Seminar on mathematical aspects of theoretical physics

Sommersemester 2012

Klaus Fredenhagen
II. Institut für theoretische Physik, Hamburg

1. Algebraic formulation of quantum theory
   Advisor: Katarzyna Rejzner (katarzyna.rejzner@desy.de)

   The talk should cover the following problems:

   - Algebraic formulation of quantum theories: observables and states in QM, Heisenberg picture, measurement, expectation values, spectrum (algebraic definition).
   - Definitions and properties of the following mathematical structures: Banach algebras with involution, C*-algebras, states, bounded and unbounded operators on Hilbert spaces (with a particular stress on the application in physics).
   - Relation between states on a topological involutive algebra and Hilbert space representations (GNS theorem).

   Literature:

   - For the mathematical definitions and properties see: O. Bratteli and D. W. Robinson "Operator Algebras and Quantum Statistical Mechanics", volume I, available at: `http://folk.uio.no/bratteli/bratrob/VOL-1S~1.PDF`
   - For the physical interpretation, see the lecture notes of K. Fredenhagen available at: "Algebraic quantum field theory":
     `http://unith.desy.de/sites/site_unith/content/e20/e72/e180/e193/infoboxContent204/AQFT.pdf`
     "Superselection sectors":
     `http://unith.desy.de/sites/site_unith/content/e20/e72/e180/e193/infoboxContent203/superselect.ps.gz`
   - You can also refer to chapter III.2 of the book of Rudolf Haag "Local quantum physics".
   - A good introduction to methods of functional analysis used in physics is provided in the book of B. Simon and M. Reed, "Methods of Modern Mathematical Physics, I: Functional Analysis", Academic Press, 1980. There you can find many examples from physics, including the quantization of the position and momentum of a particle (chapter VIII.5, example 2). There is also an extensive discussion of bounded (chapter VI) and unbounded (chapter VIII) operators, including the notion of a spectrum.

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2. KMS condition
   Advisor: Andreas Degner
   (andreas.degner@desy.de)

   - Review of description of thermal states in classical and quantum statistical mechanics
   - equivalence of KMS states and canonical ensemble for finite systems
   - KMS state for the free field and its GNS representation

   Literature:

   - R. Haag: Local Quantum Physics (chapter V)
   - O. Bratteli, D. Robinson: Operator Algebras and Quantum Statistical Mechanics II (chapter 5.3)
   - R. Haag, N.M. Hugenholtz and M. Winnink: On the equilibrium states in quantum statistical mechanics. Commun. Math. Phys. 16, 81 (1970)
   - lecture notes of K. Fredenhagen on Thermodynamics and Statistical mechanics (to be found on the site `http://unith.desy.de/research/aqft` under lecture notes)

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3. Tomita-Takesaki theory: Mathematical aspects
   Advisor: Falk Lindner (falk.lindner@desy.de)

   - road/motivation to von-Neumann algebras (by commutative algebras, weak closures of C*-algebras)
   - structures on von-Neumann algebras (standard representation, states, predual)
   - modular structure / Tomita-Takesaki theory

   Literature:

BR79 O. Bratteli and D.W. Robinson. Operator algebras and quantum statistical mechanics. Vol. 1: C* and W* algebras, symmetry groups, decomposition of states. Springer, 1979.

Sak71 S. Sakai. C*-algebras and W*-algebras. Classics in mathematics. Springer, 1971.

Sum05 S. J. Summers. Tomita-Takesaki modular theory. arXiv:math-ph/0511034, November 2005.

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4. The Haag-Kastler axiomatic framework
   Advisor: Katarzyna Rejzner (katarzyna.rejzner@desy.de)

   The talk should cover the following problems:

   - Local observables and their importance in QFT, principle of locality.
   - Presentation and detailed explanation of the Haag-Kastler axioms. A particular stress should be put on physical interpretation.
   - Examples of C*-alegbraic models (CCR, CAR algebras).
   - Representations of the observables algebra, vacuum state, superselection sectors.

   Literature:

   - An extensive discussion of the algebraic formulation of QFT can be found in the book of Rudolf Haag "Local quantum physics". In particular:
     - the principle of locality: chapter I.2,
     - the net of local observables: chapters III.1, III.3,
     - axioms: chapters III.1, III.3.3.
   - Lecture notes of K. Fredenhagen "Algebraic quantum field theory", available at: `http://unith.desy.de/sites/site_unith/content/e20/e72/e180/e193/infoboxContent204/AQFT.pdf`
   - The discussion of states on the observables algebra and superselection sectors can be found in the chapter III of the book of Rudolf Haag "Local quantum physics" and in the lecture notes of Klaus Fredenhagen "Superselection sectors", available at: `http://unith.desy.de/sites/site_unith/content/e20/e72/e180/e193/infoboxContent203/superselect.ps.gz`
   - Examples of C*-algebraic quantum models (CCR, CAR algebras) can be found in the second volume of the book by O. Bratteli and D. W. Robinson "Operator Algebras and Quantum Statistical Mechanics", available at: `http://folk.uio.no/bratteli/bratrob/VOL-2.pdf` .

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5. Tomita-Takesaki theory: Physical aspects
   Advisor: Falk Lindner (Falk.Lindner@desy.de)

   - connection between vN algebras and physical systems in general [Haa92]
   - KMS-states [Haa92, BR96]
   - geometric modular action, Bisognano-Wichmann property [Bor00]
   - perturbation theory [DJP03, BR96]

   Literature:

Bor00 H.J. Borchers. On revolutionizing quantum field theory with Tomita's modular theory. J.Math.Phys., 41:3604-3673, 2000.

BR79 O. Bratteli and D.W. Robinson. Operator algebras and quantum statistical mechanics. Vol. 1: C* and W* algebras, symmetry groups, decomposition of states. Springer, 1979.

BR96 O. Bratteli and D.W. Robinson. Operator algebras and quantum statistical mechanics. Vol. 2: Equilibrium states. Models in quantum statistical mechanics. Springer, 1996.

DJP03 J. Derezinski, V. Jaksic, and C. Pillet. Perturbation theory of W*-dynamics, Liouvilleans and KMS-states. Rev.Math.Phys., 15:447-489, 2003.

Haa92 R. Haag. Local quantum physics: Fields, particles, algebras. Springer, 1992.

Sak71 S. Sakai. C*-algebras and W*-algebras. Classics in mathematics. Springer, 1971.

Sum05 S. J. Summers. Tomita-Takesaki modular theory. arXiv:math-ph/0511034, November 2005.

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6. Partial differential equations I – causal structure and PDE with constant coefficients
   Advisors: Thomas-Paul Hack and Christian Pfeifer
   (thomas-paul.hack@desy.de, christian.pfeifer@desy.de)

   Topics:

   - globally hyperbolic manifolds, causal structure
   - linear partial differential equations with constant coefficients, hyperbolic case
   - Cauchy problem
   - local solution theory

   Literature:

   - S. Benzoni-Gavage, D. Serre - Multi-dimensional Hyperbolic Partial Differential Equations. Chapters 1.1-1.2,1.4
   - R. Wald - General Relativity. Chapters 8, 10.1
   - J. Rauch - Hyperbolic Partial Differential Equations and Geometric Optics. Chapters 1, 2
   - L. Hörmander - The Analysis of Linear Partial Differential Operators II. Chapter 12

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7. Partial differential equations II – PDE with variable coefficients and wave equation on manifolds
   Advisors: Thomas-Paul Hack and Christian Pfeifer
   (thomas-paul.hack@desy.de, christian.pfeifer@desy.de)

   Topics:

   - linear partial differential equations with variable coefficients, hyperbolic case
   - wave equation on manifolds, global solutions theory (only results, no proofs), Green's functions

   Literature:

   - S. Benzoni-Gavage, D. Serre - Multi-dimensional Hyperbolic Partial Differential Equations. Chapter 2
   - Ch. Bär, N. Ginoux, F. Pfäffle - Wave Equations on Lorentzian Manifolds and Quantization `http://arxiv.org/abs/0806.1036`. Chapter 3
   - L. Hörmander - The Analysis of Linear Partial Differential Operators III. Chapter 23

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8. Unruh effect
   Advisors: Thomas-Paul Hack and Christian Pfeifer
   (thomas-paul.hack@desy.de, christian.pfeifer@desy.de)

   - Static spacetimes
   - States on static spacetimes
   - Rindler spacetime

   Literature:

   - K. Fredenhagen script (translated)
   - R. Wald - Quantum field theory in curves spacetimes and black hole thermodynamics. Chapter 4.3, 5

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9. Hawking radiation
   Advisors: Thomas-Paul Hack and Christian Pfeifer
   (thomas-paul.hack@desy.de, christian.pfeifer@desy.de)

   - Schwarzschild spacetime
   - Unruh state
   - Hawking radiation

   Literature:

   - R. Wald - The Thermodynamics of Black Holes (`http://relativity.livingreviews.org/Articles/lrr-2001-6`)
   - R. Wald - Quantum field theory in curves spacetimes and black hole thermodynamics. Chapter 7
   - R. Wald - General Relativity. Chapter 6

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10. Quantum energy inequalities
    Advisor: Andreas Degner
    (andreas.degner@desy.de)

    - negative energy densities in QFT as a general feature
    - quasifree Hadamard states and their characterisation via wave front sets (microlocal spectrum condition, motivation from Minkowski space example)
    - rigorous definition of Wick products (and stress–energy tensor) via point–splitting approach
    - Fewsters general wordline quantum inequality
    - states of low energy on FRW spacetimes

    Literature:

    - lecture notes of Claudio Dappiaggi: Tools of quantum field theory over curved backgrounds (you can google it): In chapter 3.3 you find a good introduction into the concepts of Hadamard states and microlocal analysis
    - L. Hörmander - The Analysis of Linear Partial Differential Operators, volume I
    - C.J. Fewster: A general wordline quantum inequality. Class. Quant. Grav., 17:1897-1911, 2000
    - H. Olbermann: States of Low Energy on Robertson–Walker spacetimes. Class. Quant. Grav., 24:5011-5030, 2007