Problem 5.1

a) Consider the scalar potential of $N = 1$ supergravity in the form

$$V = e^G\left(G^{i\bar{\jmath}}G_iG_{\bar{\jmath}} - 3\right) , \qquad G = K + \ln|W|^2$$

Compute the mass matrices in a Minkowskian background and show

$$M^2_{i\bar{\jmath}} = \langle (D_iG_k\bar{D}_{\bar{\jmath}}G^k - R_{i\bar{\jmath}k\bar{l}}G^kG^{\bar{l}} + G_{i\bar{\jmath}})e^G\rangle , \qquad M^2_{ij} = \langle (G^kD_iD_jG_k + D_iG_j + D_jG_i)e^G\rangle ,$$

where $D_iG_j = \partial_iG_j - \Gamma^k_{ij}G_k,\ D_iG_{\bar{\jmath}} = G_{i\bar{\jmath}}$.

Hint: : Use $D_iV = 0$ as the condition for a Minkowski minimum.

b) Show that the fermionic mass matrix $m_{ij} = \langle (D_iG_j + \frac{1}{3}G_iG_j)\rangle m_{3/2}$ always has a zero eigenvalue and explain its physical interpretation.

Hint: Find an appropriate null-eigenvector.

Problem 5.2

The Polonyi model is defined by

$$K = \phi\bar{\phi} , \qquad W_P = m^2(\phi + \beta) , \qquad m, \beta \in \mathbb{R}$$

a) For which β is supersymmetry spontaneously broken?

b) Check that $\kappa\phi = \pm(\sqrt{3} - 1)$, $\kappa\beta = \pm(2 - \sqrt{3})$ is a Minkowskian extremum of the potential V.

c) Compute the gravitino mass and $\langle F_\phi\rangle$.

Problem 5.3

Consider the situation where an observable sector is coupled to the Polonyi model with

$$K = \phi\bar{\phi} + Q^I \bar{Q}^I \ , \qquad W = \tfrac{1}{2} m_{IJ}\, Q^I Q^J + \tfrac{1}{3} Y_{IJL}\, Q^I Q^J Q^K + W_P(\phi) \ ,$$

where Q^I are the fields of the observable sector, m_{IJ}, Y_{IJL} are constant and W_P is given in problem 5.2.

a) Compute the soft scalar masses and the A and B terms assuming that $\langle F_\phi \rangle$ is the only non-vanishing F-term. Are they universal?

b) Compute the soft gaugino masses for the two cases of a gauge kinetic function $f(\phi)$ and $f =$ constant. Are they universal?

Hint: Use the formulas given in section 15.3 of the lecture notes.

Problem 5.4

a) Rewrite the $N = 2$ Kähler potential, which for one field a is given by

$$K = i\,(\bar{a}_D a - a_D \bar{a}) \ , \qquad a_D = \partial_a F(a)$$

in the form

$$K = i \bar{V}^T \Omega V \ , \qquad \text{for} \qquad V = \begin{pmatrix} a_D \\ a \end{pmatrix}$$

and determine the 2×2 matrix Ω.

b) Show that K invariant under the transformation

$$V \to SV \ , \qquad \text{where} \qquad S = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \ , \qquad ad - bc = 1 \ , \quad a, b, c, d \in \mathbb{R} \ .$$

c) The complex gauge coupling τ transforms as

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d} \ .$$

Consider the two cases $a = d = 1, c = 0$ and $a = d = 0, b = -c = 1$. What is their physical significance in terms of τ and V?