Problem 8.1

a) Compute $\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2$ for the scattering process

$$e^-(p_1)\ \mu^-(p_2) \to e^-(p_1')\ \mu^-(p_2')$$

in the limit $m_e \approx 0$ and show

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{8e^4}{(p_1-p_1')^4}\Big(p_1\cdot p_2\ p_1'\cdot p_2' + p_1\cdot p_2'\ p_1'\cdot p_2 - m_\mu^2\ p_1\cdot p_1'\Big)$$

b) Choose $p_1 = (k,0,0,k), p_2 = (E,0,0,k)$ in the center-of-mass frame and show

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{2e^4}{k^2(1-\cos\theta)^2}\Big((E+k)^2 + (E+k\cos\theta)^2 - m_\mu^2(1-\cos\theta)\Big)\ ,$$

where θ is the scattering angle.

c) Compute the cross section $(\frac{d\sigma}{d\Omega})_{\text{CM}}$ by first showing

$$\Big(\frac{d\sigma}{d\Omega}\Big)_{\text{CM}} \ = \ \frac{1}{64\pi^2(E+k)^2}\ \frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2$$

where $|v_A - v_B| = 1 + k/E$ holds.

d) Show that in the high-energy limit $k \gg m_\mu \approx 0$ one obtains

$$\Big(\frac{d\sigma}{d\Omega}\Big)_{\text{CM}} \ = \ \frac{\alpha^2}{2E_{\text{CM}}^2(1-\cos\theta)^2}\left(4 + (1+\cos\theta)^2\right)\ .$$

What is the result in the limit $\theta \to 0$?

Problem 8.2

Show

$$\text{Tr}\big[(\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu)(\not{p}+m)(\gamma_\nu \not{k}\gamma_\mu + 2\gamma_\mu p_\nu)(\not{p}'+m)\big] = 32\big((p'k)(pk) - m^2(p'k - 2pk - pp') + 2m^4\big)\ .$$

Problem 8.3

a) For the scattering process

$$f_1(p_1) + f_2(p_2) \to f_3(p_3) + f_4(p_4)$$

one defines Mandelstam variables by

$$s := (p_1 + p_2)^2 \ , \qquad t := (p_3 - p_1)^2 \ , \qquad u := (p_4 - p_1)^2 \ .$$

Show that

$$s + t + u = \sum_{i=1}^{4} m_i^2 \ .$$

holds.

b) Express $\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$ for the scattering process

$$e^-(p_1)\ \mu^-(p_2) \to e^-(p_1')\ \mu^-(p_2') \ ,$$

which was computed in problem 8.1, in terms of Mandelstam variables in the high-energy limit $m_\mu \approx m_e \approx 0$.

c) Express $\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$ for the scattering process

$$e^-(p_1)\ e^+(p_2) \to \mu^-(p_1')\ \mu^+(p_2') \ ,$$

which was computed in class, in terms of Mandelstam variables in the high-energy limit $m_\mu \approx m_e \approx 0$.

d) Which formal operation relates the two expression from 7.2b) and 7.2c)?

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The QFT-team wishes Merry Christmas and a successful 2011 !!

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