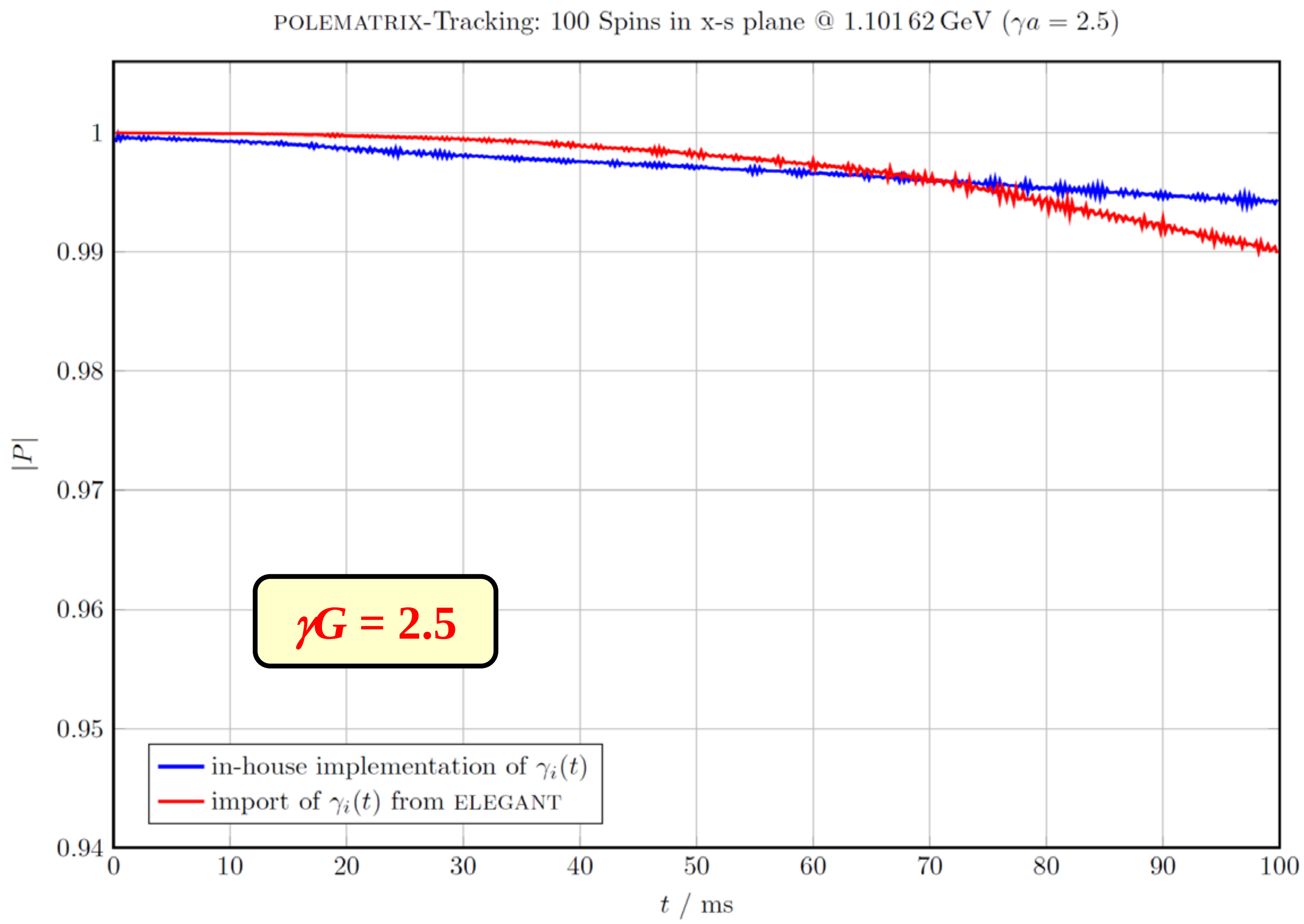
Stern-Gerlach

May the force be with us!

Spin Coherence Time @ ELSA



Spin Coherence Time @ ELSA



Spin Coherence Time @ ELSA