String inspired soft terms and the Higgs mass in the NMSSM

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Abstract

In this thesis we consider different patterns of soft terms in the next-to minimal supersymmetric Standard Model. Soft terms rely on the mechanism that mediates supersymmetry breaking at high scales and, in particular, determine the Higgs mass and spectrum of superpartners. The set of soft terms studied display features of effective theories at the GUT scale inspired by heterotic models in four dimensions. Firstly, we study the dilaton dominated scenario as an example that can explain the origin of universal soft terms. In particular, we review the dilaton domination in the MSSM and find that consistency with the Higgs mass sets a lower bound on the superpartners masses. When introducing the singlet extension the Higgs receives an additional contribution to its mass that can relax this bound and we show regimes of parameter space that illustrate both situations. In addition, we investigate gaugino mediated scenarios. These realize non-universalities in the soft masses which are phenomenologically viable. In particular, we look for a relation between the soft parameters that leads to a small hierarchy between the supersymmetry breaking scale and the electroweak scale. We discuss a simple example that could yield such a relation as a possible way of solving the little hierarchy problem. Finally, we address the opposite and intriguing possibility of having supersymmetry breaking at high scales (M_s). With the Standard Model as the low energy effective theory, the computation of the Higgs mass depends on the matching conditions of the quartic coupling at M_s. In particular, we show that for the measured value of the Higgs mass M_s can take any value up to the GUT scale. Furthermore, we find that universal soft terms at the GUT scale favor supersymetry-breaking scales close to the GUT scale.

Zusammenfasung

In dieser Dissertation untersuchen wir verschiedene Muster von weichen Terme in dem minimalen supersymmetrischen Standard Modell mit Singlet-Erweiterung (NMSSM). Die weichen Terme sind abhängig vom Mechanismus, der Supersymmetrie bricht, und bestimmen die Higgs-Masse und das Spektrum der Superpartner. Die Beispiele von weichen Terme, welche wir untersuchen, stellen Eigenschaften von effektiven Theorien an der GUT-Skala dar, die von heterotischen Modellen in vier Dimensionen inspiriert sind. Zunächst studieren wir das Szenarien, in dem das Dilaton die Supersymmetrie-Brechung dominiert. In diesem Fall erhählt man universelle weiche Terme. Insbesondere besprechen wir diese Szenarien im Kontext des MSSM und zeigen, dass Konsistenz mit der Higgs-Masse eine untere Schranke für die Massen der Superpartner impliziert. Im Fall der Singlet-Erweiterung erhält die Higgs-Masse zusätzliche Korrekturen, welche diese Schranke abschwächen können und wir zeigen explizite Parameterbereiche, welche beide Situationen illustrieren. Zusätzlich untersuchen wir Szenarios mit Gaugino Mediation. Insbesondere suchen wir nach einer Relation zwischen den weichen Termen, welche zu einer kleinen Hierarchie zwischen der Skala der Supersymmetrie-Brechung und der elektroschwachen Skala führt. Wir diskutieren ein einfaches Beispiel, welches die notwendinge Struktur aufweist und somit das kleine Hierarchieproblem löst. Zuletzt behandeln wir die Möglichkeit, dass Supersymmetrie bereits an einer hohen Skala M_s gebrochen ist. Mit dem Standardmodell als effektive Feldtheorie bei niedrigen Energien hängt die Berechnung der Higgsmasse von der Übereinstimmung der quartischen Kopplung bei M_s ab. Insbesondere belegen wir, dass für die gemessene Higgs-Masse M_s jeden beliebigen Wert bis zur GUT-Skala annehmen kann. Weiterhin finden wir dass universelle weiche Terme an der GUT-Skala Supersymmetrie-Brechung an der GUT-Skala bevorzugen.

Contents

Chapter 1

Introduction

The Standard Model synthesizes our present comprehension of Nature at the microscopic level. In particular, it introduces the electroweak and strong interactions together with the observed particles in the framework of quantum field theory. The success of its predictions through the past years has been spectacular and culminated with the discovery of a 125GeV Higgs boson [1, 2]. The LHC is the vanguard laboratory that tests the Standard Model to high precision and there is so far no proof of physics beyond it.[1] Note that the Standard Model excludes gravity from its framework. Gravitational interactions are instead addressed within General Relativity and they become relevant at scales near

$$M_p = 2.435 \cdot 10^{18}\text{GeV}\,. \tag{1.1}$$

M_p denotes the Planck scale and constitutes the intrinsic cutoff of the Standard Model.

Despite its achievements, there are reasons to believe that the Standard Model is not complete. The most compelling argument is the so called *hierarchy problem* [5–7], let us briefly outline it. The Higgs boson is a scalar field and, therefore, receives quantum corrections to its mass which are not protected in size by any symmetry. Note that if fermion masses are set to vanish the Standard Model enjoys an exact chiral symmetry. This, in turn, forbids the generation of masses by quantum corrections. Fermion masses M_f explicitly break the chiral symmetry but radiative effects are proportional to M_f and, thus, remain stable to all orders. In particular, the corrections for a scalar field involve quadratic divergences of the form [6, 7]

$$M_h^2 \simeq m_0^2 + (\lambda - y^2)\Lambda^2 \tag{1.2}$$

[1] At the end of 2015 the LHC reported a 750GeV diphoton resonance at 2σ level [3, 4]. However, the data is so far preliminary and the significance is not sufficient to drive any conclusions. The work in this thesis is prior to this announcement and therefore, we neglect the presence of this plausible experimental signature. If promoted to a discovery, some of the conclusions in this thesis might change.

where m_0 is the bare mass, λ parametrizes quartic self-interactions and y parametrizes Yukawa type of interactions with fermions. Λ can be identified with the cutoff scale of the Standard Model.[2] Recall that in the case one extrapolates the Standard Model up to arbitrary high energies, Λ is automatically set at M_p. Since the corrections in (1.2) grow quadratically with Λ, as long as $\Lambda \gg M_h$ one requires a large fine-tuning in the r.h.s of (1.2) to reproduce the (light) experimentally measured Higgs mass. This fact is known as the hierarchy problem. Furthermore, Λ is bounded from below at the TeV scale. This implies that cancellations are at least of order 10^2 and they dramatically increase when raising Λ. In the limit of $\Lambda \to M_p$ the cancellations become worse by a factor of 10^{30}.

The hierarchy problem is considered to be one of the main reason to regard Supersymmetry as a fundamental symmetry in Nature. The presence of Supersymmetry implies that quadratic radiative corrections in (1.2) systematically cancel to all orders. Supersymmetry transforms bosons into fermions, hence, to respect the invariance under supersymmetry transformations, couplings are constrained. In particular, λ and y obey $\lambda = y^2$ and the masses of scalar and fermionic fields are identical i.e. $M_h = M_f$. However, the situation of degenerate masses between the Higgs scalar and hypothetical fermionic partners (usually denoted as superpartners) is experimentally excluded. Therefore, supersymetry cannot be an exact symmetry at low energies. Supersymmetric theories, thus, introduce the so called *soft terms*. These break supersymmety explicitly while keeping the Higgs mass free of quadratic divergences [8]. In particular, soft terms give masses to the superpartners and, thus, set the scale M_s at which new particles should be found.

Soft terms appear in the Higgs potential and, hence, participate in electroweak symmetry breaking. Thus, in order not to restore the hierarchy problem, M_s was historically expected to lie at the electroweak scale ($M_{\rm ew}$) or at most in the TeV range. With M_s in this regime, supersymmetric theories have appealing consequences. In particular, the minimal supersymmetric Standard Model (MSSM) predicts gauge coupling unification at the scale $M_{\rm GUT} := 10^{16}$GeV and provides a dark matter candidate, for a comprehensive review see [9]. Furthermore, the Higgs mass measurement is consistent with the predicted value. Specifically, in the MSSM the Higgs mass at tree level is bounded by the Z-boson mass and therefore large radiate corrections are necessary to achieve the value of 125 GeV. In turn, these set $M_s \simeq$ TeV strongly favoring the appearance of superpartners in this regime. However, no supersymmetric particle was yet found and the absence of signatures in the LHC up to a few TeV [10, 11] has brought the assumption of low supersymmetry breaking scale into debate. More importantly, with the present bounds on the soft masses most models suffer from a *little hierarchy problem* [12, 13]. Singlet extensions of the minimal setup, denoted as the Next-to-Minimal Supersym-

[2] Eq.(1.2) is the most general correction one can have for a renormalizable theory of one scalar field and a Weyl fermion field in four dimensions.

metric Standard Model (NMSSM) [14], are regarded as promising alternatives to overcome the aforementioned little hierarchy problem. In particular, the Higgs mass in this framework receives an additional tree level correction which can alleviate the bound on M_s. However, the special corners of parameter space often require a fine-tuning of the soft parameters and, therefore, restore the little hierarchy. Despite the rich phenomenology of the NMSSM, the tension on the soft scale prevails.

Recently, proposals with supersymmetry breaking at very high scales have gained interest. These are denoted as *High-scale Supersymmetry* models [15–17]. Of course, these frameworks do not solve the hierarchy problem, as supersymmetric theories meant. Recall that in these scenarios superpartners are beyond experimental reach and to obtain a light Higgs in the spectrum a huge fine-tuning of soft parameters is required. The origin of the electroweak scale in this case is justified by the anthropic principle [18]. Although this argument is highly speculative, there are additional reasons to support supersymmetric realizations which are not necessarily tied to the size of the supersymmetry breaking scale. In particular, supersymmetry is present in theories derived from string theory, the prime candidate for a quantum theory of gravity. Furthermore, present data confirmed that the SM Higgs potential becomes metastable at large energy scales [19] and supersymmetric embeddings can stabilize the electroweak vacuum.[3] More interestingly, models with high-scale supersymmetry breaking yield predictions for the Higgs quartic coupling and, hence, for Higgs mass. In turn, the latter can set upper bounds on the susy breaking scale and, therefore, particular examples can be tested. In addition, let us remark that gauge coupling unification and the presence of a dark matter particle do not necessarily imply that supersymmetry-breaking is at a low scale. Split-susy models are specific examples of supersymmetric models that incorporate both features [23, 24].

In addition to addressing the size of the soft parameters, let us discuss about their origin. The structure of the soft terms induces a large degree of arbitrariness when studying phenomenological aspects, and the choices are generically biased by experimental bounds. The nature of the soft terms entirely relies on the mechanism that triggers supersymmetry breaking. The standard approach is to introduce hypothetical (*hidden*) fields, responsible for supersymmetry breaking, that couple to the low energy (*observable*) fields only via non-renormalizable interactions. Surprisingly, when computing the low energy action for the observable fields these couplings take precisely the form of the soft terms defined above. This result traces a link between the low energy phenomenology, parametrized by the soft terms, and the UV dynamics which determine their form. In particular, in supergravity the non-renormalizable interactions are generated by gravitational effects and the respective mechanism to develop soft terms received the name of *gravity mediation*. The derivation of the soft terms in gravity mediated scenarios was computed in [25–28] and the outcome is presented in terms of the

[3] See [20–22] for earlier results on the stability of the electroweak potential.

couplings in the original supergravity Lagrangian. The derivation of the soft terms for the special regime of a global (non-renormalizable) supersymmetric theories in the spirit of [27] was presented in [29]. One particular example of this mediation mechanism is known as *gaugino mediation* [30, 31].[4] These mechanisms, however, generically allow for non-universal soft terms. In particular, the non-universality of soft masses implies that the flavor structure of the theory is not protected. Thus, they can introduce flavor mixing soft scalar masses which are experimentally severely constrained [32].[5] Theoretically there is no reason to forbid them. Therefore, the choice of diagonal (and identical) soft masses seems a rather ad-hoc postulate. In sum, the structure of soft parameters derived for different mediation mechanisms is accompanied by extra assumptions only justified on phenomenological grounds. This fact can be only alleviated by embedding the theory in a UV-completion and the prime candidate is string theory.

The low energy effective actions of certain string theories are described by various 10-dimensional supergravities, which, after compactification yield, in particular, 4D theories with $\mathcal{N} = 1$ supergravity. The fields that constitute the spectrum schematically fall into two classes: moduli fields and matter fields. The latter could, in principle, convey the field content of supersymmetric extensions of the Standard Model. The former often denote flat directions of the scalar potential. Generically, additional data in the compactification is included, e.g. fluxes and non-perturbative effects to stabilize the moduli which also trigger supersymmetry breaking. This setup compel us to identify the moduli and matter fields with the hidden and observables fields respectively. Of course, in order to discuss plausible embeddings of supersymmetric extensions of the Standard Model both the fields and the couplings should be derived from the specific compactifications of the string theory considered. For an exhaustive reference on string phenomenology see [33]. However, even lacking the details of the supersymmetry breaking mechanism it is possible to investigate some model-independent statements on the structure of the soft terms by introducing generic assumptions and making use (or abuse) of general properties of the effective actions in string compactifications. This approach will prevail throughout the work in this thesis which we outline as follows.

In this thesis we investigate the phenomenological implications of particular sets of soft terms within the NMSSM. The soft terms chosen in each case highlight particular aspects of effective theories in four dimensions inspired by heterotic string compactifications. In particular, we study the situation where the dilaton field is responsible for supersymmetry breaking. This is known as the *dilaton dominated scenario* [27, 34, 35]. The dilaton field belongs to the class of the moduli fields and appears at leading order in the Lagrangian with specific couplings

[4] Gauge mediation is an additional example of this class of theories, however, it requires a particular gauge structure and additional matter.

[5] Gaugino mediation can suppress flavor mixing but relies on additional features of the theory, in particular, the presence of extra dimensions and the convenient localization of fields.

which do not rely on the compactification involved. Therefore, the predictions obtained in the dilaton dominated setup involve a large class of heterotic models. Furthermore, we anticipate that the soft terms in this scheme are universal and hence, phenomenologically appealing. The dilaton domination is usually realized by assuming that gaugino condensation induces a non-trivial potential for the dilaton that breaks supersymmetry. In particular, gaugino condensation is a non-perturbative effect which can generate a hierarchy between the gravitino mass i.e. the soft scale and the Planck scale. In turn, low energy supersymmetry is possible in this framework. Assuming the gravitino mass can take values of $\mathcal{O}(\mathrm{TeV})$, we study the phenomenology of the dilaton domination in both the MSSM and NMSSM [36]. More specifically, following [36] we calculate the Higgs mass and the supersymmetric spectrum. Using the present experimental constraints we derive the respective bounds on the soft parameters. The phenomenology of dilaton domination in the MSSM was previously studied in [34, 35, 37], we review some of the earlier results and introduce the singlet extended version.

In addition, we discuss soft terms obtained under the assumption that the NMSSM is embedded in a higher dimensional orbifold grand unified theory (GUT) [38–44]. Higher dimensional theories are non-renormalizable and therefore can be only regarded as effective theories with an intrinsic cutoff Λ. As long as $\Lambda < M_{p,d}$ they can be approximated as non-renormalizable global supersymmetric theories.[6] In the heterotic string, these theories appear as special intermediate regimes in asymmetric orbifold compactifications. Without specifying the mechanism that introduces the scale Λ as long as $\Lambda > M_{\mathrm{GUT}}$ the effective theory enjoys the features stated above.[7] Explicit realizations of higher dimensional orbifold GUTs in the heterotic string were derived in [45–49]. In this work following [29] we consider a special structure of soft terms which only depend on the localization of fields in the extra dimensions and study the respective phenomenology. In addition we address the question of whether a special relation between the soft parameters can solve the little hierarchy problem. Certainly, such a relation should be derived from the UV-theory and slight variations of this condition completely spoil the outcome. We use the example in [50] to show an explicit case that realizes this possibility and, even though the assumptions are speculative, it motivates the conjecture that the little hierarchy problem could be an artifact of our ignorance.

Finally, we study the implications of universal set of soft terms in the regime of high supersymmetry breaking scales. The stabilization of the moduli generically leads to supersymmetry breaking at high-scales. Therefore, it is interesting to address whether it is possible to make predictions and eventually test this class of models. The determination of the Higgs mass in high-scale supersymmetry yields bounds on the parameters of the supersymmetric theory, in particular on the scale of supersymmetry breaking. The study of high-scale supersymmetry

[6] Here $M_{p,d}$ denotes the Planck mass in the d-dimensional theory.

[7] Strictly speaking the bound is $\Lambda > R^{-1}$, with R the size of the extra dimension. Here and throughout this work we assume $R^{-1} = M_{\mathrm{GUT}}$.

within the MSSM was done in [15–17, 51–53]. There it was found that the quartic coupling can only take non-negative values. In turn, the latter implies that the supersymmetry breaking scale is bounded from above by $\simeq 10^{10}$GeV. In this work, following [54] we redo the analysis for the NMSSM. Interestingly, the coupling of the singlet to the Higgs introduces additional contributions to the quartic coupling which can severely modify the size of the Higgs mass as a function of M_s. In particular, we show that in the singlet-extended MSSM the susy-breaking scale can take values up to M_{GUT}.

This thesis is organized as follows. In chapter 2 we introduce the notation and relevant formulae in supersymmetric theories and in chapter 3 we review the Next-to-Minimal Supersymmetric Standard Model. The dilaton dominated scenario is studied in chapter 4 followed by the gaugino mediated scenario analyzed in chapter 5. The calculation of the soft terms in this case implements a model-independent derivation given in appendix A. In chapter 6 we investigate high-scale supersymmetry in the NMSSM. The NMSSM renormalization group equations used in the computations can be found in appendix B. Finally, in chapter 7 we present the conclusions. As already emphasized the work in this thesis is based on the following publications [29, 36] and [54]. While the last was written by the author herself, the treatment of the first two tries to highlight the author's own contribution to those references. Let us remark that the author participated in another two collaborations [55, 56], some of the results in [55] will be mentioned in this thesis but will not be expanded in any detail. The work in [56] will not be presented here.

Chapter 2

Supersymmetric Theories

It is the purpose of this chapter to set the notation and display the relevant formulae that will be used throughout this thesis. In the first part following [9, 57] we present the Lagrangian of global supersymmetric theories together with the soft supersymmetry breaking terms. In the second part, using [57, 58] we write down the supergravity Lagragian and following [27, 28] we give the explicit form of soft terms generated by the gravity mediation mechanism. In addition, based on [55] we give a brief outlook on a special class of supergravity theories known as no-scale models.

2.1 $\mathcal{N} = 1$ Global supersymmetry

Coleman and Mandula show that the most general symmetry in a QFT is a direct product of the Poincare algebra with an internal symmetry [59]. The novelty of supersymmetry is that it incorporates fermionic fields as generators of an hypothetical symmetry. The corresponding commutation relations given as follows were worked out in [60], and constitute the supersymmetry algebra

$$
\begin{aligned}
&\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu\,, &&\{Q_\alpha, Q_\beta\} = 0 = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\}\\
&[\bar{Q}_{\dot{\alpha}}, P_\mu] = 0\,, &&[Q_\alpha, P_\mu] = 0\\
&[Q_\alpha, L^{\mu\nu}] = \tfrac{1}{2}(\sigma^{\mu\nu})_\alpha^\beta Q_\beta\,, &&[\bar{Q}_{\dot{\alpha}}, L^{\mu\nu}] = \tfrac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \bar{Q}_{\dot{\beta}}\,.
\end{aligned}
\tag{2.1}
$$

Here Q_α are the generators of supersymmetry which transform as Weyl fermions and $P_\mu, L^{\mu\nu}$ are the generators of the Poincare algebra. The indices $\mu, \nu = 0, ..3$ are space-time indices, $\alpha, \beta = 1, 2$ label the components of the Weyl fermions and $\dot{\alpha}, \dot{\beta} = 1, 2$ the respective complex conjugates. Finally, $\sigma^\mu = (-\mathbb{I}, \sigma^i), \bar{\sigma}^\mu = (-\mathbb{I}, -\sigma^i)$ where σ^i denote the Pauli matrices and $\sigma^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)$.

By means of the supersymmetry algebra one can construct the massless (and massive) representations, called *multiplets*, of a supersymmetric theory in four dimensions. The corresponding $\mathcal{N}=1$ massless representations are denoted as *chiral* multiplets and *vector* multiplets. The chiral multiplet contains a spin-0 and a spin-1/2 components while the vector multiplet has a spin-1 and a spin-1/2 components. More importantly, using the susy algebra one can show that if supersymmetry is unbroken, the masses of the fermionic and bosonic components of the same multiplet are degenerate. This means that supersymmetry cannot be an exact symmetry at low energies. The construction of (renormalizable) supersymmetric theories that incorporate spontaneous supersymmetry breaking are, however, not phenomenologically viable (see section 2.1.1). Therefore, one is compelled to introduce interactions in the Lagrangian that break supersymmetry explicitly. In particular, the class of operators included are those that do not introduce quadratic divergences. These are called *soft terms* and preserve the most appealing feature of supersymmetric embeddings of the Standard Model as explained in the introduction. In the next section we give the explicit form of the most general renormalizable supersymmetric Lagrangian for chiral multiplets coupled to vector multiplets and in section 2.1.2 we write down the corresponding soft supersymmetry breaking terms.

2.1.1 Super Yang-Mills theories

Let us begin this section by setting the notation for the respective quantum field representations of the multiplets defined in the previous section. In particular, the chiral multiplet has a spin-0 and spin-1/2 components which are realized by a complex field ϕ and a Weyl fermion ψ_α. In addition, the massless vector multiplet has a spin-1 and a spin-1/2 components which we identify with a vector boson v_μ and a Weyl fermion λ_α respectively. Let us comment, that often off-shell representations of the supersymmetric multiplets are introduced. These include auxiliary fields, F and D for the chiral and vector multiplets respectively, whose equations of motions are purely algebraic and on-shell yield the same theory.

Let us introduce a vector field transforming in the adjoint representation of a gauge group G. Generically, $v_\mu = v_\mu^a T^a$, with T^a the generators of G. These satisfy the algebra

$$[T^a, T^b] = i f^{abc} T^c \ , \ \mathrm{Tr}(T^a T^b) = k\delta^{ab} \ , \ k > 0 \tag{2.2}$$

with $a, b, c = 1, ...\mathrm{dim}(adj(G))$. The generators of G commute with the supersymmetry generators, therefore all components of the multiplet have the same gauge index. The vector multiplet is denoted by

$$(v_\mu^a, \lambda^a)\,. \tag{2.3}$$

The latter couples to n_c chiral multiplets in a given representation $r(G)$

$$(\phi^i, \psi^i) \tag{2.4}$$

$i = 1, ..n_c$ where $n_c = dim(r(G))$.[8]

The Lagrangian of a supersymmetric theory is completely determined by the Kähler potential K, the superpotential W and the gauge kinetic function f, which are specified in terms of the complex scalar fields $\phi, \bar{\phi}$. K is a real function, in particular, $K_{i\bar{j}} := \frac{\partial^2 K}{\partial\phi^i \partial\bar{\phi}^{\bar{j}}}$ is called the Kähler metric and determines the form of the kinetic terms of the components of the chiral field. In renormalizable theories $K(\phi,\bar{\phi})$ is constrained to be

$$K(\phi,\bar{\phi}) = \sum_i |\phi^i|^2 \ , \quad K_{i\bar{j}} = \delta_{i\bar{j}} \ . \tag{2.5}$$

and the kinetic terms are canonical. Analogously, the gauge kinetic function $f_{ab}(\phi)$ is a holomorphic function of the scalar fields that specifies the kinetic terms of the components of the vector multiplet. In renormalizable theories

$$f_{ab} = \delta_{ab} \tag{2.6}$$

and hence, the kinetic terms are canonical. In turn, the superpotential W is an holomorphic function which determines the interactions in the Lagrangian. In renormalizable theories $W(\phi)$ can be at most cubic and the interactions should respect the gauge invariance. From this one infers the most general form of $W(\phi)$ reads

$$W(\phi) = \xi_i\phi^i + \tfrac{1}{2}\mu_{ij}\phi^i\phi^j + \tfrac{1}{3}y_{ijk}\phi^i\phi^j\phi^k. \tag{2.7}$$

ξ_i, μ_{ij} and y_{ijk} are parameters with mass dimension two, one and zero respectively. Note that the linear terms can only involve singlet chiral fields.

In sum, the most general renormalizable Lagrangian with global supersymmetry is given by

$$\begin{aligned}\mathcal{L}_{\text{global}} = &-\delta_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} - i\delta_{i\bar{j}}\bar{\psi}^{\bar{j}}\not{D}\psi^i - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\, a} - i\bar{\lambda}^a\not{D}\lambda^a - V(\phi,\bar{\phi}) \\ &+ i\sqrt{2}g(\bar{\phi}^{\bar{i}}T^a_{\bar{j}i}\psi^j\lambda^a - \bar{\lambda}^a T^a_{i\bar{j}}\phi^i\bar{\psi}^{\bar{j}}) - \tfrac{1}{2}W_{ij}\psi^i\psi^j - \tfrac{1}{2}\bar{W}_{\bar{i}\bar{j}}\bar{\psi}^i\bar{\psi}^{\bar{j}} \ .\end{aligned} \tag{2.8}$$

The first line shows the kinetic terms for the components of the chiral and vector multiplet respectively together with the scalar potential $V(\phi,\bar{\phi})$. D_μ denote the covariant derivatives given by

$$D_\mu\phi^i = \partial_\mu\phi^i + igv^a_\mu T^{ai}_j\phi^j \ , \qquad D_\mu\psi^i = \partial_\mu\psi^i + igv^a_\mu T^{ai}_j\psi^j \tag{2.9}$$

and g is the gauge coupling constant. In addition, we used $\not{D} := \bar{\sigma}^\mu D_\mu$, and

$$D_\mu\lambda^a = \partial_\mu\lambda^a - gf^{abc}v^b_\mu\lambda^c \ , \quad F^a_{ab} = \partial_\mu v_\nu - \partial_\nu v_\mu - gf^{abc}A^b_\mu A^c_\nu \ . \tag{2.10}$$

[8] Note that to avoid clutter we did not explicitly write the Weyl indices in λ_α and ψ_α.

The second line specifies the interactions involving fermionic components. In particular, we defined $W_i := \frac{\partial W}{\partial \phi^i}$ and $W_{ij} := \frac{\partial^2 W}{\partial \phi^i \partial \phi^j}$. Note, by direct computation of the second derivatives of the superpotential

$$W_{ij} = \mu_{ij} + 2y_{ijk}\phi^k \,, \tag{2.11}$$

that μ_{ij} is the mass matrix of the fermions of the chiral multiplets while y_{ijk} induces the Yukawa couplings. Finally, the scalar potential is given by

$$\begin{aligned} V(\phi, \bar{\phi}) &= \tfrac{1}{2} D^a D^a + \sum_i |F^i|^2 \\ &= \tfrac{1}{2} g^2 (\bar{\phi}^{\bar{j}} T^a_{i\bar{j}} \phi^i)^2 + |W_i|^2 \end{aligned} \tag{2.12}$$

where F^i and D^a are defined via

$$D^a = -g(\bar{\phi}^{\bar{j}} T^a_{i\bar{j}} \phi^i) \,, \qquad F^i = -\bar{W}_{\bar{i}} \,. \tag{2.13}$$

It is immediate to see that the scalar potential in (2.12) is positive semi-definite. Furthermore, note that gauge induced terms given by the first factor in (2.12) introduce additional quartic interactions. Let us comment that if G has an abelian factor one can introduce an additional coupling, named Fayet-Illiopolos (FI) term

$$\mathcal{L}_{FI} = \xi_{FI} D \,, \tag{2.14}$$

which is invariant under supersymmetric transformations. In the following we omit this term.

Note that the expressions above hold at tree level and receive quantum corrections which we did not display here. One of the prime features of supersymmetric theories at the quantum level is the *non-renormalization theorem*, the respective proof can be found in [61], see also [62]. The latter states that in $\mathcal{N} = 1$ supersymmetric theories the superpotential is not renormalized to any order in perturbation theory. It can only receive non-perturbative corrections. The Kähler potential is corrected to all orders. The gauge kinetic function, and hence the gauge coupling, is only renormalized at one loop. Both the Kahler potential and the gauge kinetic function also admit non-perturbative corrections. As a result, the renormalization group equations in supersymmetric theories take a very simple form, these are crucial to study the low energy predictions of theories defined at high scales.

Let us conclude this section by recalling that spontaneous symmetry breaking occurs when

$$\langle F^i \rangle \neq 0 \,, \quad \text{and(or)} \quad \langle D^a \rangle \neq 0 \tag{2.15}$$

where the former condition receives the name of *F-term breaking* and the latter *D-term breaking.* Mechanisms to trigger spontaneous symmetry breaking have been long studied in the literature, the prime examples are the O'Raifeartaigh model [63] and the Fayet-Illiopoulos model [64] for F-term and D-term supersymmetry breaking respectively. However, even a supersymmetric framework with spontaneous supersymmetry breaking is not phenomenologically viable. Recall that the presence of a massless fermion in nature, the Goldstone fermion, was not yet observed. In addition, a closer look to (2.8) shows that it is not possible to engineer masses to fermionic components of vector multiplets, there is simply no term of the form $\propto \lambda\bar{\lambda}$. Provided we insist with keeping the Standard Model supersymmetric, there is no other choice but to introduce an ad-hoc explicit breaking of supersymmetry.

2.1.2 Soft supersymmetry breaking interactions

As already anticipated, to construct phenomenologically viable scenarios it is compulsory to add an explicit breaking piece to the supersymmetric Lagrangian. This procedure might appear theoretically unjustified, however, the requirement of embedding such an effective description of the low energy dynamics in a UV-setup will be elegantly solved when introducing supergravity and generic non-renormalizable supersymmetric frameworks. The standard procedure is to introduce a generic parametrization of the supersymmetry breaking sector. In the following we provide its most generic form.

The most appealing feature of supersymmetric theories is that it protects the weak scale from large quadratic radiative corrections. To maintain this property after incorporating explicit supersymmetry breaking, the allowed supersymmetry breaking terms take a very special form. The classification of operators that explicitly break supersymmetry and generate no quadratic divergences was performed in [8] and are denoted as *soft breaking terms*. The most generic soft Lagrangian is of the form

$$-\mathcal{L}_{\text{soft}} = m^2_{i\bar{j}}\phi^i\bar{\phi}^{\bar{j}} + (b_{ij}\phi^i\phi^j + A_{ijk}\phi^i\phi^j\phi^k + c.c.) + \tfrac{1}{2}M_a\lambda^a\lambda^a + c.c. \tag{2.16}$$

$m_{i\bar{j}}$ are called soft scalar masses, b_{ij} are bilinear terms usually denoted as b-terms, A_{ijk} are trilinear couplings usually denoted as A-terms and M_a are the masses of the gauginos of the different gauge factors labeled by a. Note that the holomorphic terms in the scalar fields in (2.16) are quadratic (ϕ^2) and cubic (ϕ^3), higher powers are forbidden since they generate quadratic divergences at one loop.

To conclude, the introduction of the soft Lagrangian given in (2.16) introduces enough freedom to overcome the problems mentioned in the previous section. In particular, the masses of supersymmetric particles can be arbitrary high. One should not omit that the origin and size of this parameters remains unexplained and we return to this point when introducing supergravity. In addition, recall that the number of free parameters in (2.16) is generically too large to make model-independent statements. One customary assumption is to take the soft terms in (2.16) to be *universal*. In this case they satisfy

$$m^2_{i\bar{j}} = m_0^2\,\delta_{i\bar{j}}\,, \quad M_a = M_0\;\forall\,a, \quad A_{ijk} = A_0\,y_{ijk}\,. \tag{2.17}$$

This assumption considerably reduces the number of independent parameters and simplifies the analysis. More importantly, a structure of the form in (2.17) has some desirable phenomenological features that we mention in the next chapter.

2.2 $\mathcal{N}=1$ Supergravity

So far we considered theories with *global* supersymmetry. Theories which are invariant under local $\mathcal{N}=1$ supersymmetry transformations are denoted as $\mathcal{N}=1$ supergravity theories [57, 65–69].[9] The corresponding representations of the supergravity algebra include the chiral and vector multiplet together with the *gravity* multiplet which has a spin-2 and spin-3/2 components. The corresponding quantum fields are identified with the graviton and the gravitino. The respective action of supergravity reproduces the Eistein-Hilbert action coupled to matter. Recall that gravitational interactions are defined at the Planck scale. Thus, this framework is relevant when discussing dynamics of supersymmetric theories at high energy scales.

The spontaneous breaking of supersymmetry in supergravity has an analogous Higgs-mechanism known as the super-Higgs effect. In this case, the Goldstone fermion is eaten by the gravitino which in turn becomes massive. In contrast to the global case, supergravity embeddings of the minimal supersymmetric SM (or extensions) with spontaneous symmetry breaking are phenomenologically viable. In particular, assuming supersymmetry is broken by a separate (or hidden) sector at a high scale one can compute an effective action for the low energy fields. Interestingly, keeping the leading order terms in the gravitino mass one obtains that the low energy effective theory is given by a globally supersymmetric piece as in (2.8) plus a set of soft terms as in (2.16). Moreover, the scale of the soft terms is parametrized by the gravitino mass. This mechanism to generate the soft terms is known as gravity mediation. In the next section we give the explicit form of the supergravity Lagrangian for chiral multiplets coupled to vector multiplets. In addition, in section 2.2.2 we give an outlook on a special class of supergravity theories known as no-scale models. Finally, in section 2.2.3 we provide the soft terms in gravity mediated scenarios.

[9] Recall that $\mathcal{N}=1$ supergravity in four dimensions is non-renormalizable, thus, it should be regarded as an effective theory with an intrinsic cutoff at M_p.

2.2.1 $\mathcal{N}=1$ Supergravity coupled to matter

In the following we display the couplings of chiral and vector multiplets to $\mathcal{N}=1$ supergravity [70, 71]. Specifically, the Lagrangian in supergravity reads[10]

$$\begin{aligned}\tfrac{1}{\sqrt{g}}\mathcal{L} = &-\tfrac{1}{2\kappa^2}R - \frac{1}{4}\mathrm{Re}f_{ab}(\phi^i)F^{a\mu\nu}F^b_{\mu\nu} + \frac{1}{4}\mathrm{Im}f_{ab}(\phi^i)F^{a\mu\nu}\tilde{F}^b_{\mu\nu} \\ &- K_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} - V(\phi,\bar{\phi}) + \text{fermionic terms}\,.\end{aligned} \tag{2.18}$$

Here R is the Ricci scalar, $g := |\det g_{\mu\nu}|$ with $g_{\mu\nu}$ the space-time metric and we defined the gravity coupling $\kappa^{-2} := M_p^2$. Note that the first term is the Einstein-Hilbert action. The remaining terms in the first line are the respective kinetic terms of the gauge field strength and $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. The corresponding covariant derivatives in the kinetic terms of the scalar fields are covariant with respect to gravity and the gauge group. Finally, the scalar potential is given by

$$\begin{aligned}V &= K_{i\bar{j}}F^i\bar{F}^{\bar{j}} - 3\kappa^2 e^{\kappa^2 K}|W|^2 + \tfrac{1}{2}g^2\mathrm{Re}f_{ab}D^aD^b \\ &= e^{\kappa^2 K}\left(D_iWK^{i\bar{j}}D_{\bar{j}}\bar{W} - 3\kappa^2|W|^2\right) + \tfrac{1}{2}g^2\mathrm{Re}f_{ab}D^aD^b\end{aligned} \tag{2.19}$$

where $K^{i\bar{j}} := K^{-1}_{i\bar{j}}$ denotes the inverse Kähler metric and $D_iW := (W_i + \kappa^2 K_i W)$. In the first line D^a denote the generalization to supergravity of the D-terms in (2.13) and depend on the Killing vectors of the metric, for their explicit form see e.g. [57]. $\bar{F}^{\bar{j}}$ is defined via

$$\bar{F}^{\bar{j}} = -e^{\kappa^2 K/2}K^{\bar{j}i}(W_i + \kappa^2 K_i W)\,. \tag{2.20}$$

Note that after cannonically renormalizing the kinetic terms for the gauge bosons one can read of the gauge coupling constant. In particular, for the particular case in which the gauge kinetic function is diagonal, the gauge coupling reduces to

$$f_{ab} = f\delta_{ab}\,, \quad g^2 := \mathrm{Re}f^{-1} \tag{2.21}$$

where f denotes an overall function that can depend on the scalar fields.

In supergravity, as in the global case in (2.15), $\langle F^i\rangle$ (and $\langle D^a\rangle$) are the order parameters of supersymmetry breaking. In this case, the *Super Higgs effect* [72–75] states the Goldstone fermion is eaten by the gravitino which in turn becomes massive. The gravitino mass parametrizes the scale of supersymmetry breaking and it depends on the Kähler potential and superpotential via the following equation

$$m_{\frac{3}{2}} = \kappa^2 e^{\langle K\rangle/2}|W|\ . \tag{2.22}$$

[10] The fermionic terms can be found in [57, 58].

Let us finish this section with some remarks. Recall that in the limit $\kappa^2 \to 0$, i.e. $M_p \to \infty$, called the *rigid limit*, gravity decouples and the Lagrangian in (2.8) is restored. However, provided there is an intermediate scale Λ between the electroweak scale and the Planck scale, non-renormalizable operators supressed by negative powers of Λ might be relevant for the effective low energy theory. In this situation, taking the rigid limit but keeping Λ fixed does not restore (2.8) but a *non-renormalizable theory with global supersymmetry.* In paticular, ignoring D-terms the scalar potential is given by

$$V \simeq \tilde{K}_{i\bar{j}} \tilde{F}^i \tilde{\bar{F}}^{\bar{j}} \,, \quad \text{with} \quad \tilde{\bar{F}}^{\bar{j}} = -\tilde{K}^{\bar{j}i} \tilde{W}_i \tag{2.23}$$

where $\tilde{K}_{i\bar{j}}$ can have arbitrary non-renormalizable operators suppressed by Λ. This class of embeddings has interesting phenomenological applications. In particular, as in gravity mediation, one can assume a hidden sector responsible for supersymmetry breaking at high scales and compute the effective action for low energy fields. The outcome is again the Lagrangian in (2.8) plus soft terms as in (2.16). The explicit form of the soft terms is computed in the appendix A.[11] We will study this situation with a particular example in chapter 5.

2.2.2 No-scale supergravity

No-scale models constitute a class of matter-coupled supergravities in four dimensions with vanishing or positive-(negative)-semi-definite scalar potentials [78, 79]. In particular, ignoring D-terms, no-scale type of supergravities are defined via the condition

$$G_i G^{i\bar{j}} G_{\bar{j}} = p, \quad p \in \mathbb{R} \,, \tag{2.24}$$

G denotes the *generalized Kähler potential* and is given by $G = K + \log |W|^2$. Note that the scalar potential in (2.19) can be written in terms of G as

$$V = e^G \left(G_i G^{i\bar{j}} G_j - 3 \right) \,. \tag{2.25}$$

From (2.25) we learn that for generalized Kähler potentials that satisfy (2.24) $V = 0$ if $p = 3$. In the particular case that the theory enjoys a Peccei-Quinn shift symmetry, W is constant and therefore (2.24) coincide with the condition

$$K_i K^{i\bar{j}} K_{\bar{j}} = p \,, \quad p \in \mathbb{R} \,. \tag{2.26}$$

In the shift-symmetric case solving (2.26) is equivalent to solving a differential equation known as the real homogeneous *Monge-Ampère* equation. The general solution was presented in a

[11] In particular, models that incorporate higher derivatives can be embedded in global supersymmetric frameworks [76, 77] and could contribute to the soft terms.

semi-explicit form by the author in a collaboration in [55]. One special family of solutions is displayed as follows

$$K = -p \log(Y) \tag{2.27}$$

with Y an homogeneous function of degree one. The no-scale property of (2.27) was already pointed out in [80]. In [55] we presented non-homogeneous solutions which were so far unknown in the literature, and constructed explicit examples. In this thesis we will not expand on no-scale supergravity any further, for more details on the general solution and a discussion on Kähler potentials of the type of (2.27) realized in effective actions that descend from string compactifications see [55].

2.2.3 Gravity mediation

One of the most beautiful results in supergravity shows that if one formulates supersymmetry breaking by an hypothetic (*hidden*) sector, the effective theory for low energy (*observable*) fields, which is computed by taking the rigid limit while leaving $m_{\frac{3}{2}}$ fixed, reduces to

$$\lim_{M_p \to \infty,\, m_{\frac{3}{2}}\,\text{fixed}} \mathcal{L} = \mathcal{L}_{\text{global}} + \mathcal{L}_{\text{soft}} \ . \tag{2.28}$$

In particular, the structure of $\mathcal{L}_{\text{soft}}$ takes precisely the form of (2.16) and the parameters are completely specified by the Kähler potential the superpotential and the gauge kinetic function. This mechanism of generating $\mathcal{L}_{\text{soft}}$ is named of *gravity mediation*, for a pedagogical example see the *Polonyi model* [81]. The fact that generic string compactifications yield supergravity theories as their low energy description encouraged the belief that one can ultimately compute the precise structure of the soft terms by means of the knowledge of the specific Lagrangian (see e.g. chapter 4). The derivation of the soft terms was performed in [27, 28] introducing generic (unknown) couplings in the Lagrangian. In this section we will reproduce their structure and describe the specific assumptions applied in the derivation.

Consider a generic $\mathcal{N} = 1$ supergravity theory consisting of a hidden sector and an observable sector. The former is parametrized by chiral fields t^i while the latter is given in terms of the chiral matter fields A^I. Provided that the scale of the relevant dynamics of the observable fields is suppressed with respect to the scale at which the supergravity Lagrangian is specified, and $\langle A^I \rangle = 0$, the Kähler potential can be expanded around this minimum. Namely,

$$K(t, \bar{t}, A, \bar{A}) = \kappa^{-2} \hat{K}(t, \bar{t}) + Z_{I\bar{J}}(t, \bar{t}) A^I \bar{A}^{\bar{J}} + (\frac{1}{2} H_{IJ}(t, \bar{t}) A^I A^J + c.c.) + \dots \tag{2.29}$$

where the dots indicate higher order terms in A. Analogously, the superpotential is expanded in powers of A^I

$$W(t,A) = \hat{W}(t) + \frac{1}{2}\tilde{\mu}_{IJ}(t)A^I A^J + \frac{1}{3}\tilde{Y}_{IJK}(t)A^I A^J A^K + \ldots \ . \tag{2.30}$$

The (renormalized) gauge couplings can be generically written as follows[12] [82–86]

$$\begin{aligned} g_a^{-2}(t,\bar{t},p) =& \mathrm{Re} f_a + \tfrac{b_a}{8\pi^2}\log\frac{M_p}{p} + \frac{c_a}{16\pi^2}\hat{K}(t,\bar{t}) \\ &+ \tfrac{T(G)}{8\pi^2}\log g_a^{-2}(t,\bar{t},p) - \sum_r \tfrac{T_a(r)}{8\pi^2}\log\det Z^{(r)}(t,\bar{t},p) \end{aligned} \tag{2.31}$$

where p indicates the renormalization scale, the index a runs over the gauge group factors.[13]

The derivation of the effective potential for the observable fields given as follows holds under the following assumptions

1. $\langle F^t \rangle \neq 0$, i.e. supersymmetry breaking is triggered in the hidden sector.
2. $\langle F^A \rangle = 0$, i.e. there is no supersymmetry breaking in the observable sector.
3. $\langle V \rangle = 0$, i.e. the cosmological constant is set to vanish.

Using the expression for the scalar potential defined in (2.19) and taking the limit $M_p \to \infty$, while keeping the leading contributions in $m_{\frac{3}{2}}$, one recovers the following effective potential

$$\begin{aligned} V(A,\bar{A}) =& \sum_a \frac{g_a^2}{4}(\bar{A}^{\bar{I}} Z_{\bar{I}J} T_a A^J)^2 + \partial_I W^{\mathrm{eff}} Z^{I\bar{J}} \bar{\partial}_{\bar{J}} \bar{W}^{\mathrm{eff}} \\ &+ m^2_{I\bar{J}} A^I \bar{A}^{\bar{J}} + (\frac{1}{2} b_{IJ} A^I A^J + \frac{1}{3} A_{IJK} A^I A^J A^K + \text{c.c.}) \ , \end{aligned} \tag{2.32}$$

where W^{eff} denotes an effective superpotential defined as follows

$$W^{\mathrm{eff}}(A) = \frac{1}{2}\mu_{IJ} A^I A^J + \frac{1}{3} Y_{IJK} A^I A^J A^K \ , \tag{2.33}$$

with

$$\mu_{IJ} = e^{\hat{K}/2}\tilde{\mu}_{IJ} + m_{\frac{3}{2}} H_{IJ} - \bar{F}^{\bar{j}} \bar{\partial}_{\bar{j}} H_{IJ} \tag{2.34}$$

and

$$Y_{IJK} = e^{\hat{K}/2}\tilde{Y}_{IJK} \ . \tag{2.35}$$

In (2.32) we identify the first line with a global supersymmetric scalar potential while the second line denotes the soft supersymmetry breaking terms, as defined in (2.12) and (2.16)

[12] The *non-renormalization theorem* remains valid in supergravity.

[13] For completeness we provide the numerical coefficients, $b_a = \sum_r n_r T_a - 3T(G_a)$, $c_a = \sum_r n_r T_a - T(G_a)$, $T_a(r) = \mathrm{Tr}_r(T_a^2)$ and $T(G_a) = T_a(\text{adjoint})$ where r denotes the representations of the observable gauge group and n_r the corresponding number of chiral fields A^I transforming under r.

respectively. These soft terms depend on the original parameters of the Kähler function and the superpotential via

$$m^2_{I\bar{J}} = m^2_{\frac{3}{2}} Z_{I\bar{J}} - F^i \bar{F}^{\bar{j}} R_{i\bar{j}I\bar{J}}\,, \tag{2.36}$$
$$b_{IJ} = \ F^i D_i \mu_{IJ} - m_{\frac{3}{2}} \mu_{IJ}\,, \tag{2.37}$$
$$A_{IJK} = \ F^i D_i Y_{IJK}\,, \tag{2.38}$$

with

$$\begin{aligned} R_{i\bar{j}I\bar{J}} &= \partial_i \bar{\partial}_{\bar{j}} Z_{I\bar{J}} - \Gamma^N_{iI} Z_{N\bar{L}} \bar{\Gamma}^{\bar{L}}_{\bar{j}\bar{J}}\,, \quad \Gamma^N_{iI} = Z^{N\bar{J}} \partial_i Z_{\bar{J}I}\,, \\ D_i Y_{IJK} &= \partial_i Y_{IJK} + \tfrac{1}{2} \hat{K}_i Y_{IJK} - \Gamma^N_{i(I} Y_{JK)N}\,, \\ D_i \mu_{IJ} &= \partial_i \mu_{IJ} + \tfrac{1}{2} \hat{K}_i \mu_{IJ} - \Gamma^N_{i(I} \mu_{J)N}\,. \end{aligned} \tag{2.39}$$

Finally, the canonically normalized gaugino masses yield

$$M_a = F^i \partial_i \log g_a^{-2} + \tfrac{1}{16\pi^2} b_a m_{\frac{3}{2}} \tag{2.40}$$

where the first term is the tree level contribution and the second term is known as *anomaly mediation* [87].

Let us finish this chapter by making a few remarks:

- The condition of vanishing cosmological constant is assumed for phenomenological reasons. Generically, a vacuum energy originated by the dynamics of the hidden sector would yield values inconsistent with observation. The generalization of the results above allowing for an arbitrary cosmological constant was computed in [28].

- On dimensional grounds all parameters are in mass units of $m_{\frac{3}{2}}$. In particular, the assumption of vanishing cosmological constant i.e. $\langle V \rangle = 0$, in eqs. (2.19) and (2.20) yield a relation between the gravitino mass and the F-term given by

$$m_{\frac{3}{2}} = \langle \tfrac{\kappa^2}{3} K_{i\bar{j}} F^i \bar{F}^{\bar{j}} \rangle^{\frac{1}{2}}\ . \tag{2.41}$$

  Note that as long one does not specify the dynamics of the hidden sector $m_{\frac{3}{2}}$ remains unfixed.

- The soft terms are generically not universal.

- There are two sources of supersymmetric bilinear interactions in (2.34), in particular, recall that the second term is induced by Kähler potential. This mechanism of generating a μ-term is called the *Giudice-Massiero mechanism* [88].

Chapter 3

The NMSSM

In this chapter we introduce the Next-to-Minimal Supersymmetric Standard Model. The NMSSM extends the minimal supersymmetric Standard Model by a gauge singlet and therefore has distinct features with respect to the minimal case. In particular, following [14] we present the Lagrangian, the mechanism of electroweak symmetry breaking and the spectrum. Furthermore, guided by [9, 14] and [12, 13], we discuss generic achievements and difficulties of low energy supersymmetry together with current experimental constraints. In particular, we study the theoretical bound on the Higgs mass and the little hierarchy problem.

3.1 The particle content

In supersymmetric extensions of the Standard Model, each of the known fundamental fields is promoted to a supermultiplet. In particular, scalar bosons (Higgs sector) and fermions (quarks and leptons) are embedded into chiral supermultiplets while gauge bosons are incorporated into vector supermultiplets. The complete spectrum is given in Tables 3.1 and 3.2, where the superfields are classified according to their transformation properties under the SM gauge group $SU(3) \times SU(2) \times U(1)$.

Note that two Higgs doublets H_u, H_d were introduced. The presence of an additional Higgs is necessary in order to cancel a gauge anomaly. In addition, as will become clear in the next section, to generate the appropriate Yukawa interactions while maintaining the holomorphicity of the superpotential the introduction of an additional Higgs is compulsory. The singlet S superfield denotes an extra singlet chiral field incorporated in the next-to minimal version of the supersymmetric standard Model.

The spin-0 superpartners of quarks and leptons are called by an *s-* preceding the name, i.e. squarks and sleptons, or generically sfermions and are denoted by the same symbol that the respective quark and lepton with an additional tilde. The generic nomenclature for the spin-1/2 superpartner of a scalar field is to append an *-ino* at the end of the name of the

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q_L	$(\tilde{u}_L\ \ \tilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times$3 families)	U_R	$\tilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	D_R	$\tilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{1}{3})$
sleptons, leptons	L_L	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times$3 families)	E_R	$\tilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(h^+_u\ \ h^0_u)$	$(\tilde{h}^+_u\ \ \tilde{h}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(h^0_d\ \ h^-_d)$	$(\tilde{h}^0_d\ \ \tilde{h}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
Singlet, singlino	S	s	$\tilde{s}$	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

Table 3.1: *Chiral supermultiplets in the Next-to-Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.*

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\tilde{W}^\pm\ \ \tilde{W}^0$	$W^\pm\ \ W^0$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\tilde{B}^0$	B^0	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

Table 3.2: *Gauge supermultiplets in the (Next-to-) Minimal Supersymmetric Standard Model.*

Standard Model scalar and are also written with an additional tilde. Thus in the Higgs and singlet superfields we have the higgsinos and singlino respectively. The same rule applies to the spin-1/2 superpartners of the gauge bosons, which are referred to as the gauginos.

3.2 The Lagrangian

The structure of the Lagrangian is of the form (2.8) and is completely specified by the field content specified in Table 3.1, the gauge sector in Table 3.2 and the superpotential. The singlet couples to the Higgs sector via a Yukawa coupling and, furthermore, together with the MSSM participates in electroweak symmetry breaking. The most general (renormalizable) superpotential that incorporates the singlet is given by

$$W_{\text{NMSSM}} = W_{\text{Yukawa}} + (\mu_h + \lambda S)H_u H_d + \mu_s S^2 + \kappa S^3 + \xi S\,, \tag{3.1}$$

where S is the NMSSM singlet and H_u, H_d are the MSSM Higgs doublets (see Table 3.1). W_{Yukawa} denotes the Yukawa interactions of the MSSM, which read

$$W_{\text{Yukawa}} = y_u Q U_R H_u + y_d Q_L D_R H_d + y_e L_L E_R H_d\,. \tag{3.2}$$

y_u, y_d, y_e are the Yukawa couplings which are matrices in the family space. Q are the quark doublets, U_R and D_R are the quark singlets, L_L are the lepton doublets and E_R are the lepton singlets, these are vectors in the family space (see Table 3.1). λ and κ are dimensionless couplings, in particular, the sign of λ can be always taken positive after a field redefinition while κ can take both signs. μ_h and μ_s stand for the supersymmetric masses of the Higgs and singlet respectively and are usually denoted as $\mu-$terms. Finally, ξ represents a tadpole term of mass dimension two.

Note that mass terms of the form $H_u^* H_u$, $H_d^* H_d$ and $S^* S$ are forbidden in the superpotential which must be holomorphic in the chiral fields. In addition, from (3.2) it becomes clear why it was mandatory to include two Higgs in order to give masses to all the SM fermions. The Yukawa couplings belong to the superpotential which is holomorphic in the chiral fields. Thus, terms like $Q_L H_d D_R$ or $L_L H_d E_R$ cannot be replaced by interactions of the form $Q_L H_u^* D_R$ and $L_L H_u^* E_R$.

Recall that from a UV-perspective the dimensionful couplings in the superpotential need not be M_{ew} but could in principle be defined in M_p units, the minimum scale where new physics appear. In the MSSM, μ_h is the only dimensionful parameter and arguing for a light $\mu-$parameter is known as the *mu-problem*. As (3.1) shows, this problem is worsen in the singlet-extension where additional dimensionful parameters are introduced. However, specific scenarios have been constructed in the literature [89] where these are generated after supersymmetry breaking and therefore, naturally appear in units of the soft scale. In this work we will introduce different setups within the NMSSM under the assumption that the corresponding dimensionful terms can naturally take values of the order of the supersymmetry-breaking scale.

It is worth pointing out that often times the NMSSM appears in the literature as the $\mathbb{Z}_3$ (or scale)-invariant version of (3.1). The latter corresponds to the following superpotential

$$W_{\text{scale-inv}} = W_{\text{Yukawa}} + \lambda S H_u H_d + \kappa S^3 \tag{3.3}$$

which can be recovered from (3.1) by setting $\mu_h = \mu_s = \xi = 0$. The respective Lagrangian enjoys a $\mathbb{Z}_3$-symmetry that corresponds to multiplying all components of the chiral superfields by $e^{i2\pi/3}$. Note that any dimensionful parameter in (3.1) breaks the $\mathbb{Z}_3$-symmetry explicitly. This particular setup has the appealing feature that the μ-term is generated dynamically when the singlet acquires a VEV and hence, elegantly solves the mu-problem. However, this scenario suffers from tadpole [90] and domain wall [91] problems. Here we will always discuss the general NMSSM, the corresponding expressions in the constraint version can be derived by setting the dimensionful parameters to zero.

The respective soft terms in the NMSSM can be obtained from the general definition in (2.16). Specifically, we consider the particle content of the NMSSM and write down all possible soft terms in (2.16) invariant under the gauge symmetry. For simplicity we assume the matrix of soft scalar masses is diagonal. In turn, we choose A-terms to be diagonal in the flavor index.

After these considerations, the soft interactions in the Lagrangian read

$$\begin{aligned} -\mathcal{L}_{\text{soft}} =& \tfrac{1}{2}(M_a \lambda^a \bar{\lambda}^a + \text{c.c.}) + m_{h_u}^2 |H_u|^2 + m_{h_d}^2 |H_d|^2 + m_s^2 |S|^2 \\ &+ (\lambda A_\lambda H_u H_d S + \tfrac{1}{3}\kappa A_\kappa S^3 + \frac{1}{2} b_s S^2 + \xi_s S + b_h\ H_u H_d + \text{c.c.}) \\ &+ \sum_{\text{generations}} m_q^2 |Q|^2 + m_u^2 |U_R|^2 + m_d^2 |D_R|^2 + m_l^2 |L|^2 + m_e^2 |E|^2 \\ &+ (y_u A_u Q H_u U_R - y_d A_d Q H_d D_R - y_e A_e L H_d E_R + \text{c.c.})\ . \end{aligned} \tag{3.4}$$

In particular, we identify the scalar higgses, singlet, left(and right)-handed squarks and sleptons by $m_{h_u}, m_{h_d}, m_s, m_q, m_u, m_d, m_l$ and m_e respectively. For convenience, A-terms are written proportional to the Yukawa couplings with coefficients $A_\lambda, A_\kappa, A_u, A_d, A_e$. Note that A_u, A_d, A_e are vectors in the family space, in particular, the corresponding components for the third-generation are identified with A_t, A_b and A_τ respectively. In addition, in the NMSSM there are two b-terms b_h and b_s respectively and one soft tadpole term ξ_s for the singlet.

3.2.1 R-parity

Note that (3.1) is not the most general gauge-invariant and renormalizable superpotential one can write down. Additional gauge-invariant interactions remain that were not included in (3.1) because they violate either Baryon number (B) or Lepton number (L). These include

$$W_{\Delta L=1} = \tfrac{1}{2}\lambda L_L L_L E_R + \lambda' L_L Q_L D_R + \mu' L_L H_u\,, \tag{3.5}$$

$$W_{\Delta B=1} = \tfrac{1}{2}\lambda'' U_R D_R D_R\,, \tag{3.6}$$

where each term should be understood as a sum in family space. Recall that Q_L and U_R, D_R chiral superfields have Baryon number $B = +1/3$ and $B = -1/3$ respectively, while all other superfields have $B = 0$. The lepton number assignements are $L = 1$ for all L_L and $L = -1$ for E_R while all others have vanishing lepton number. In sum, as previously anticipated, the contributions in (3.5) and (3.6) violate total lepton and baryon number respectively.

The argument to exclude these terms from the (N)MSSM Lagrangian is based on pure phenomenological grounds. B-(and L-)violating processes have not been observed in Nature. The most compelling bound comes from proton decay, for further details see [9] and for an extensive review see [92]. In the MSSM (and extensions thereof) one introduces an extra symmetry that forbids (3.5) and (3.6) while keeping the desirable terms in (3.1). This new symmetry is a $U(1)$ symmetry called R-parity or matter parity.[14] The R-parity assignment is

[14] Requiring $R-$parity conservation in the Lagrangian is in practice equivalent to imposing matter-parity conservation.

defined for each field according to

$$P_R = (-1)^{3(B-L)-2j} \tag{3.7}$$

where B and L are as before and j denotes the spin. Note that the different components of the chiral superfields have different $R-$parities. From (3.7) it follows that SM particles together with the Higgs bosons and the singlet have R-parity $P_R = +1$ whereas squarks, sleptons, gauginos, higgsinos and the singlino have R-parity $P_R = -1$. One crucial phenomenological consequence from having this additional symmetry is that the lightest supersymmetric particle with $P_R = -1$, usually denoted as LSP, is stable [9]. If the LSP is weakly interacting it becomes a good dark matter candidate [93, 94]. Let us add that R-parity is not a phenomenologically necessary requirement, explicit viable examples exist with R-parity violation. From now on we will assume R-parity is part of the definition of the (N)MSSM.

3.3 Electroweak symmetry breaking

As presented in Table 3.1, the Higgs sector in the NMSSM consists of two Higgs doublets and a singlet. These involve ten real degrees of freedom and three of these constitute the Goldstone bosons that are eaten by the gauge bosons $W^{\pm}, Z^0$ after electroweak symmetry breaking. Recall that we denote the components of the Higgs doublets H_u and H_d by $H_u = (h_u^+, h_u^0)^T$ and $H_d = (h_d^0, h_d^-)^T$ and the singlet by s. In particular, the fields responsible for supersymmetry breaking are h_u^0, h_d^0 and s. Let us proceed to compute the scalar potential.

The scalar potential for the Higgs sector results from (2.12) and (3.4) by setting all other scalar fields to zero. As indicated in (2.12) the supersymmetric scalar potential is composed by the D-term piece together with the superpotential (or F-term) contribution. The D-terms are

$$D^a_{SU(2)} = -\tfrac{1}{2} g_2 (\bar{H}_u \sigma^a H_u + \bar{H}_d \sigma^a H_d)\,, \quad D_{U(1)} = -\tfrac{1}{2} g_1 (\bar{H}_u^2 - H_d^2) \tag{3.8}$$

where σ are the Pauli matrices and the index a runs over the adjoint of $SU(2)$. g_1 and g_2 denote the gauge couplings of $U(1)_Y$ and $SU(2)$ respectively. Inserting (3.1) (with $W_{\text{Yukawa}} = 0$) and (3.8) in (2.12) yields[15]

$$\begin{aligned} V_{\text{susy}} &= \frac{g_1^2 + g_2^2}{8} \left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2 \right) + \frac{g_2^2}{2} |h_u^+ h_d^{0*} + h_u^0 h_d^{-*}|^2 \\ &+ |\mu_h + \lambda s|^2 \left(|h_u^0|^2 + |h_d^0|^2 + |h_u^+|^2 + |h_d^-|^2 \right) + |\lambda \left(h_u^+ h_d^- - h_u^0 h_d^0 \right) + \kappa s^2 + \mu_s s + \xi|^2 \end{aligned} \tag{3.9}$$

[15] In this expression, we replaced the Higgs doublets by the explicit component form.

In turn, (3.4) yields

$$V_{\rm soft} = m_{h_u}^2\left(|h_u^0|^2 + |h_u^+|^2\right) + m_{h_d}^2\left(|h_d^0|^2 + |h_d^-|^2\right) + (b_h + \lambda A_\lambda s)\left(h_u^+ h_d^- - h_u^0 h_d^0\right) + \\ + m_s^2 |s|^2 + (\tfrac{1}{2} b_s s^2 + \xi_s s + c.c.)\ . \tag{3.10}$$

In order to calculate the electroweak symmetry breaking minimum we assume real values for $\langle h_u^0\rangle$, $\langle h_d^0\rangle$ and $\langle s\rangle$.[16] Note that the condition $\langle h_u^+\rangle = \langle h_d^-\rangle = 0$ is necessary to leave the $U(1)$ symmetry unbroken, this cannot be guaranteed a priori and should be checked for each model. In the following we will assume the above is true, and compute the scalar potential responsible for supersymmetry breaking by replacing (3.9) and (3.10) by $\langle h_u^+\rangle = \langle h_d^-\rangle = 0$. This gives [14]

$$V_{\rm higgs} = \frac{1}{8}(g_1^2 + g_2^2)(|h_u|^2 - |h_d|^2)^2 + (m_{h_u}^2 + \mu^2)|h_u|^2 + (m_{h_d}^2 + \mu^2)|h_d|^2 + \\ + \lambda^2 |h_u|^2 |h_d|^2 + (-b\, h_u h_d + c.c.) + V_s \tag{3.11}$$

where we supress the upper index 0 in h_u, h_d and μ and b are defined as

$$\mu = \mu_{\rm eff} + \mu_h, \qquad \mu_{\rm eff} = \lambda s\,, \qquad b = \mu_{\rm eff}\, b_{\rm eff} + b_h + \lambda(\mu_s s + \xi), \qquad b_{\rm eff} = A_\lambda + \kappa s\,. \tag{3.12}$$

V_s incorporates self interactions of the singlet and yields

$$V_s = (m_s^2 + \mu_s^2)|s|^2 + \kappa^2 |s|^4 + (\frac{\kappa}{3} A_\kappa s^3 + \frac{1}{2} b_s s^2 + \xi_s s + \text{h.c})\,. \tag{3.13}$$

The VEVs $\langle h_u\rangle, \langle h_d\rangle, \langle s\rangle$ can be computed by minimizing (3.11).

The introduction of the singlet makes a model-independent analysis of the existence of a minimum in (3.11) rather involved. However, since $\langle s\rangle$ is of the order of the soft terms and we assume $M_s \gg \langle h_u\rangle, \langle h_d\rangle$ the VEV of the singlet can be obtained from minimizing V_s in (3.13). In turn, $\langle s\rangle$ appears as an additional parameter in (3.11) and the situation becomes effectively identical to the MSSM case. Under this assumption, we proceed to outline the conditions for which a minimum for the MSSM Higgs exists.

1. The potential must be bounded from below for $|h_u|\,, |h_d|$ large. In this regime the dominant terms in the potential are the quartic terms. Note that as long as $\lambda \neq 0$ this is automatically guaranteed.

2. The solution $\langle h_u^0\rangle = \langle h_d^0\rangle = 0$ should not be a stable minimum.

[16] In the MSSM where the Higgs sector contains only the Higgs doublets one can make use of the $SU(2)$ symmetry to align the VEV of the Higgses with the real part of the respective complex field. In the NMSSM the reality of the singlet cannot be guaranteed but depends on the parameters of the superpotential. In the following we will assume that this is possible.

3. The minimization, or tadpole, equations read

$$\begin{aligned}
\left.\frac{\partial V}{\partial h_u}\right|_{\min} &= v_u\left((m_{h_u}^2+\mu^2)+\lambda^2 v_d^2+\tfrac{g_1^2+g_2^2}{4}(v_u^2-v_d^2)\right)-bv_d=0\,,\\
\left.\frac{\partial V}{\partial h_d}\right|_{\min} &= v_d\left((m_{h_d}^2+\mu^2)+\lambda^2 v_u^2-\tfrac{g_1^2+g_2^2}{4}(v_u^2-v_d^2)\right)-bv_u=0\,,\\
\left.\frac{\partial V}{\partial s}\right|_{\min} &\simeq s\left(m_s^2+\mu_s^2+b_s+2\kappa\xi+\kappa A_\kappa s+2\kappa^2 s^2+3\kappa s\mu_s\right)+\xi_s+\xi\mu_s=0\,,
\end{aligned} \tag{3.14}$$

where the partial derivatives are evaluated at the minimum, and we used the definitions $\langle h_u\rangle = v_u, \langle h_d\rangle = v_d, \langle s\rangle = s$. In the last equation we approximated $\frac{\partial V}{\partial s} \simeq \frac{\partial V_s}{\partial s}$.

The following definitions are conventionally used

$$v_u = v\sin\beta\ , \quad v_d = v\cos\beta\ , \quad v^2 = v_u^2+v_d^2 = (174\text{GeV})^2 \tag{3.15}$$

and

$$\tan\beta = \frac{v_u}{v_d}\ , \quad M_z^2 = \frac{(g_1^2+g_2^2)}{2}v^2\ . \tag{3.16}$$

with $\beta \in [\frac{\pi}{4}, \frac{\pi}{2}]$. By means of (3.16), the minimization conditions for the MSSM Higgses given by the first two equations in (3.14) can be conveniently rewritten to determine M_z and $\tan\beta$, via the following relations[17]

$$M_z^2 = 2\left(-\mu^2+\hat{m}^2\right), \tag{3.17}$$

where we defined

$$\hat{m}^2 := \frac{m_{h_d}^2-\tan^2\beta\, m_{h_u}^2}{\tan^2\beta-1}\ , \tag{3.18}$$

and

$$\sin(2\beta) = \frac{2b}{m_{h_u}^2+m_{h_d}^2+2\mu^2+\lambda^2 v^2}\ . \tag{3.19}$$

Recall that the value of $\tan\beta$ and the (running) top mass M_t fix the top Yukawa coupling via

$$M_t = y_t v_u = y_t v\sin\beta\,. \tag{3.20}$$

The minimum of (3.13) is quite involved and model dependent. In particular, in the $\mathbb{Z}_3$-invariant case $\langle s\rangle$ is given by

$$\langle s\rangle \simeq \tfrac{1}{4\kappa}\left(-A_\kappa-\sqrt{A_\kappa^2-8m_s^2}\right)\ . \tag{3.21}$$

Notice that to have a global VEV for the singlet it is required that $A_\kappa \gtrsim 9m_s^2$. Depending on the parameters, the Higgs potential of the scale-invariant NMSSM can have several local

[17] Note that the experimental constraint $M_z \simeq 91$GeV fixes one parameter in the Higgs sector.

minima and one should make sure to choose the global one.

Threshold corrections

As the supersymmetry-breaking scale is pushed to higher values, the effect of threshold corrections to the scalar potential become relevant. At one loop they can be computed from the Coleman-Weinberg potential [14]

$$\Delta V = \frac{1}{64\pi^2}\text{Str } M^4 \left(\log\left(\frac{M^2}{M_s^2}\right) - \frac{3}{2}\right). \tag{3.22}$$

where Str denotes the supertrace. M_s is conventionally taken at

$$M_{\rm s} := \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \tag{3.23}$$

with $m_{\tilde{t}_{1,2}}$ being the eigenvalues of the stop mass matrix. These explicitly read

$$m^2_{\tilde{t}_{1,2}} = M_t^2 + \tfrac{1}{2}(m^2_{q_3} + m^2_{u_3}) \mp \sqrt{W} \tag{3.24}$$

where $W = \frac{1}{4}(m^2_{q_3} - m^2_{u_3})^2 + M_t^2(A_t - \mu\ h_d/h_u)^2$ is the mixing parameter. The dominant contribution in ΔV comes from the top sector and shifts the soft Higgs masses as follows[18]

$$m^2_{h_u} \to m^2_{h_u} + -\frac{3}{16\pi^2} y_t^2 m^2_{\tilde{t}_1}\ , \quad m^2_{h_d} \to m^2_{h_d}\ , \quad m^2_s \to m^2_s\ . \tag{3.25}$$

3.4 Renormalization Group Equations

Provided a set of boundary conditions for the parameters of the NMSSM are defined at a high scale, one needs to evolve gauge and superpotential couplings together with the soft terms via their renormalization group equations (RGEs) to extract the corresponding parameters at the electroweak scale. The structure of the RGEs in supersymmetric theories is particularly special as a result of the *non-renormalization theorem*, see section 2.1.1. The RGEs of the NMSSM are given at one loop in appendix B, the complete two loop expressions can be found in [14].

Regarding the gauge couplings, it is well known *gauge coupling unification* is achieved in the MSSM at scales near

$$M_{\rm GUT} \simeq 10^{16}\text{GeV} \tag{3.26}$$

defined as the GUT scale. Let us comment that it is customary to use the $SU(5)$ normalization convention for the gauge couplings, i.e. $\alpha_0 = \alpha_2 = \alpha_3 = \frac{3}{5}\alpha_1 \simeq 0.04$. Note that in the NMSSM gauge coupling unification is not spoiled, the evolution of the gauge couplings at one loop is

[18] In particular, we assume $m_{\tilde{t}_1} \simeq m_{\tilde{t}_2}$.

defined in (B.1) and is identical to the respective β-functions in the MSSM. The NMSSM Yukawa couplings (λ and κ) only appear in the beta functions of the gauge couplings at two loop level and, thus, the numerical effect upon the scale of unification is negligible.

Concerning the Yukawa couplings, one needs to make sure that the perturbative expansion holds, i.e. that the chosen boundary conditions do not introduce singularities known as *Landau poles* in the running of the couplings.[19] In the MSSM, the top and bottom Yukawa couplings are fixed at the top mass scale M_t by $\tan\beta$ and the measured top and bottom quark masses respectively. In particular, the top Yukawa increases with low values of $\tan\beta$. The absence of a Landau pole at the GUT scale sets a lower bound on $\tan\beta$ of $\tan\beta \gtrsim 1.5-2$. Conversely, the bottom Yukawa increases with large values of $\tan\beta$. In turn, avoiding a Landau singularity at the GUT scale sets an upper bound on $\tan\beta$ of $\tan\beta \lesssim 80$. Furthermore, notice that in the large $\tan\beta$ regime $y_t \simeq y_b$ at the electroweak scale. On the other hand, the upper bound on the NMSSM Yukawa coupling λ was calculated in [14] and reads $\lambda \lesssim 0.7$. This bound strongly depends on $\tan\beta$ (or y_t) and on κ. In particular, low values of $\tan\beta$ yield large values of y_t at low energies enhancing the growth of λ at large scales and thus, the bound becomes tighter ($\lambda \lesssim 0.6$). Larger values of κ introduce an analogous effect, they boost the running of λ at large scales and as a result, the upper bound on λ at the low scale is severely reduced see [14].

In addition, it is common practice to only take into account the top Yukawa and neglect all other Yukawa couplings. From the above discussion it follows that this approximation holds as long as $\tan\beta$ is not too large [9]. The NMSSM Yukawa couplings (λ,κ) are unknown and thus cannot be ignored in the RGEs unless one is interested only in particular corners of the parameter space where $\lambda, \kappa \ll 1$. Notice that only λ appears in the MSSM Higgs β-functions and the effect upon the evolution of the couplings is mild. However, λ (and also κ) dominate the evolution of the soft terms associated to the singlet. Thus, if the NMSSM Yukawa couplings are small the corresponding soft terms for the singlet will stay constant and equal to their boundary conditions at $M_{\rm GUT}$.

The evolution of the soft parameters is more involved and fully depends on the chosen boundary condition at $M_{\rm GUT}$. The amount of independent soft parameters is large and the analysis becomes quite involved if all the parameters are completely arbitrary. A minimal approach, motivated by the absence of FCNC (see section 3.5), is to assume universal soft terms at the GUT scale. In this case one can make generic statements upon the structure of soft terms at the electroweak scale as we describe as follows.

To begin with, the one-loop β functions for the gaugino masses defined in (B.6) can be easily integrated and yield [9, 14]

$$M_3 : M_2 : M_1 \simeq 5.5, 1.9, 1\,. \tag{3.27}$$

[19]The following results quoted from the MSSM remain valid in the NMSSM.

Furthermore, the one-loop β-functions only depend on the gauge couplings, hence, the result is identical both in the MSSM and NMSSM. Note that the one loop β-functions of the gaugino masses (and A-terms) do not depend on the soft masses while the soft masses do depend on both gaugino and soft masses. In turn this implies that a theory with non-vanishing M_0 at the GUT-scale and the remaining soft parameters set to zero, i.e. $M_0 \gg m_0$, is phenomenologically viable. Conversely, if the universal soft masses m_0 where the only non-vanishing parameter at the GUT scale, i.e. $m_0 \ll M_0$, gaugino masses would stay massless at tree level and hence, would lead to phenomenologically excluded regions.

Regarding soft scalar masses, the β-functions of the scalar masses of the first two families have only gauge interactions. This implies that if they satisfy universal boundary condition at $M_{\rm GUT}$, they remain diagonal in the family space at low energies. Furthermore, the effect of the renormalization is negligible and therefore stay almost degenerate at low energies. Notice that keeping the first two generations degenerate, as required to avoid FCNC, does not imply that all the soft scalar masses should unify at the GUT-scale. From the observation above, we learn that it is sufficient that the first two generations do. In turn, the third generation of sfermions and the Higgs sector can take different soft masses at the GUT scale.

The soft (MSSM) Higgs masses and third generation of sfermions receive contributions to their β-functions induced by the third generation Yukawa couplings and gauge couplings. In particular, in the Higgs sector the terms proportional to the Yukawas have a positive sign and therefore decrease at lower energies. Note that the dominant term in the running of the soft up-Higgs mass is induced by the top Yukawa. Furthermore, this contribution is sufficiently large to drive $m^2_{h_u}$ to negative values and thus, trigger electroweak symmetry breaking (EWSB). In other words, electroweak symmetry breaking is induced by quantum corrections. This analysis is not modified by the presence of the singlet. As we anticipated above, the presence of the Yukawa coupling λ has a mild effect on the running of the soft MSSM Higgs masses. Moreover, in the EWSB the VEV of the singlet essentially relies on the soft parameters associated to the singlet. The third generation squarks instead receive large radiative corrections induced by the top Yukawa. However, the mixing mass terms can be also large and thus, the eigenstates are not aligned with the original squarks and can be light.

3.5 Spectrum and experimental bounds

In this section we present the spectrum of supersymmetric particles at tree level. We follow [14] in the mass matrices of the Higgs sector, gluinos, charginos and neutralinos, and [95] in the respective masses of squarks and sleptons. In addition we present some relevant experimental constraints, these can be all found in [96, 97].

3.5.1 Masses in the Higgs sector

To compute the tree level masses in the Higgs sector one expands the scalar potential around $\langle h_d\rangle, \langle h_u\rangle, \langle s\rangle$ and extract the quadratic terms, i.e.

$$M^2_{ij} = \left.\frac{\partial^2 V}{\partial h^i \partial h^j}\right|_{min} . \tag{3.28}$$

M_{ij} contains ten (real) fields and decomposes into two 3×3 blocks that mix the three real and imaginary components of the three neutral scalars, and a 4×4 block for the complex charged Higgs components. After subtracting the three Goldstone bosons, the eigenstates are denoted by $(h, H, h_s, A, a_s, H^{\pm})$.

Let us write down the respective mass matrices at tree level. This derivation follows [14] where the soft masses are replaced with the VEVs by means of the tadpole equations in (3.14). In [14] the μ_h term was taken to vanish here we reproduce the result by including the μ_h term. For the CP-even scalars, in the basis $h = \{\mathrm{Re}h^0_d, \mathrm{Re}h^0_d, \mathrm{Re}s\}$ it reads[20]

$$M_{h^ih^j} = \begin{pmatrix} g^2v_d^2 + b\,\tan\beta & (2\lambda^2 - g^2)v_uv_d - b & \Delta_1 \\ & g^2v_u^2 + b\cot\beta & \Delta_2 \\ & & m^2_{h_s} + \delta \end{pmatrix}, \tag{3.29}$$

where we defined

$$\begin{aligned} m^2_{h_s} &= \kappa s(A_\kappa + 4\kappa s + 3\mu_s) - (\xi_s + \xi\mu_s)/s\,, \\ \delta &= \lambda(A_\lambda + \mu_s)v_uv_d/s - \lambda\mu_h\, v^2/s\,, \end{aligned} \tag{3.30}$$

and the mixing terms

$$\begin{aligned} \Delta_1 &= \lambda 2\mu\, v_d - \lambda(b_{\mathrm{eff}} + \kappa s + \mu_s)v_u\ , \\ \Delta_2 &= \lambda 2\mu\, v_u - \lambda(b_{\mathrm{eff}} + \kappa s + \mu_s)v_d\ . \end{aligned} \tag{3.31}$$

By rotating the mass matrix by the angle β, one of the eigenvalues aligns with the SM-like Higgs whose mass bounds are currently 125.15 ± 15GeV[1, 2]. The computation of the rotated squared mass matrix is straightforward and the corresponding diagonal element provides an upper bound to the mass of the Higgs at tree level. This reads

$$M_h^{\mathrm{tree}} \simeq M_z^2\cos^2 2\beta + \lambda^2v^2\sin^2 2\beta\,, \tag{3.32}$$

where (3.15) and (3.16) were used. From (3.32) one recovers a well known result in the MSSM, that states that the tree level Higgs mass is very light and one needs to have large radiative corrections to achieve the 125GeV. In addition, note from (3.32) that this constraint is absent in the NMSSM where, in principle, one can go to the low $\tan\beta$ and large λ regime to match

[20] Recall from (3.28) that the mass matrix is symmetric.

(3.32) with the experimentally measured mass.

Notice that the mixing terms in (3.31) are proportional to λ. Thus in the limit $\lambda \to 0$ the mass matrix becomes block diagonal with a 2×2 block that is identical to the MSSM Higgs mass matrix and an extra 1×1 block that yields the mass of the singlet. Furthermore, in the decoupling limit, $\lambda \to 0$ and $s \to \infty$, $\delta \to 0$ and, thus the mass of the singlet is given by $m^2_{h_s}$. In the Higgs doublet sector the decoupling regime implies $M_s \to \infty$ and hence the light Higgs scalar approaches (3.32) while the heavy Higgs yields

$$m_H^2 \simeq \frac{2b}{\sin 2\beta}\,. \tag{3.33}$$

The CP-odd mass matrix in the basis $a = \{A, a_s\}$ reads[21]

$$M_{a^i a^j} = \begin{pmatrix} \frac{2b}{\sin 2\beta} & \Delta_{Aa_s} \\ & m_{a_s} + \delta_{a_s} \end{pmatrix} \tag{3.34}$$

with

$$\begin{aligned} \Delta_{Aa_s} &= \lambda(A_\lambda - 2\kappa s - \mu_s)v\,, \\ \delta_{a_s} &= \lambda(b_{\text{eff}} + 3\kappa s + \mu_s)h_u h_d/s\,, \end{aligned} \tag{3.35}$$

and

$$m^2_{a_s} = -3\kappa A_\kappa s - 2b_s - \kappa\mu_s s - \xi(4\kappa + \mu_s/s) - \xi_s/s. \tag{3.36}$$

Provided the mixing can be neglected, the eigenstates have masses

$$m_A^2 \simeq \frac{2b}{\sin 2\beta} \tag{3.37}$$

and $m_{s_I} \to m_{a_s}$.

Finally, one finds the charged Higgs masses are given by

$$m^2_{H^\pm} = \frac{2b}{\sin 2\beta} + v^2(\tfrac{1}{2}g^2 - \lambda^2)\ . \tag{3.38}$$

Note, that the masses receives an additional contribution from the singlet and they can be smaller than in the MSSM case. In the limit $\lambda \to 0$ one recovers $m^2_{H^\pm} \simeq m^2_H \simeq m^2_A$.

3.5.2 Squark and slepton masses

The mass terms of squarks and sleptons appear in the Lagrangian with the form

$$\mathcal{L} = -UM_U^2U^\dagger - DM_D^2D^\dagger - EM_E^2E^\dagger \tag{3.39}$$

[21] A is defined via $A = (\cos\beta\,\mathrm{Im}h_u^0 + \sin\beta\,\mathrm{Im}h_d^0)$ and $a_s = \mathrm{Im}s$.

with

$$U = (\tilde{u}_L, \tilde{u}_R^*)\,, \quad D = (\tilde{d}_L, \tilde{d}_R^*)\,, \quad E = (\tilde{e}_L, \tilde{e}_R^*)\,, \tag{3.40}$$

where the family index was omitted. Assuming M_U^2 is diagonal in the family space, the latter reduces to three blocks of 2×2 matrices $M_{U_i}^2$ of the form

$$M_{U_i}^2 = \begin{pmatrix} m_{q_i}^2 + M_{u_i}^2 + L_{u_i} & M_{u_i} X_{u_i}^* \\ M_{u_i} X_{u_i} & m_{u_i}^2 + M_{u_i}^2 + R_{u_i} \end{pmatrix} \tag{3.41}$$

where i labels the generation, $M_{u_i} = y_{u_i} v_u$ is the mass of the i-th generation up-quark and

$$L_{u_i} = \tfrac{1}{6}(4M_w^2 - M_z^2)\cos 2\beta\,, \quad R_{u_i} = \tfrac{2}{3}(-M_w^2 + M_z^2)\cos 2\beta\,, \quad X_{u_i} = A_{u_i} - \mu^* \cot\beta\ . \tag{3.42}$$

Recall that M_w and M_z are the W-boson and Z-boson masses respectively. Similarly, for the down quarks

$$M_{D_i}^2 = \begin{pmatrix} m_{q_i}^2 + M_{d_i}^2 + L_{d_i} & M_{d_i} X_{d_i}^* \\ M_{d_i} X_{d_i} & m_{d_i}^2 + M_{u_i}^2 + R_{d_i} \end{pmatrix}\ . \tag{3.43}$$

Analogously, $M_{d_i} = y_{d_i} v_d$ and

$$L_{d_i} = -\tfrac{1}{6}(2M_w^2 + M_z^2)\cos 2\beta\,, \quad R_{d_i} = \tfrac{1}{3}(M_w^2 - M_z^2)\cos 2\beta\,, \quad X_{d_i} = A_{d_i} - \mu^* \tan\beta\ . \tag{3.44}$$

Note that the mixing terms for the first and second generation of squarks are negligible, furthermore, provided the soft masses are at the TeV-scale one learns that the masses are practically equal to the respective soft masses. Thus, one expects them to be heavy and present data set a generic bound of [96, 97]

$$m_{\tilde{q}} \gtrsim 1.3\text{GeV}\ . \tag{3.45}$$

Conversely, note that the mixing terms for the third generation of squarks can be considerably large. This implies that they can be splitted into large and heavy eigenstates, the current experimental bounds set a lower bound of [96, 97]

$$m_{\tilde{q}_3} \gtrsim 500\text{GeV}\ . \tag{3.46}$$

Finally, for the sleptons one has

$$M_{E_i}^2 = \begin{pmatrix} m_{l_i}^2 + M_{e_i}^2 + L_{e_i} & M_{e_i} X_{e_i}^* \\ M_{e_i} X_{e_i} & m_{e_i}^2 + M_{e_i}^2 + R_{e_i} \end{pmatrix} \tag{3.47}$$

with

$$L_{e_i} = \tfrac{1}{2}(M_z^2 - 2M_w^2)\cos 2\beta\,, \quad R_{e_i} = (M_w^2 - M_z^2)\cos 2\beta\,, \quad X_{e_i} = A_{e_i} - \mu^* \tan\beta\ . \qquad (3.48)$$

For the selectron and smuon it is found [96, 97]

$$m_{\tilde{e},\tilde{\mu}} \gtrsim 275\text{GeV}. \qquad (3.49)$$

3.5.3 Gluino, Neutralinos and Charginos

Gluinos are already written in terms of eigenstates in the Lagrangian, hence, the masses are given by M_3. The experimental bound on the gluino is the most constrained and reads[22] [96, 97]

$$m_{\tilde{g}} \gtrsim 1.5\text{TeV}\ . \qquad (3.50)$$

The Winos $\tilde{W}^{\pm}, \tilde{W}^0, \tilde{B}^0$ together with the Higgsinos $\tilde{h}^0_{u,d}, \tilde{h}^-_d, \tilde{h}^+_u$ and the Singlino $\tilde{s}$ mix in a five-vector of neutral fermions $N = (\tilde{B}^0, \tilde{W}^0, h^0_u, h^0_d, s)$ and two pairs of charged fermions $C^- = (\tilde{W}^-, \tilde{h}^-_d)$ and $C^+ = (\tilde{W}^+, \tilde{h}^+_d)$. The mass matrix are defined as follows

$$\mathcal{L} = -C^- M_C (C^+)^T - \tfrac{1}{2} N M_N N^T + c.c.\,, \qquad (3.51)$$

with M_N and M_C being symmetric matrices

$$M_C = \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2} g_2 v_u} \\ & \mu \end{pmatrix}, \qquad (3.52)$$

and

$$m_N = \begin{pmatrix} M_1 & 0 & -\frac{g_1 h_d}{\sqrt{2}} & \frac{g_1 h_u}{\sqrt{2}} & 0 \\ & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 h_u}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda h_u \\ & & & 0 & -\lambda h_d \\ & & & & 2\kappa s + \mu_s \end{pmatrix}. \qquad (3.53)$$

The eigenstates of M_N are known as the *neutralinos* and usually labeled by $\chi^a,\ a = 1,..,5$ while the eigenstates of M_C receive the name of *charginos* and are denoted by $\chi^{\pm}$. Recall that in the limit of $M_i, \mu \ll M_W$, which is true for most models consistent with present bounds,

[22] This bound assumes that squarks of first and second generation squarks have similar masses. In the case the squarks are much heavier the bound can be relaxed to $m_{\tilde{g}} \gtrsim 1.2$TeV.

the matrices become diagonal. Present bounds on charginos lie above

$$m_{\chi^{\pm}} \gtrsim 330\mathrm{GeV}. \tag{3.54}$$

As we already anticipated the lightest neutralino represents a good dark matter candidate, its mass is usually identified as m_{χ^0}. The respective upper bound on its mass is generically derived indirectly by requiring that it yields a relic abundance in agreement with the experimental value. Notice that the chargino spectrum is identical to the MSSM whereas the neutralino sector is enlarged by the singlino. This yields new possibilities for the dark matter candidate that were thoroughly studied in the literature, a review of dark matter in the NMSSM can be found in the NMSSM [14] (see also references therein).

3.5.4 Flavor and CP violation

Before finishing this section let us comment that flavor and CP violation set stringent bounds on the soft parameters, these are reviewed in [13]. For a comprehensive study of flavor and CP violations in supersymmetric models see [98]. In particular, recall from the definition of the soft terms in (3.4) that the absence of flavor mixing soft masses and Yukawa couplings is not theoretically forbidden. However, family mixing terms in the mass matrix are experimentally highly suppressed and hence, they are set ad-hoc to vanish. The structure of soft terms relies upon the supersymmetry breaking mechanism and, from the known frameworks, only gauge mediation is flavor blind. In gravity mediation, flavor mixing is not protected and the universality of soft terms is assumed based on the above phenomenological arguments. Regarding CP violation, the most compelling bounds originate from the electric dipole moments of the electron and neutron. These originate from complex phases in the soft parameters, in particular, the μ and b terms but also in the $A-$terms and gaugino masses. To circumvent the bounds, soft terms are generically assumed to be real although there is no theoretical reason for complex phases to be suppressed.

3.6 The upper bound on the Higgs mass

In the decoupling limit, i.e. under the condition that the singlet is sufficiently heavy (or provided the mixing terms are negligible) the Higgs mass at one loop can be approximated by [99][23]

$$M_h^2 \simeq M_z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \frac{3M_t^4}{4\pi v^2} \ln \frac{M_{\mathrm{s}}^2}{M_t^2} + \frac{3M_t^4}{4\pi v^2} \tilde{X}_t^2 \left(1 - \tfrac{1}{12}\tilde{X}_t^2\right) \tag{3.55}$$

[23]This expression is derived for the $\mathbb{Z}_3$ version of the NMSSM but the result holds for general versions of the NMSSM after replacing $\lambda\langle s\rangle$ for μ.

where $m^2_{\tilde{t}_{1,2}} \sim m^2_{q_3} \sim m^2_{u_3} \gg M_t^2$ is assumed and $\tilde{X}_t$ is the mixing parameter in the stop sector defined via

$$\tilde{X}_t = (A_t + \mu \cot\beta)/M_s. \tag{3.56}$$

The first two terms in (3.55) represent the tree level Higgs mass, the terms logarithmic in M_s corresponds to the dominant one loop quantum corrections induced by the stop sector and are identical to the MSSM. Note that by setting $\lambda = 0$ the tree level Higgs mass in the MSSM is bounded from above by the Z-boson mass. The upper bound is reached for large values of $\tan\beta$ and requires large radiative corrections to reach the 125GeV. Furthermore, in this regime the measured Higgs mass automatically sets the scale of the susy scale to be $M_s \simeq 4$TeV.

From (3.55) one learns that in the NMSSM, to agree with the experimentally measured Higgs mass, there are several possibilities.

- Adjust M_s as a function of $\tan\beta$.

This possibility is of course incompatible with the assumption of low energy supersymmetry breaking, i.e. $M_s \simeq \mathcal{O}(\text{TeV})$ and the expression for the Higgs mass in (3.55) is no longer valid. We will investigate this setup in chapter 6.

- Introduce large stop mixing.

The contribution from the stop mixing is maximized for $\tilde{X}_t^2 = 6$ [100]. In the large $\tan\beta$ regime the lower bound on M_s can be shifted to $M_s \simeq 1$TeV. However, to realize such values for $\tilde{X}_t$ one typically requires very large A-terms.

- Go to the NMSSM regime

Note from (3.55) that in the NMSSM the Higgs mass receives an additional contribution at tree level induced by the Yukawa interaction that couples the singlet to the MSSM Higgs doublets. As can be read directly from (3.55), this piece is maximized for low values of $\tan\beta$ and large values of λ. Recall from section 3.4, that to stay perturbative up to the GUT scale λ is bounded at low energies by $\lambda \simeq 0.6 - 0.7$ while the pertubativity of the top Yukawa sets a lower bound on $\tan\beta$ of $\tan\beta \gtrsim 1.5 - 2$. Nontheless, the sole NMSSM piece is sufficient to reach the 125GeV without relying on large radiative corrections. However, notice from the Higgs mass matrix that the mixing parameters between the Standard Model Higgs and the singlet in the CP-odd mass matrix scale with λ. Thus, for large values of λ the approximation (3.55) is no longer valid and the respective Higgs mass has to be calculated by diagonalizing the mass matrix. In this situation, unless delicate cancellations between the soft parameters appearing in the mixing terms occur, the mixing severely lowers the Higgs mass with respect to the upper bound dictated by (3.55). A possible way to avoid the effect of the mixing terms is to go to the regime where the singlet is significantly heavier. The upper bound on the Higgs within the NMSSM was studied in [101–104].

3.7 The Little Hierarchy problem

In the previous section we learn that to accommodate the Higgs mass one needs to introduce large radiative corrections. Furthermore, the latter fix the scale of supersymmetry breaking M_s, i.e. the scale of the soft parameters, at a few TeV scale. This statement is generically true for the MSSM and when introducing the singlet the situation can be relieved only for special corners of the parameter space. However, notice from (3.17) that soft terms participate in electroweak symmetry breaking via the value of $\hat{m}$ in (3.18). Thus, provided soft parameters are of order M_s, a cancellation between μ and $\hat{m}$ is required to match M_z with its experimental value. This fact is known as the *little hierarchy problem*.

In models where supersymmetry breaking is mediated at a high scale Λ, e.g. the GUT scale, the supersymmetry-breaking parameters at low scales can be parametrized in terms of the soft terms defined at the GUT scale by means the RGEs. In particular, $m_{h_u}^2(Q)$ reads[24]

$$-2m_{h_u}^2(Q) = c_{M_aM_b}M_aM_b^2 + c_{M_aA_\alpha}M_aA_\alpha + c_{m_i^2}m_i^2\,, \tag{3.57}$$

where the c's are numerical coefficients obtained by means of the RGEs, the subindices denote $a, b = 1, 2, 3$, $\alpha = u, d, e$ and i runs over all the scalars and Q is the renormalization scale.[25] Note that these coefficients are determined by the dimensionless couplings of the theory, i.e. gauge and Yukawa couplings, together with the cutoff scale Λ. Let us make a few remarks

- $m_{h_u}^2$ is not sensitive to those parameters whose respective coefficients are suppressed. In turn, not all soft masses are equally constraint.
- Generically one expects that for lower Λ the tension between the electroweak and the susy scale is more relaxed. However the choice of the cutoff scale is motivated by a UV-setup.

To give some insights in (3.57) the coefficients calculated with the RGEs at the two loop level, within the MSSM with $\tan\beta = 10$ and $Q = 400\text{GeV}$ in [105] are given as follows

$$\begin{aligned}-2m_{h_u}^2(Q) =&3.84M_3^2 + 0.32M_3M_2 - 0.42M_2M_2 - 0.15M_2A_t - 0.65M_3A_t\\ &- 1.27m_{h_u}^2 + 0.73m_{q_3}^2 + 0.57m_{u_3}^2 - 0.11m_{u_2}^2 - 0.11m_{u_1}^2 + ..\end{aligned} \tag{3.58}$$

where the ellipsis denote suppressed contributions. Note that even though H_u does not directly couple to gluinos the latter appear in (3.57) via radiative corrections and with a large coefficient. In addition, for the first and second generation of sfermions, the Yukawa couplings are so small that their main impact on the Higgs potential is through hypercharge D-term contributions.

[24] In this expression it is implicit that soft terms on the r.h.s are defined at the GUT scale.
[25] Recall that $\hat{m}^2 \simeq -m_{h_u}^2$ as long as $\tan\beta$ is not too small.

These manifest in the small coefficients that parametrize the dependence on the corresponding soft scalar masses.

For the particular case of universal soft terms at the GUT scale the expression in (3.58) reduces to

$$-2m_{h_u}^2(Q) = 3.79\,M_0^2 + 0.013\,m_0^2 - 0.82 M_0\,A_0 + 0.22 A_0^2\ . \tag{3.59}$$

In this case the dependence on scalar masses is almost vanishing. This implies that $m_{h_u}^2(Q)$ becomes insensitive to the soft masses. Moreover, the first and second generation of squarks have a mild effect on this result, more specifically, they only contribute with $0.014 m_0^2$. This implies that the assumption of universal soft masses in this sector does not affect the conclusions about the sensitivity of the weak scale with respect to m_0.

3.7.1 Measuring Naturalness

So far there is no hint of supersymmetric particles at the LHC and yet there is no clear consensus on whether the absence of superpartners is enough evidence to abandon the assumption of supersymmetry in nature, or at least its low-energy versions. In order to find an agreement upon this point a *fine-tuning measure* [106] was introduced. The latter quantifies the sensitivity of the electroweak scale with respect to the variations of the fundamental parameters of the theory. The standard prescription to calculate the fine-tuning measure can be summarized by the following procedure [106]

1. Define the fundamental parameters $\{a_i\}$.

2. Calculate the sensitivity parameters Δ_i defined as follows

$$\Delta_i := \left|\frac{\partial \log M_z^2}{\partial \log a_i^2}\right| = \left|\frac{a_i}{M_z^2}\right|\left|\frac{\partial M_z^2}{\partial a_i^2}\right|\ . \tag{3.60}$$

3. Determine the overall measure of naturalness $\Delta := \max\{\Delta_i\}$.

The accepted tolerance for $\Delta_{\max}$ has shifted over time, historically set at $\Delta_{\max} \lesssim 10$ and nowadays increased towards $\Delta_{\max} \lesssim 100$. Let us comment on some of the caveats identified in the procedure outlined above.

(i) The implications of naturalness bounds strongly depend on the choice of framework. However, without the knowledge of the mechanism of supersymmetry breaking the latter is an ad-hoc step. Some economical options minimize the number of fundamental parameters and hence, simplify the conclusions upon the mass scale of the superpartners. However, possible hierarchies or correlations between soft masses that could relax the small hierarchy problem are neglected in these setups.

(ii) Concerning the definition of the fundamental parameters, it is customary to take the (GUT-scale) soft parameters. Instead, other approaches also include the dimensionless couplings of the Standard Model, e.g. gauge and Yukawa couplings. However, in UV-completed scenarios Yukawa couplings are determined independently of the supersymmetry breaking mechanism. In particular, there are known examples constructed from string theory where the Yukawa couplings can only take discrete values that depend on the compactification geometry. In this case, varying the couplings continuously to calculate the fine-tuning would lead to a wrong conclusion. Specifically the top Yukawa is large and therefore, it typically yields a large fine-tuning value.

From the above discussion it becomes clear that the fine-tuning measure should not be taken pragmatically. Though, it can be useful to give estimates on our expectations. The fine-tuning was studied in the MSSM and NMSSM in [107–112] and the situation is similar in both frameworks. In particular, recall from section 3.6 that in the NMSSM there is an additional contribution to the Higgs mass which, in the regime of low $\tan\beta$ and large λ, is sufficiently large to account for the 125GeV. However, as we remarked before large λ usually leads to dramatic mixing of the Higgs with the singlet whose effect can be only alleviated by going to the decoupling limit, $\langle s\rangle \to \infty$. This implies that the $\mu-$term is pushed to larger values and, thus, the little hierarchy problem is restored. In addition, note that $\tan\beta$ is fixed by the soft parameters in (3.19). Generically low values of $\tan\beta$ are obtained at the price of fine-tuning the $b-$term. Finally, it is worth pointing out that the parametrization of $\hat{m}$ in terms of the soft parameters mildly changes when going from the MSSM regime (small λ and moderate $\tan\beta$) to the NMSSM regime (large λ and small $\tan\beta$). Therefore, the coefficient parameterizing the gluino dependence in (3.57) remains large and induces a fine-tuning similar to the one expected in the MSSM.

Chapter 4

Dilaton domination

In this chapter we consider soft terms for a specific gravity mediated scenario where the hidden field responsible for supersymmetry breaking is the dilaton, named the *dilaton dominated* scenario [27, 28]. The dilaton belongs to the spectrum of effective supergravity theories obtained from compactifications of the heterotic string and the respective soft terms are universal at leading order. We study the phenomenology of the dilaton domination both in the MSSM and NMSSM [36]. In particular, in the MSSM we review earlier results given in [34, 37] and we introduce new constraints on the soft parameters and the supersymmetric spectrum according to the present experimental bounds [1, 2, 96, 97]. In the NMSSM, we concentrate on the general case which was not previously studied in the literature. The analysis in this chapter highlights the author's own contribution to [36]. Furthermore, some of the results presented here do not appear in [36] and are the author's own addition to the work in that reference.

4.1 The effective supergravity action

The spectrum of $\mathcal{N} = 1$ heterotic string compactifications in four dimensions fall into two families: matter fields and moduli fields. The first could comprise the field content of the MSSM (or extensions thereof) and we identify them with observable fields. The second often trigger supersymmetry and therefore, are natural candidates for hidden fields. A special member of this family is the dilaton. The specific form of the effective action relies on the details of the compactification considered. However, the dependence of the action on the dilaton at leading order is model-independent. In the remainder of this section we give the respective generic form of the Kähler potential, superpotential and gauge kinetic function where the dependence on the dilaton is manifest. These expressions will be used in the next section to derive the soft terms in the dilaton dominated scenario.

In the following, we collectively denote the matter fields by A, moduli fields t and distinguish the dilaton field by ϕ. The generic structure of the Kähler potential reads [113]

$$K = -\log(\phi+\bar{\phi}) + K^{(0)}(t,\bar{t},A,\bar{A}) + \sum_{n=1}^{\infty}\frac{K^{(n)}(t,\bar{t},A,\bar{A})}{(\phi+\bar{\phi})^n} + K^{(np)}(\phi,\bar{\phi},t,\bar{t})\,, \tag{4.1}$$

where the leading order contributions are given by the first two terms, $K^{(n)}$ are the perturbative quantum corrections generated in an expansion of inverse powers of the dilaton [114, 115] and $K^{(np)}$ denote non-perturbative corrections.[26] The superpotential takes the form

$$W = W^{(0)}(t,A) + W^{(np)}(\phi,t)\ . \tag{4.2}$$

In particular, it has no perturbative corrections [61] but admits non-perturbative effects $W^{(np)}$ which are relevant for supersymmetry breaking. As the notation indicates, $W^{(0)}$ is independent of the dilaton. On the other hand, the gauge kinetic function can be written as follows [114, 116] [27]

$$f_a = k_a\phi + f_a^{(1)}(t,A) + f_a^{(np)}(\phi,t) \tag{4.3}$$

where $k_a = 1$ for each factor of the SM gauge group labeled by a, $f_a^{(1)}$ denotes the one loop radiative corrections and $f_a^{(np)}$ the contribution from non-perturbative effects. In particular, $f_a^{(1)}$ does not depend on the dilaton. Note that, as promised, the dependence on the dilaton in the Kähler potential, superpotential and gauge kinetic function is universal at leading order.

Let us emphasize that a prime example of non-perturbative effect used to break supersymmetry in the hidden sector is *gaugino condensation* [117–123]. The simplest realizations of gaugino condensation fail to generate a potential for the dilaton that breaks supersymmetry. However, more involved constructions exist where this was accomplished [124, 125]. In the next section, we will not specify the dynamics that induce $\langle F^\phi\rangle \neq 0$ but conceive this possibility and proceed to compute the soft terms.

4.2 The dilaton domination

In this section we compute the soft terms for the particular situation where the dominant source of supersymmetry breaking in the hidden sector is the dilaton field, this is known as the *dilaton dominated scenario*. In particular, we start from an $\mathcal{N}=1$ supergravity theory with the Kähler potential, superpotential and gauge kinetic functions specified in (4.1), (4.2) and (4.3) respectively. In the spirit of section 2.2.3, we do not specify the mechanism that triggers supersymmetry breaking but instead derive the soft terms from a few generic assumptions that we recall as follows.

[26] Note that the dependence on the dilaton at leading order is of the no-scale type, see (2.27).
[27] Note that here we use the definition of f given in (2.21).

1. Supersymmetry is broken in the hidden sector, in particular,

$$\langle F^{\phi}\rangle \gg \langle F^{t}\rangle \tag{4.4}$$

   as in the dilaton dominated scenario.

2. $\langle F^A\rangle = 0$, i.e. there is no supersymmetry breaking in the observable sector.

3. $\langle V\rangle = 0$. The cosmological constant vanishes in the minimum.

To compute the soft terms we follow section 2.2.3.[28] To begin with, we expand the Kähler potential defined in (4.1) in powers of the observable fields

$$K(\phi,\bar\phi,A,\bar A) = -\log(\phi+\bar\phi) + Z_{I\bar J}A^I A^J + (+\frac{1}{2}H_{IJ}A^I A^J + c.c.) + .. \tag{4.5}$$

where we only consider the leading order terms in (4.1). Importantly, note that $Z_{I\bar J} \neq Z_{I\bar J}(\phi,\bar\phi)$ and $H_{IJ} \neq H_{IJ}(\phi,\bar\phi)$, i.e. they encode unknown couplings which are not functions of the dilaton. Analogously, we expand the superpotential in (4.2) as a function of the observable fields, this gives

$$W(\phi,A) = W^{np}(\phi) + \frac{1}{2}\tilde\mu_{IJ}A^I A^J + \frac{1}{3}\tilde Y_{IJK}A^I A^J A^K + .. \, . \tag{4.6}$$

Here $\tilde\mu_{IJ}, \tilde Y_{IJK}$ are unknown couplings which do not depend on the dilaton i.e. $\tilde\mu_{IJ} \neq \tilde\mu_{IJ}(\phi)$ and $Y_{IJK} \neq Y_{IJK}(\phi)$. Finally, for completeness we rewrite the tree level gauge kinetic function and gauge coupling respectively as follows

$$f_a = \phi\;, \quad g_a^{-2} = \tfrac{1}{2}(\phi+\bar\phi) \;\; \forall\, a. \tag{4.7}$$

We can now proceed to compute the soft terms by means of the expressions given in section 2.2.3. Specifically, using (4.7) in (2.40) and (4.5) in eqs.(2.36) and (2.38), direct computation yields

$$M_a = \hat F^{\phi}\,, \quad m^2_{I\bar J} = m^2_{\frac{3}{2}} Z_{I\bar J}\,, \quad A_{IJK} = -\hat F^{\phi} Y_{IJK} \tag{4.8}$$

where we defined $\hat F^{\phi} := \frac{F^{\phi}}{(\phi+\bar\phi)}$. The bilinear terms are computed using (4.5) in (2.37). In particular, note that it is necessary to derive $m_{\frac{3}{2}}$ with respect to the dilaton, at this point we use the definition in (2.22). After some manipulations these read

$$b_{IJ} = 2m^2_{\frac{3}{2}}H_{IJ} - (\hat F^{\phi} + m_{\frac{3}{2}})e^{-\frac{1}{2}\log(\phi+\bar\phi)}\tilde\mu_{IJ}\,. \tag{4.9}$$

[28] From now on the moduli fields will be omitted in the discussion. In particular, $K(\phi,\bar\phi,A,\bar A) = K(\phi,\bar\phi,\langle t\rangle,\langle\bar t\rangle,A,\bar A), W(\phi,A) = W(\phi,\langle t\rangle,A)$ and $f_a(\phi) = f_a(\phi,\langle t\rangle)$.

Y_{IJK} and μ_{IJ} are computed from (2.35) and (2.34) and yield

$$Y_{IJK} = e^{-\frac{1}{2}\log(\phi+\bar{\phi})}\tilde{Y}_{IJK}\,, \quad \mu_{IJ} = e^{-\frac{1}{2}\log(\phi+\bar{\phi})}\tilde{\mu}_{IJ} + m_{\frac{3}{2}}H_{IJ}\,. \tag{4.10}$$

Let us make a few remarks. Firstly, provided $\tilde{Y}_{IJK}, \tilde{\mu}_{IJ}$ and H_{IJ} are unknown, Y_{IJK} and μ_{IJ} can be regarded as free parameters. Secondly, recall that the soft scalar masses here are not canonically normalized, furthermore, $m^2_{I\bar{J}} \propto Z_{I\bar{J}}$ and thus, we can already anticipate that they are universal. Note that this is a direct implication of the form of the Kähler potential in (4.5). In particular, the fact that $Z_{I\bar{J}}$ does not depend upon the dilaton implies that $R_{\phi\bar{\phi}I\bar{J}}$ appearing in (2.36) vanishes. Furthermore, the soft gaugino masses are also universal. This could be directly inferred from the gauge coupling constant in (4.7) which does not depend on the gauge group label. Finally, recall that as long as one does not specify the dynamics of the dilaton, the gravitino mass is left unfixed. In turn, it can be considered a free parameter.

The third of our assumptions allow us to further relate the soft parameters. Specifically, F^ϕ is fixed by the gravitino mass via (2.41). Inserting $K^{\phi\bar{\phi}}$, computed from (4.5), in (2.41) yields

$$\hat{F}^\phi = \sqrt{3}\, m_{\frac{3}{2}} \tag{4.11}$$

where we assumed F^ϕ is real. Replacing with (4.11) in (4.8) and using the notation for universal soft terms in (2.17) yields

$$M_0 = \sqrt{3} m_{\frac{3}{2}}\,, \quad m_0 = m_{\frac{3}{2}}\,, \quad A_0 = -M_0\,. \tag{4.12}$$

For the bilinear term b_{IJ} there are two possibilities depending on the origin of the μ_{IJ} term. If $H_{IJ} \neq 0$ and $\tilde{\mu}_{IJ} = 0$, i.e. the mu-term is generated by the Giudice-Massiero mechanism, then (4.9) simplifies to

$$b_{IJ} = 2 m_{\frac{3}{2}} \mu_{IJ} \tag{4.13}$$

where we used that $\mu_{IJ} = m_{\frac{3}{2}} H_{IJ}$. Note that in this case the free parameters of the dilaton dominated scenario reduce to $m_{\frac{3}{2}}$, μ_{IJ} and Y_{IJK}.[29] Finally, if $H_{IJ} \neq 0$ and $\tilde{\mu}_{IJ} \neq 0$ then b_{IJ} can be considered unfixed. Therefore, alltogether one has $m_{\frac{3}{2}}$, μ_{IJ}, b_{IJ} and Y_{IJK} as the free parameters. It is worth mentioning that (4.12) holds at leading order and receive further loop corrections. In particular, recall from (2.40) the gaugino masses get anomaly mediated terms of the form

$$\delta M_a \simeq \frac{b_a}{16\pi^2} m_{\frac{3}{2}} \tag{4.14}$$

with b_a the corresponding one-loop coefficient as defined for (2.31). In addition, note from (4.1) that the Kähler potential receives perturbative corrections, which induce non-universal

[29] Note that if $H_{IJ} = 0$ and $\tilde{\mu}_{IJ} \neq 0$, i.e. the mu-term is only generated as a coupling in the superpotential, then (4.9) reduces to $b_{IJ} = -(\sqrt{3}+1) m_{\frac{3}{2}} \mu_{IJ}$ and the free parameters are as in the constraint case.

contributions to all soft parameters in (4.12). Their magnitude is model-dependent although generically one expects that they are suppressed with respect to the tree level result. The phenomenological implications of the dilaton domination after including these corrections were studied in [35, 115]. In the next chapter we will study the phenomenological implications of these set of soft terms both in the MSSM and NMSSM.[30]

4.3 Phenomenology

For the numerical analysis we use SPheno [128, 129] created by SARAH [130–132]. This incorporates complete one-loop calculations of SUSY and Higgs masses. For the scalar Higgs masses dominant two loop corrections are also included. As the theoretical bound for the Higgs mass we consider the conventional uncertainty of 3GeV [133].

4.3.1 Dilaton domination in the MSSM

Using (3.4) we can identify the corresponding soft terms in the dilaton dominated scenario in the MSSM.[31] Recall that the Yukawa couplings y_u, y_d, y_e are fixed and there is only one mu-term μ_h with soft bilinear b_h. In sum, taking the soft scalar masses m_0 as a free parameter and using (4.12) we have

$$M_0 = \sqrt{3} m_0 \,, \quad A_0 = -M_0 \tag{4.15}$$

where the subindex 0 indicates that this boundary condition is set at the GUT scale and we omit additional indices. On the other hand, there are the two possibilities for the b_h-term which depend on the origin of the μ_h-term in the scalar potential.

Case I:

The b_h term is given by (4.13), i.e.

$$b_{h_0} = 2\mu_{h_0} m_0 \ . \tag{4.16}$$

Note that before electroweak symmetry breaking the dilaton domination consists of two independent parameters μ_{h_0} and m_0, furthermore, the spectrum is symmetric under $\mu_h \to -\mu_h$, therefore one can always choose $\mu \geq 0$. However, requiring the correct Z-boson mass through (3.17) reduce these to only one free parameter.

Case II:

b_h is considered a free parameter (see discussion in section 4.2). In this case one has three independent parameters μ_{h_0}, m_0 and b_{h_0}, that after electroweak symmetry breaking reduce to two. It is customary to use the μ-parameter to fix the electroweak scale and trade b_h by $\tan\beta$

[30] In [126] it was suggested that in the dilaton domination in the MSSM global charge and color breaking vacua are generated. However, the lifetime of the (local) electroweak vacuum is significantly longer than the age of the Universe [127] and thus, the dilaton domination can be considered as phenomenologically viable.

[31] Recall that to recover the MSSM from the NMSSM definitions it is sufficient to set $S = 0$.

defined in (3.19).[32] In sum, the free parameters of this scenario are: m_0, $\tan\beta$ and the sign of μ_h.

Let us begin by studying the Higgs mass in this setup. As already anticipated the case I has only one free parameter m_0. μ_h is fixed by (3.17), this relation forces μ_h to take large values, i.e. of the order of (or above) m_0. This can be seen from the fact that the RGEs drive $\hat{m}^2$ to large values which, in turn, need to be canceled by μ_h^2 to yield the observed Z-boson mass. Interestingly, the Giudice-Massiero relation in (4.16) together with μ completely fix β in (3.19). In particular, the value of $\sin 2\beta$ in (3.19) is of order one and, in turn, $\tan\beta$ is very low ($\tan\beta \simeq \mathcal{O}(1)$). Recall that in the MSSM the Higgs mass at one loop can be approximated by (3.55) with $\lambda = 0$. From this expression one learns that for low $\tan\beta$ the tree level contribution to the Higgs mass is almost vanishing. If the supersymmetry breaking scale is expected to lie within the TeV scale, radiative corrections cannot achieve the 125GeV. In conclusion, this constrained scenario cannot provide a sufficiently large Higgs mass, as already concluded in [37].

In the more general case II, the freedom to set $\tan\beta$ to large values relaxes the tension discussed above. In this regime, the tree level contribution in (3.55) reaches its upper bound and the Higgs mass becomes independent of $\tan\beta$ and effectively just a function of m_0. This behaviour was also observed in [37]. Note that the dependence upon m_0 is induced via the radiative corrections. Furthermore, requiring that the Higgs mass lies within the experimental bound completely fixes m_0. In Figure 4.1 we show the Higgs mass as a function of m_0 for large $\tan\beta$ and both possibilities for the sign(μ_h). Ignoring any fine-tuning discussion, one observes that the radiative corrections can push the Higgs mass up to the observed value. Moreover, the effect of the mixing between stops introduced in (3.55) and parametrized by (3.56) is also manifest. The distinct signs of μ_h yield different values for the mixing parameter in (3.56) and therefore, induce differences in the Higgs mass. In particular, the Higgs mass is larger (smaller) for positive (negative) sign of μ_h respectively. In addition, let us point out that the top Yukawa coupling induces large contributions to the running of the stop soft masses. Recall from (3.20), that the top Yukawa is completely fixed by the experimental value of the top mass. Therefore, varying the top mass within the uncertainty bounds yield differences in the stop masses at low energies and hence on M_s given in (3.23). As seen in (3.55), slight changes of M_s can shift the Higgs mass. More precisely, for larger (smaller) M_t lead to smaller (larger) M_h respectively. In the following we always use the central value for M_t and hence do not vary the top Yukawa coupling. In [36] we give the range of m_0 consistent with the measured Higgs mass, this can be obtained by looking for the minimum (maximum) values of m_0 respectively

[32] Note that M_z depends on the squared value of μ_h and thus, it leaves the sign of μ_{h_0} unfixed.

that yield M_h in 125 ± 3GeV. We can extract these from Figure 4.1, they read[33]

$$550\text{GeV} \lesssim m_0 \lesssim 1620\text{GeV}\ (\mu_{h_0} > 0) \qquad 620\text{GeV} \lesssim m_0 \lesssim 1800\text{GeV}\ (\mu_{h_0} < 0)\ . \tag{4.17}$$

Regarding the heavy Higgs, m_H obeys (3.33) and is expected to lie in the TeV range. Furthermore, recall from the discussion in section 3.5 that in this regime the mass of the heavy Higgs is degenerate with pseudoscalar boson mass and the chargino masses, i.e.

$$m_H \simeq m_A \simeq m_{H^\pm}\ . \tag{4.18}$$

In Figure 4.2 (left) we show the numerical result for m_A as a function of the Higgs mass.

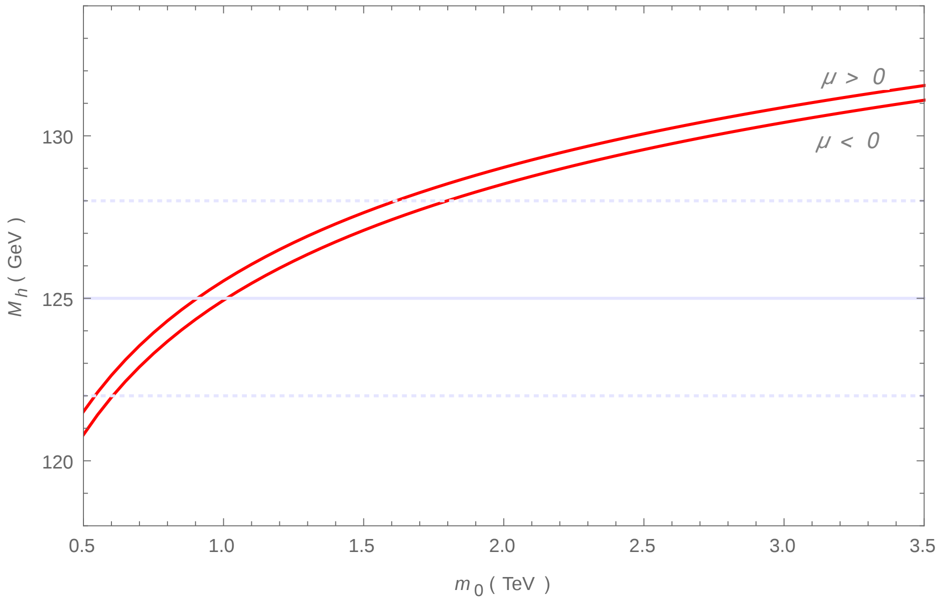

Figure 4.1: *Higgs mass as a function of the soft mass parameter for* $\tan\beta = 15$ *and* $\mu_h < 0\,(> 0)$. *Indicated is the central value for the Higgs mass together with the theoretical uncertainty (dotted lines).*

Let us now turn to the neutralino sector. This consists of the bino and wino gauginos together with the two higgsinos. Using (3.53) one can extract the MSSM mass matrix of the neutralinos by setting the row and column that involves the singlino to vanish. Futhermore, note that the mixing terms are negligible and therefore the eigenvalues can be approximated by

$$m_{\chi^1} \simeq M_1\,, \quad m_{\chi^2} \simeq M_2\,, \quad \text{and} \quad m_{\chi^{3,4}} \simeq \mu_h\ . \tag{4.19}$$

In particular, M_1 and M_2 are determined at low energies via (3.27). By means of (3.27)

[33] Let us point out that the central value of the Higgs mass together with the central value of the top Yukawa coupling used here correspond to the latest LHC data [1, 2] and are slightly shifted from the ones used in [36]. Therefore the bound reported in this thesis does not coincide with the one published.

together with the relation between M_0 and m_0 in the dilaton dominated scenario in (4.15), one can derive the expression for the gaugino masses in terms of the gravitino, or m_0, mass [37]. In particular, the lightest neutralino is an almost pure bino-neutralino and its mass reads

$$m_{\chi^1} \simeq 0.8 m_{\frac{3}{2}} \ . \tag{4.20}$$

Analogously, the mass of the wino yields

$$m_{\chi^2} \simeq 1.5 m_{\frac{3}{2}} \ . \tag{4.21}$$

The higgsinos scale with μ_h and hence are the heaviest neutralinos. Note that one loop corrections to $m^2_{h_u}$ induce relevant corrections to the outcome of μ_h in (3.17). The numerical results for the neutralino masses are shown in Figure 4.2 (right) as a function of the Higgs mass. Note that the Higgs mass sets a lower bound on the bino mass of

$$m_{\chi^1} \gtrsim 500\text{GeV} \, . \tag{4.22}$$

This can be derived from Figure 4.2 (right) by requiring that $M_h \gtrsim 122$GeV.

The gluino mass coincides with the soft gaugino parameter M_3. Following the same procedure as for the gauginos, M_3 can be parametrized in terms of the gravitino mass [37]. Specifically, employing (3.27) and (4.15) one learns that the gluino mass behaves like

$$m_{\tilde{g}} \simeq 4 m_{\frac{3}{2}} . \tag{4.23}$$

The numerical result of the gluino mass is shown in Figure 4.2 (left) as a function of the Higgs mass. Using Figure 4.2 and requiring that the Higgs mass is in the range $M_h \gtrsim 122$GeV one can extract a lower bound for the expected gluino mass [36]. This yields

$$m_{\tilde{g}} \gtrsim 2.3\text{TeV} \tag{4.24}$$

and thus, it can be observed in the near future. Finally, sleptons are generically lighter than squarks and the spectrum of masses is at the TeV scale. In Table 4.1 we show a benchmark point for large $\tan\beta$ and m_0 within the bound in (4.17). In particular, one observes that sleptons are within $800 - 1300$GeV whereas squarks are found between $2 - 3$TeV. The lightest squarks are the stops as a result of the large mixing.

Before proceeding let us make a few remarks

- By looking at the higgsino masses in Figure 4.2 (right) in the window consistent with the Higgs mass one learns that the values of μ_h are found within $1.5 - 3.5$TeV. Having a μ-term above the TeV scale signals a little hierarchy problem (see section 3.7) which cannot be avoided in the dilaton dominated scenario. m_0 is fixed by the Higgs mass, therefore the soft parameter $m^2_{h_u}$ is also fixed. As anticipated $m^2_{h_u} \simeq \mathcal{O}(M_s^2)$ and the μ_h term needs to cancel this large contribution to the Z-boson mass in (3.17) leading to a little hierarchy problem.

- The bino neutralino is the LSP. The latter is a good dark matter candidate and the study of the implications of having a bino LSP where presented by the authors collaborators in [36]. Here we will not expand on this subject any further. For a review of bino dark matter in the MSSM see [134].

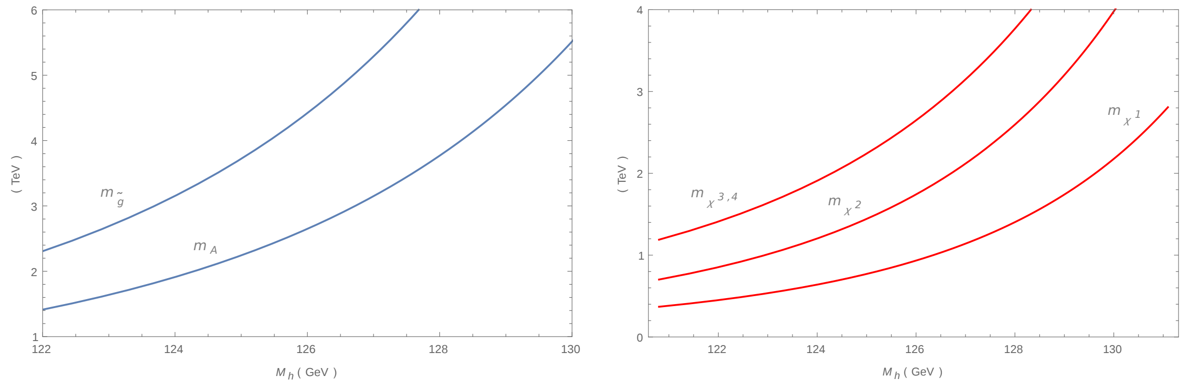

Figure 4.2: *(left) Gluino and heavy Higgs masses as a function of the Higgs mass. (right) Neutralino masses as a function of the Higgs mass.*

To conclude, the dilaton dominated scenario in the MSSM is phenomenologically viable. Furthermore, the entire spectrum is effectively a function of only one parameter: the gravitino mass. By demanding that the Higgs mass is consistent with observation we derived a bound for the gravitino mass given in (4.17). The latter fixes the masses for all superpartner masses which are mainly found at the TeV scale, we give a benchmark example in Table 4.1. As a result the model is very predictive and some regions of the parameter space could be tested in the near future. In particular, the lower bounds on gluino and squark masses are within the regions might be soon probed at the LHC [96, 97].

4.3.2 Dilaton domination in the NMSSM

Proceeding as in section 4.3.1, we begin by identifying the free parameters in the dilaton dominated scenario in the NMSSM. In particular, in this case there are two free Yukawa couplings κ, λ and two mu-terms μ_h, μ_s with soft bilinear couplings b_h and b_s respectively. Soft terms are as in (4.15) and we consider two possibilities for the b-terms. One option takes the

b-terms as additional free parameters, the second uses the relation in (4.13) i.e.

$$b_{h_0} = 2\mu_{h_0} m_0\,, \quad b_{s_0} = 2\mu_{s_0} m_0. \tag{4.25}$$

Altogether, before electroweak symmetry breaking the free parameters in the NMSSM are $(\lambda,\kappa,m_0,\mu_{h_0},b_{h_0},\mu_{s_0},b_{s_0},\xi_0,\xi_{s_0})$ when b-terms are free parameters, $(\lambda, \kappa, m_0, \mu_{h_0}, \mu_{s_0}, \xi, \xi_{s_0})$ when the b-terms obey (4.25). As in the MSSM electroweak symmetry breaking fixes one parameter. Again the tadpole equations allow to trade $\tan\beta$ and here also the vev of the singlet s by two of the soft parameters. Furthermore, in the case where b-terms are independent one can shift the vev of s and remove one of the dimensionful parameters, we adopt $\xi = 0$. In sum we have

$$(m_0, \tan\beta, \lambda, \kappa, \mu_s, b_s, s) \tag{4.26}$$

and

$$(m_0, \tan\beta, \lambda, \kappa, \mu_s, \mu_h) \tag{4.27}$$

in the constrained version. The phenomenology of the dilaton domination in the singlet-extended MSSM was studied for the particular $\mathbb{Z}_3$ version in [135]. The Higgs mass in this setup was found below the experimental bound and therefore the model is not viable. In the remaining of the chapter we study the dilaton domination in the general NMSSM following [36]. Generically, due to the large amount of free parameters a model-independent analysis is quite involved. However, one can recognize special regions of parameter space with distinct features in the spectrum. We discuss these as follows and present them with explicit examples in Table 4.1.

Let us begin by identifying those features that are identical to the MSSM case. As explained in section 3.4, gaugino masses are completely fixed by the gauge parameters and M_0 via the relations in (3.27). The Yukawa couplings associated to the singlet appear in the running only at the two loop level and their effect on gaugino masses is negligible. In turn, the gluino, bino and wino masses satisfy the relations in (4.23), (4.20) and (4.21) respectively. Interestingly, by means of (4.23) and demanding that the gluino mass satisfies the experimental lower bound given in (3.50), we derive a lower bound on m_0 of

$$m_0 \gtrsim 340\text{GeV}. \tag{4.28}$$

Note that this is an absolute lower bound on m_0 and it is independent upon the Higgs mass.

Let us now turn to the Higgs sector. In particular, we distinguish between the two limiting cases of small and large values of λ. In the regime of large λ the mixing terms in the Higgs mass matrix given in (3.31) significantly increase. In turn, the Higgs mass obtained after diagonalizing the mass matrix in (3.29) severely decreases. In order to suppress the mixing one can look for the regions in the parameter space where the singlet becomes very heavy and decouples, e.g. by chosing large μ_s. In this situation the mass of the singlet can be approximated at tree leve by $m^2_{h_s}$ in (3.30). Assuming the singlet is sufficiently heavy, the Higgs mass can be parametrized at one loop via (3.55). Furthermore, for low values of $\tan\beta$ the tree level Higgs mass can be considerably larger than in the MSSM case. In this regime, the Higgs mass as a function of m_0 does no longer behave as in Figure 4.1 but it is heavier and the lower bound on m_0 is set by (4.28). The lower and upper bounds on $\tan\beta$ and λ respectively are completely fixed by the absence of Landau poles in the RGEs (see discussion in section 3.4). These read $\tan\beta \gtrsim 1.5$ and $\lambda \lesssim 0.65$.[34] We show explicit examples of this scenario in the benchmark points NMSSM1, NMSSM2 and NMSSM3 in Table 4.1. In particular, note that m_0 is consistent with (4.28) but lower than the MSSM bound in (4.17). Furthermore, the Higgs mass agrees with observation. On the other hand, in the limit of $\lambda \to 0$ the mixing terms are extremely suppressed and the analysis of the Higgs mass resembles the MSSM case. In particular, the dependence upon m_0 is identical to Figure 4.1 and the bounds derived in (4.17) prevail. An explicit example of this case is given in the benchmark point NMSSM4 in Table 4.1. Furthermore, in this regime the mass of the singlet is again approximated by (3.30) and depending on the values of the soft parameters associated to the singlet, the latter can be lighter or heavier than the Higgs.

The neutralino sector involves the bino and wino gauginos together with the higgsinos and the singlino. Provided the mixing terms between higgsinos and the singlino in the mass matrix defined in (3.53) can be neglected in the diagonalization, the respective higgsino and singlino masses yield

$$m_{\chi^{3,4}} \simeq \mu \quad \text{and} \quad m_{\chi^5} \simeq 2\kappa s + \mu_s \; . \tag{4.29}$$

This behavior can be obtained in two different regimes: by requiring that λ is extremelly small or by demanding that $m_{\chi^5} \gg M_{\rm ew}$. The former case allows the singlino mass to take any value in the spectrum. In particular it can be the lightest neutralino and there is no lower bound for its mass. The specific value of the singlino mass is fixed by the particular values chosen for the soft parameters appearing in (4.29). This result is completely different from the situation in the MSSM where the lightest neutralino is always the bino. This situation is presented in the benchmark point NMSSM4 in Table 4.1, specifically note that the lightest neutralino is the singlino. Furthermore, in this regime the effective mu-term induced by the singlet $\mu_{\rm eff}$ defined in (3.12) vanishes and $\mu \simeq \mu_h$. In turn, the dependence of the higgsino masses as a function of m_0 is as in Figure 4.2.[35] On the other hand, the situation where the singlino is heavy is more model dependent. The particular value of λ and the soft parameters

[34]Note that this bound corresponds to λ at low energies. The respective value of λ at $M_{\rm GUT}$ is $\lambda_0 \simeq 1.5$ and was used in the examples in Table 4.1.

[35]Note that the derivation of this relation assumed relatively large values of $\tan\beta$ which is necessary to achieve the experimentally observed Higgs mass. This analysis prevails in the limit $\lambda \to 0$

in the singlet sector determine both the higgsino and singlino masses via (4.29). In this regime, the lightest neutralino is again the bino neutralino. However, note that it can be lighter than in the MSSM case. In particular, for large λ and low $\tan\beta$ one can use (4.28) and (4.20) to calculate the respective lower bound on the bino mass in this regime. This yields

$$m_{\chi^1} \gtrsim 270\text{GeV} \tag{4.30}$$

which, as promised, is lower than in the MSSM. See the examples NMSSM2 (3,4) in Table 4.1, in these the lightest neutralino is a pure bino but with a lighter mass than in the MSSM example given in the same table.

Let us conclude by commenting on the spectrum of sfermions. The β-functions of the first and second generation of soft squark and slepton masses does not receive any contribution associated to the soft parameters in the singlet sector. As shown in section 3.5, for the first two generations the mixing is negligible and the physical masses can be approximated by the soft masses. Therefore, the computation is identical to the MSSM case. However, recall that within the MSSM the dilaton domination has only one mass parameter m_0 that fixes the entire spectrum. The range for sfermion masses obtained in this case correspond to values of m_0 within the bound consistent with the Higgs mass given in (4.17). As we discussed above, in the NMSSM lower values of m_0 are allowed in the regime of large λ and small $\tan\beta$. Therefore, in the NMSSM the mases of sfermions for the first two generations can be lighter than in the MSSM. The analysis of the third generation sfermions is more subtle. For non-negligible λ there is a small effect on the running of the soft masses. In addition, the mixing terms in the mass matrix depend on μ and $\tan\beta$. In particular, in the low $\tan\beta$ regime the outcome for the stop and sbottom masses can be quite different. As an example consider the benchmark point NMSSM1 (and also $2, 3$) in Table 4.1, there squarks and sleptons are lighter than in the MSSM benchmark point given in the same table. Note that m_0 in the NMSSM case is lower than the minimum in (4.17).

Let us finish with a few additional remarks

- The analysis of the little hierarchy problem in this scenario is more involved due to the large amount of free parameters. However, generically we expect it to be as severe as in the MSSM (see discussion in 3.7). This is manifest in the examples provided in Table 4.1 where, for the different regimes of λ and $\tan\beta$, the μ_h parameter is found above the TeV scale.

- In the MSSM the LSP is the bino neutralino. In the NMSSM the singlino can be the LSP. The latter is also a good dark matter candidate and can change the constraints derived for the bino LSP [36]. For an overview on singlino dark matter see e.g. [14] and [136].

To conclude, the dilaton domination in the NMSSM is phenomenologically viable. The

	MSSM	NMSSM1	NMSSM2	NMSSM3	NMSSM4
m_0 [GeV]	860	500	500	500	1400
$\tan\beta$	36.3	2.7	2.8	2.7	33
μ_h [GeV]	tad	1060*	1000	1350*	2700*
b_h [GeV^2]	tad	$2.7\cdot10^5$*	$2\,m_0\,\mu$	$9.7\cdot10^5$*	$2.6\cdot10^5$*
λ	–	1.5	1.5	1.5	0.003
κ	–	-0.5	-0.58	-0.5	0.14
s [GeV]	–	500	1330*	-102*	3440
μ_s [GeV]	–	-5000	5400	-5000	-128
b_s [GeV^2]	–	2850^2	$2\,m_0\,\mu_s$	$8.2\cdot10^6$	$6.3\cdot10^5$
ξ [GeV^2]	–	0	$-2.5\cdot10^6$*	0	0
ξ_s [GeV^3]	–	$-3.7\cdot10^9$*	$-3.3\cdot10^9$*	0	$-4.3\cdot10^9$*
$m_{\tilde{g}}$ [GeV]	3100	1900	1900	1900	4800
m_{squark} [GeV]	2100-2900	1600-1800	1300-1750	1300-1750	3300-4500
m_{slepton} [GeV]	800-1300	600-750	600-750	600-750	1300-2050
$m_{\tilde{\chi}^\pm}$ [GeV]	1250	710	710	710	2050
M_h [GeV]	123.4	125.7	123.6	127.2	125.9
m_{h_2} [GeV]	1385	1600	1550	1580	1140
m_{A_1} [GeV]	1410	785	785	785	1680
$m_{\tilde{\chi}^0}$ [GeV]	700	390	390	390	540
$\tilde{\chi}^0$ bino part	0.999	0.998	0.998	0.998	$\mathcal{O}(10^{-9})$
$\tilde{\chi}^0$ wino part	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-10})$
$\tilde{\chi}^0$ higgsino part	0.001	0.002	0.002	0.002	$\mathcal{O}(10^{-8})$
$\tilde{\chi}^0$ singlino part	–	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	1

Table 4.1: *Benchmark points for the MSSM and the NMSSM. All input parameters except s and $\tan\beta$ are given at the GUT scale. Values marked with a * are output values at the electroweak scale determined by the electroweak symmetry breaking conditions.*

main differences with the dilaton domination in the MSSM are summarized as follows. Firstly, we found that the bound on m_0 can be lower than in the MSSM case. This is possible in the large λ and small $\tan\beta$ regime due to the additional contribution to the Higgs mass at tree level. In turn, the spectrum can be lighter. Secondly, for $\lambda \to 0$ and large $\tan\beta$ we recovered the results of the previous section. However, the lightest scalar and neutralino can be the singlet and the singlino respectively. This is remarkably different from the MSSM case where the lightest neutralino is always the bino. Explicit examples of both regimes were given in Table 4.1.

Chapter 5

Hiding the little hierarchy problem

In this chapter we consider soft terms that can be naturally embedded in higher dimensional orbifold GUTs. Assuming that the respective cutoff scale is below the Planck scale, we discuss the effective four dimensional theories in global supersymetry. In this context, soft terms arise from non-renormalizable couplings between observable and hidden fields and were computed in [29]. Here we study a specific scenario known as gaugino mediation. Firstly, we look for a special relation between soft gauginos and soft scalar masses that could solve the little hierarchy problem. Furthermore, based on the work in [50] we show a naïve example that yields this relation. Finally, we analyzed the phenomenology of this class of models. The results of this chapter follow [29] and are focused on the author's own contribution to that reference.

5.1 Generalities of higher dimensional orbifold GUTs

In this chapter we consider embeddings of the NMSSM where the underlying theory is a non-renormalizable theory with global supersymmetry. Particular realizations of this class of theories are orbifold compactifications of higher dimensional GUTs [38–43]. These were mostly studied in the context of effective field theory and are regarded as intermediate descriptions of a more fundamental UV-setup. Explicit examples of higher dimensional GUTs in asymmetric orbifold compactifications of the heterotic string were constructed see e.g.[45–48].

Higher dimensional GUTs are d-dimensional theories, usually with $d = 5$ or 6, that are compactified on orbifolds to yield supersymmetric embeddings of the Standard Model in four dimensions. These higher dimensional theories incorporate grand unified gauge groups e.g. $SU(5)$ or $SO(10)$ and for $d = 5, 6$ they have eight supercharges. The compactification reduces half of the supersymmetry and also breaks the unified gauge group down to the SM gauge group, i.e.

$$G_{\text{GUT}} \xrightarrow{M_c} G_{SM} \,. \tag{5.1}$$

M_c is the compactification scale defined via $M_c \simeq R^{-1}$ with R being the radius of the compactification. Note that M_c fixes the scale of gauge coupling unification, $M_{\rm GUT} = M_c$, which is matched to the theoretical prediction of 10^{16}GeV. In sum, at $M_{\rm GUT}$ the effective four dimensional theory has $\mathcal{N} = 1$ supersymmetry and the gauge content of the SM. Furthermore, the details of the compactification are chosen such that the spectrum is composed by the observable sector, i.e. the MSSM or extensions, and a presumed hidden sector responsible for supersymmetry breaking. Extra matter fields receive masses of $\mathcal{O}(M_{\rm GUT})$ and decouple from the low energy theory.

Importantly, higher dimensional theories are inherently non-renormalizable and, hence, they can only be regarded as effective theories valid up to a given cutoff Λ. As long as

$$\Lambda \ll M_{p,d} \tag{5.2}$$

gravitational interactions are sub-leading and the effective four dimensional theory can be discussed in the framework of global supersymmetry. In particular, the computation of the effective action relies on the localization of fields in the extra dimensions. Specifically, fields that are confined to a four dimensional subspace (e.g. brane or fixed point) are denoted as brane fields whereas fields that propagate in d-dimensions are denoted as bulk fields. The standard setup is to assume gauge and Higgs fields live in the bulk while the remaining matter fields and also the hidden field sit at different fixed points. The localization of fields has implications for supersymmetry breaking and, more importantly, for the structure of the soft terms. In particular, soft terms are generated by local non-renormalizable couplings between the hidden field and observable fields in the bulk. In section 5.2 we proceed to compute the soft terms for the particular configuration of fields specified above.

5.2 Gaugino mediation

In this section we compute the soft terms for the NMSSM under the special assumption that the latter is embedded in a higher dimensional orbifold GUT. Our starting point is a four dimensional theory at the GUT scale with global supersymmetry. The derivation that follows is independent of the numbers of extra dimensions and the choice of unified gauge group, but only relies on the localization of fields in the extra dimensions. In particular, we follow [30, 31, 137] in that gauge fields and MSSM Higgses propagate in the bulk whereas the three families of fermions together with the singlet multiplets sit at an orbifold fixed point. It is worth pointing out that the first and second generation of fermions need not be located at the same fixed point and this choice has no effect upon the conclusions presented in this section. In addition, we consider that a hidden field Σ is confined to a fixed point, different from the one where the singlet and 3rd generation of fermions are localized. Furthermore, we presume

Σ gets a VEV that triggers supersymmetry breaking. Recall that Σ couples via local operators to the fields in the bulk, i.e. gauge and Higgs superfields. However, couplings between Σ and fields located at a spatially separated fixed point, i.e. the singlet and matter fermions, can only develop radiatively and are generically suppressed. In turn, from the localization of fields described above the latter implies that gaugino and soft scalar Higgs masses are sizable at tree level while soft terms for the remaining NMSSM fields vanish. This mechanism is known in the literature as *gaugino mediation* [30, 31, 137].

The soft terms originate from non-renormalizable couplings that involve Σ, F_Σ and the observable fields in the bulk. These couplings depend on the dynamics of supersymmetry breaking and, provided the supersymmetric theory at $M_{\rm GUT}$ is known, they can be explicitly calculated. In the spirit of [27], we do not specify the mechanism responsible for supersymmetry breaking but rather parametrize its effects via the supersymmetry breaking parameter $\langle F_\Sigma \rangle$. The computation of the soft terms for non-renormalizable global supersymmetric theories is, thus, performed in a model-independent way and the outcome is given in terms of the Kähler potential, the superpotential and the gauge kinetic function. Assuming the hidden field decouples at low energies, the computation of the effective scalar potential for the observable fields yields the renormalizable supersymmetric piece as in (2.8) plus the soft terms in (2.16). The scale of the soft terms in this case is defined via

$$m_{\rm soft} = \frac{F_\Sigma}{\Lambda} \ . \tag{5.3}$$

The explicit expressions can be found in appendix A together with the details of the calculation.

We proceed to specify the soft terms for the setup described above. We consider universal couplings between the hidden fields and the bulk fields and hence, specify the Kähler potential, superpotential and gauge kinetic function at leading order via

$$\begin{aligned} K &= Z(\hat\Sigma, \hat{\bar\Sigma})\ (\,|H_u|^2 + |H_d|^2) + (\frac{1}{2}\mu_K\,(\hat\Sigma, \hat{\bar\Sigma}) H_u H_d + \text{c.c.})\ , \\ f_a &= h(\hat\Sigma)\ , \quad \mathrm{W} = \mu_{\rm W}\,(\hat\Sigma) H_u H_d\ , \end{aligned} \tag{5.4}$$

where we defined $\hat\Sigma = \frac{\Sigma}{\Lambda}$ and a labels the gauge factors of the Standard Model gauge group. Without further details on the higher dimensional theory, the functions $Z, h, \mu_K, \mu_{\rm W}$ are unknown. The last term in K gives rise to the μ_h and b_h parameters via the Giudice-Masiero mechanism [88]. μ_h receives an additional contribution from $\mu_{\rm W}$ and thus does not have to be of order $m_{\rm soft}$. Therefore we treat μ_h as a free parameter (i.e. do not address the μ-problem).

By means of (A.15) and (A.10) in appendix A we calculate the soft gaugino and soft scalar masses. Written in terms of canonically normalized fields, these yield

$$M_0 = F_\Sigma\, \partial_\Sigma \log \mathrm{Re}(h)\,, \quad m_0^2 = -|F_\Sigma|^2 \partial_\Sigma \bar\partial_{\bar\Sigma} \log Z\ \ . \tag{5.5}$$

From (5.5) we derive a relation between the soft gaugino and soft higgs masses to be

$$M_0 = k\, m_0\,, \quad k = \frac{\partial_\Sigma \log \mathrm{Re}(h)}{(-\partial_\Sigma \bar{\partial}_{\bar{\Sigma}} \log Z)^{\frac{1}{2}}}\ . \tag{5.6}$$

Furthermore, the leading order contribution of this relation is obtained by expanding Z and h in powers of $\hat{\Sigma}, \hat{\bar{\Sigma}}$

$$Z \simeq 1 + \rho\,|\hat{\Sigma}|^2 + \ldots\ , \quad h \simeq 1 + \gamma\hat{\Sigma} + \ldots\ , \tag{5.7}$$

where the numerical coefficients ρ, γ are unknown constants. A linear term in Z, $\rho_1\,(\hat{\Sigma} + \hat{\bar{\Sigma}})$ can be absorbed in ρ by a field redefinition of the form $H_{u,d} \to (1+\rho_1)H_{u,d}$ and $\rho \to \rho - \rho_1^2$. Inserting (5.7) into (5.6) yields

$$k = \frac{1}{2}\frac{\gamma}{(-\rho)^{\frac{1}{2}}}\ . \tag{5.8}$$

5.3 Hiding the little hierarchy problem

5.3.1 A low electroweak scale from a special gaugino-scalar mass relation

In this section we will compute the values of k that solves (or hide) the little hierarchy problem. In particular, we will calculate the k for which $\hat{m}$ is suppressed with respect to the supersymmetry breaking scale, i.e.

$$\hat{m} \ll M_{\rm s}\ . \tag{5.9}$$

Note, however, that this does not imply that the fine tuning is relieved. In order to have this suppression one requires a very precise value for k and small deviations from this value would spoil the necessary cancellations. Therefore, such a relation should be regarded as the outcome of a UV completed theory. In section 5.5 and following [50] we will show an example where the k can be of the desired magnitude.

The behaviour in (5.9) where certain parameter's renormalization group trajectories meet for a family of ultraviolet boundary conditions is referred to as *focus point* [138]. Recall from (3.59) in section 3.7 that for the customary choice of universal boundary conditions at the GUT scale, $\hat{m}$ becomes almost indifferent to variations in the GUT-scale soft masses, see (3.59). This fact motivated the idea that if gauginos were much lighter than soft scalar masses, supersymmetric models would achieve (5.9) [138]. The present bounds exclude this possibility however, new versions of the focus point idea were exploited [139, 140]. In particular, within the MSSM correlations between non-universal gaugino masses [141] were studied. In addition, a special relation between soft scalar masses and gaugino masses assuming universal soft terms was investigated in [50].

5.3.2 Calculation of k

Following the derivation in section 5.2 we consider the following non universal soft terms at the GUT scale

$$\begin{aligned} &m_0^2 = m_{h_u}^2 = m_{h_d}^2 \ , \qquad m_q^2 = m_u^2 = m_d^2 = m_l^2 = m_e^2 = m_s^2 = 0, \\ &M_0 = M_{a=1,2,3} \ , \qquad A_\lambda = A_\kappa = A_u = A_d = A_e = 0 \ , \quad b_s = \xi_s = 0 \end{aligned} \tag{5.10}$$

while the parameters b_{h_0} and μ_{h_0} are left free. Furthermore, we assume the ($\mathbb{Z}_3$) NMSSM superpotential (3.3) in addition to a non-vanishing μ_h-term. Note that the parameters given in (5.10) are flavor-diagonal but they are non-universal in that the soft Higgs masses differ from the soft sfermion masses.

To compute the soft terms at M_s, the one-loop renormalization group equations (RGEs) given in appendix B and threshold corrections of the soft Higgs masses given in (3.25) are also included. The gauge couplings are fixed at the GUT scale by $\alpha_0 = \alpha_2 = \alpha_3 = \frac{3}{5}\alpha_1 \simeq 0.04$ and only the top Yukawa (y_t) and the NMSSM Yukawa couplings (λ,κ) are taken into account while all other Yukawa couplings are neglected. This approximation holds as long as $\tan\beta$ is not too large [9]. In sum, the free parameters before electroweak symmetry breaking are

$$M_0, \ m_0, \ \mu, \ \tan\beta, \ \lambda_0, \ \text{and} \ \kappa_0 \ , \tag{5.11}$$

where μ_h and b_h have been traded for μ defined in (3.12) and $\tan\beta$ defined in (3.19) respectively. From the RGE one obtains the soft Higgs mass parameters at low energy in terms of the GUT parameters. Explicitly one finds

$$m_{h_{i=u,d}}^2 = \alpha_i(\lambda_0, \kappa_0, \tan\beta)\, M_0^2 + \beta_i(\lambda_0, \kappa_0, \tan\beta)\, m_0^2 \ , \tag{5.12}$$

where α_i, β_i are functions of the Yukawa couplings which can be computed numerically and we replaced the top Yukawa by $\tan\beta$ using (3.20).

Using the assertion

$$M_0 = k\, m_0 \ , \tag{5.13}$$

we computed the values of k for which $\hat{m}$ in (3.18) is suppressed with respect to the supersymmetry breaking scale, i.e.

$$\hat{m} \ll M_\text{s} \ . \tag{5.14}$$

Inserting the soft Higgs masses (5.12) and m_0 from (5.13) into (3.18) one obtains

$$\hat{m}^2 = c(\lambda_0, \kappa_0, \tan\beta, k) \ M_0^2 \ , \tag{5.15}$$

where c can be expressed in terms of α_i, β_i, k and $\tan\beta$. For the Yukawa coupling κ at low energy we use $\kappa \sim 0.4 - 0.6$ which corresponds to $\kappa_0 \sim O(1)$. Thus, effectively $\hat{m}^2$ at low energy is parametrized by

$$\hat{m}^2 = c(\lambda_0, \tan\beta, k)\ M_0^2. \tag{5.16}$$

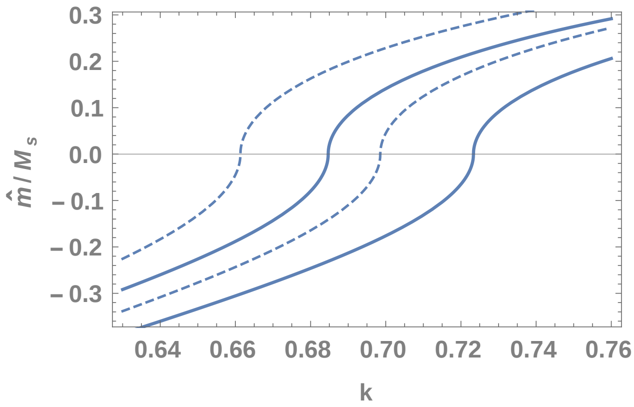

Figure 5.1: $\hat{m}/M_s$ *for different values of* λ_0*, from left to right* $\lambda_0 = 0.4, 0.001$, $\tan\beta = 6, 15$ *(dashed,thick) and fixed* $\kappa \simeq 0.4 - 0.6$*,* $M_s = 3$ *TeV.*

For $\lambda_0 \ll 1$ the singlet decouples and the Higgs sector is effectively the Higgs sector of the MSSM. In this case the Higgs mass reaches its upper tree level bound for large values of $\tan\beta$ and thus allows for $M_s = \mathcal{O}(1\text{TeV})$. From Figure 5.1 we see that in the regime $\lambda_0 \ll 1$ and for $0 \lesssim \hat{m} \lesssim 0.2\, M_s$, the range of the required k take values in the narrow range

$$0.70 \lesssim k \lesssim 0.76 \quad \text{(effective MSSM)}\ . \tag{5.17}$$

On the other hand, a phenomenologically interesting region in the NMSSM corresponds to low $\tan\beta$ and large λ_0. In this regime the tree level value of the Higgs mass is maximized and can take larger values than in the MSSM case. However, too small values of $\tan\beta$ imply a large cancellation of the two terms that contribute to $\hat{m}$ in (3.18), due to the fact that $m^2_{h_d}$ is large at low energies. Hence we only consider moderate values of $\tan\beta\,(\simeq 10)$, for which $\hat{m}^2 \simeq -m^2_{h_u}$. Analogously, too large values of λ_0 induce large μ_{eff}, e.g. for $M_s = 3$TeV the upper bound $\lambda_0 \lesssim 0.4$ corresponds to $\lambda \lesssim 0.33$ and $\mu_{\text{eff}} \lesssim 500$GeV. Moreover, the upper bound on λ_0 is lowered for larger M_s.[36] Notice that these constraints exclude the appealing regime of the NMSSM where the Higgs mass can get a larger tree level contribution. Requiring $0 \lesssim \hat{m} \lesssim 0.2\, M_s$, $5 \lesssim \tan\beta \lesssim 15$ and $0.01 \lesssim \lambda_0 \lesssim 0.4$ the range of k widens

$$0.66 \lesssim k \lesssim 0.76 \quad \text{(NMSSM)}. \tag{5.18}$$

[36] The parameter A_κ is negligible at low energy and thus can be disregarded in the calculation of $\langle s \rangle$. However, ξ_s can get sizable radiative corrections provided λ_0 is not too small. Similarly, m_s^2 and b_s are the dominant contribution to $\langle s \rangle$ when $\lambda_0 \to 0$. $\langle s \rangle$ is computed from (3.13).

The b_h parameter can be adjusted to give the desired values of $\tan\beta$. Using (3.19) and (5.18) the values of b_h that give $5 \lesssim \tan\beta \lesssim 20$ are within the range

$$0 \lesssim b_h/M_0^2 \lesssim 0.4\,. \tag{5.19}$$

Finally, in Figure 5.2 we show that as promised $\hat{m} \ll M_\mathrm{s}$ for different values of $\tan\beta$ and k.

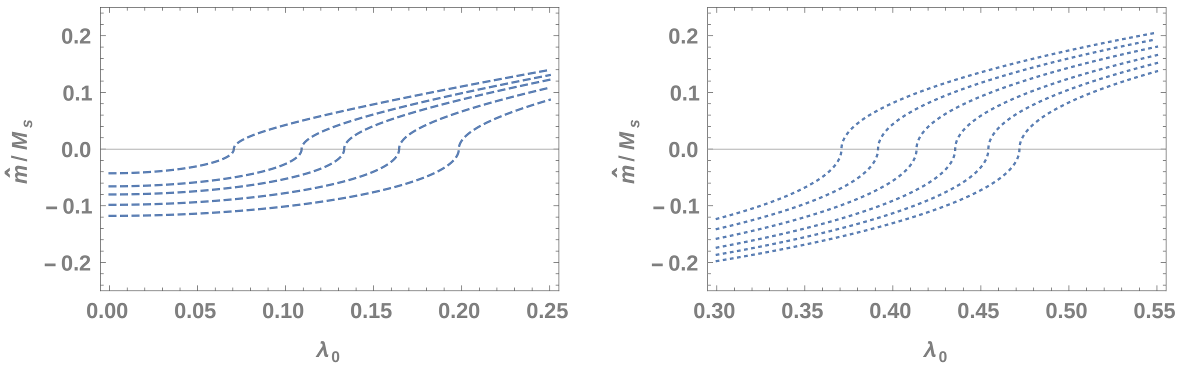

Figure 5.2: *$\hat{m}/M_\mathrm{s}$ plotted as a function of λ_0, with $k = 0.71$ (left plot), $k = 0.67$ (right plot), increasing $\tan\beta$ between 6 and 12 from left to right and with $M_\mathrm{s} \simeq 3$TeV. We see that $\hat{m}$ is $\mathcal{O}(M_z)$ (region between -0.2 and 0.2) for a broad range of $\tan\beta$ and λ_0.*

5.4 Phenomenology

Using (3.55) we find that for $M_\mathrm{s} \sim 3-6\,$TeV, $\tan\beta \simeq 10$ the Higgs mass is consistent with the measured value [1, 2] $M_h = 125.1$GeV within an uncertainty of 3 GeV and we checked that the mixing of the singlet with the Higgs is negligible in this range of parameters. The above values of M_s correspond to $M_0 \sim 1.5-3.5$TeV. The gluino mass M_3 obtained from the RGEs and stop masses calculated from (3.24) are

$$M_3 \sim 4-8\text{TeV}\ , \qquad m_{\tilde{t}_{1,2}} \sim 3-6\text{TeV}\ . \tag{5.20}$$

From M_3 the wino and bino masses are computed via the standard relations [9, 14] in (3.27) giving

$$M_2 \sim 1400-3000\,\text{GeV}, \qquad M_1 \sim 700-1600\,\text{GeV}\,. \tag{5.21}$$

As we already discussed the Higgsino masses scale with μ, which is bounded from below by 100 GeV. Since $\hat{m}$ is of the order of the electroweak scale we need to have μ in a similar range to obtain the correct Z boson mass. As a consequence the Higgsino masses turn out to be a few hundred GeV.

On the other hand, in the effective MSSM region we find that for very small λ_0 ($\mathcal{O}(10^{-4})$) the neutral singlet can become lighter than the Higgs. In this regime, the singlino is the lightest neutralino (see Figure 5.3). Moreover, a light singlet can yield significant changes in the Higgs decay constants that are consistent with the present LHC bounds. Experimental signatures have been recently studied and provide predictions for the next run [142–144]. For larger values of λ_0 the singlet and singlino become heavy $\mathcal{O}(\text{TeV})$.

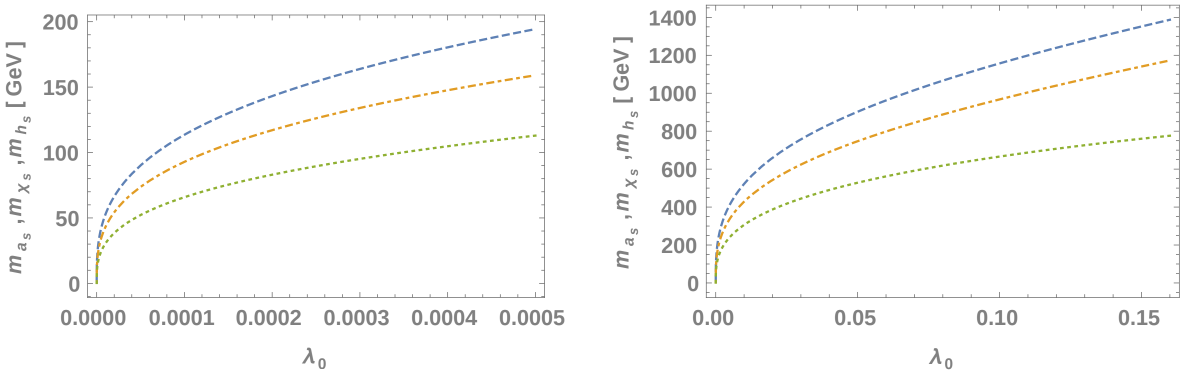

Figure 5.3: *In the left figure we show the masses of the singlet scalar (dashed), singlino (dashed-dotted) and singlet pseudoscalar (dotted) for low values of λ_0 and $M_s = 3$TeV. In the right plot the masses of the singlet and singlino are plotted for the same values of M_s and larger values of λ_0. One can see that they rapidly increase with λ_0.*

The singlet pseudoscalar turns out to be also very light and its mass strongly depends on the λ_0 coupling (see Figure 5.3). In the scale invariant version of the NMSSM, the potential exhibits an approximate global $U(1)$ R-symmetry. This symmetry was first discussed in [145] and it is exact at the GUT scale with $A_\lambda = A_\kappa = 0$ as set in (5.10) and becomes approximate at low energies via the radiative corrections to the A-terms. The symmetry is spontaneously broken when the scalars, h_u, h_d and s get a VEV. The corresponding pseudo-Goldstone boson is the singlet pseudoscalar. Here the symmetry is already broken by the μ_h and b_h terms terms at the GUT scale, however, provided that these together with λ_0 are small, the mass is slightly corrected from the scale invariant NMSSM case. Moreover, it can modify the Higgs boson decays, the collider signatures of this scenario have been studied and are consistent with present LHC bounds [142–144, 146, 147].

The spectrum of sleptons and squarks of the first and second generation resembles that of the MSSM in gaugino mediated scenarios [30, 148]. In particular, both generations stay nearly degenerate with squarks are heavier than sleptons. The lightest sleptons are the right-handed ones and their masses lie below the bino neutralino within $m_{\tilde{e}_R} \simeq 600 - 1300\,\text{GeV}$. However, recall from section 3.7 that the soft masses first and second generation of squarks have mild influence on the determination of $\hat{m}$. In turn, the requirement of universal soft masses is unnecessary for these fields. Hence, for a different boundary condition they could be found anywhere in the spectrum, not forbidden by experimental bounds.

The masses of the heavy Higgs scalars can be approximated by (3.37) Notice that the

Parameters	P1	P2	P3	P4
λ_0	0.33	10^{-4}	0.1	10^{-3}
M_0 [GeV]	2000	2500	3000	3500
m_0^2 [GeV^2]	$7 \cdot 10^6$	$9.5 \cdot 10^6$	$1.35 \cdot 10^7$	$1.75 \cdot 10^7$
m_{h_s} [GeV]	1850	114.5	907.4	178.3
M_h [GeV]	123.6	126	125.7	127.9
$m_H, m_{H^\pm, m_A}$ [GeV]	2824	3434	4067	4660
m_{a_s} [GeV]	1040	66.65	561	108.8
$m_{\tilde{\chi}_s}$ [GeV]	1659	93.65	814.4	147.8
$m_{\tilde{\chi}_{\mu_1}}$ [GeV]	491	695	693	766.2
$m_{\tilde{\chi}_{\mu_2}}$ [GeV]	497	700	696	770
$m_{\tilde{\chi}_{\rm bino}}$ [GeV]	880	1106	1335	1569
$m_{\tilde{\chi}_{\rm wino}}$ [GeV]	1642	2056	2473	2893
$m_{\tilde{g}}$ [GeV]	4070	5145	6104	7047
$m_{\rm squark}$ [GeV]	2680-3760	3330-4630	3930-5480	4540-6310
$m_{\rm slepton}$ [GeV]	667-1300	840-1620	1000-1940	1180-2250

Table 5.1: *Examples of mass spectra computed with SPheno [128, 129] created by SARAH [130–132]. We used* $\tan\beta = 15$, $\kappa_0 = 1$ *and* $M_s = 3, 3.8, 4.5, 5$*TeV (from left ro right in the table).*

largest contribution is given by $m_{h_d}^2$. For moderate values of $\tan\beta$, $m_{h_u}^2$ and μ^2 bounded by the requirement in (5.9). Furthermore, in this regime $m_{h_d}^2$ is roughly RGE invariant and stays near its boundary condition at the GUT scale i.e. $m_{h_d}^2 \simeq m_0^2$. Using the range of M_0 and k derived above we obtain

$$m_A \simeq m_H \simeq m_{H^\pm} \simeq 2 - 5\text{TeV}\,. \tag{5.22}$$

We cross check the results with a modified version of SPheno [128, 129] created by SARAH [130–132].[37] This performs a complete one-loop calculation of all SUSY and Higgs masses and includes the dominant two-loop corrections for the scalar Higgs masses. We show several benchmark points in Table 5.1, in particular, the spectrum for large λ_0 in P1 and P3, for small λ_0 in P2 and an intermediate value of λ_0 in $P4$.

5.5 A naïve example

In the following we would like to show an naive example where the k obtained in the previous section can be realized under specific assumptions on the hidden sector. Recall that couplings in the four-dimensional theory, including those that induce the soft terms, have a different mass dimension in the higher dimensional theory. The most natural choice is to set them to be of $\mathcal{O}(1)$ in units of the cutoff, which is the only mass scale in the higher dimensional effective field theory description. In principle, there is no argument that prevents these coefficients to

[37] We thank Kai Schmidt-Hoberg and Florian Staub for helping with the program.

be suppressed with respect to Λ, but without the knowledge of the UV setup this information remains unknown. Nevertheless, by direct computation of the quantum loop corrections at scales near Λ and, furthermore, requiring that these are $O(1)$ one can naively set an upper bound on the couplings. Interestingly, if the coupling in the 4D theory at the matching scale was fixed, this relation can be used to set an upper bound on the cutoff. Conversely, by fixing Λ one could use this bound to set a maximum size for the 4D couplings. Using this procedure to estimate either Λ or the couplings in the effective 4D theory receives the name of Naïve Dimensional Analysis (NDA) [149, 150]. Note that NDA relies on the assumption that near Λ the effective field theory approach breaks down and the theory becomes strongly coupled. For a comprehensive review on NDA together with the prescription on how to implement it see [151].

The corresponding ratio between gaugino and soft scalar masses by means of NDA was computed in [50]. Furthermore, it is completely determined in a d-dimensional theory in terms of Λ and the volume of the extra dimensions V_{d-4}. This relation is explicitly given by

$$M_0 = k\, m_0\,, \quad \text{with } k = \left(\frac{l_d}{l_4 \Lambda^{d-4} V_{d-4}}\right)^{\frac{1}{2}}, \tag{5.23}$$

with l_d a numerical factor $l_d = 2^d \pi^{d/2} \Gamma(d/2)$. Λ is bounded from above by the Planck scale in d-dimensions, which is defined via

$$M_{p,d} = \left(\frac{M_p^2}{V_{d-4}}\right)^{\frac{1}{d-2}}. \tag{5.24}$$

We calculate k replacing $V = (2\pi R_l)^d$ with $R_l^{-1} \simeq M_{\text{GUT}}$ and $1.24 M_{\text{GUT}} \lesssim \Lambda \lesssim M_{p,d}$ (the lower bound of Λ is determined by the absence of FCNC, see discussion below). For $d = 5$ it yields

$$0.3 \lesssim k \lesssim 0.8 \tag{5.25}$$

and thus provides the coefficient in (5.18) with the expected size. For $d = 6$ or larger the out coming k is smaller than the required values. Notice that k decreases with Λ, in particular, if Λ takes the value of the Planck mass in 5 dimensions k is too small to account for the necessary values.

Let us mention that FCNC in these models are absent as long as the cutoff is sufficiently large. More precisely, dangerous terms are generated through loops in the extra dimensions and scale like $\propto e^{-\Lambda L}$ with L the distance between the branes [137], here $L = 2\pi R$. A suppression consistent with experimental bounds ($\lesssim 4 \cdot 10^{-4}$) implies $\Lambda L \gtrsim 7.8$, see [30]. Thus, we must require a lower bound on Λ of $\Lambda \gtrsim 1.24\, R^{-1}$. On the other hand, as stated above Λ is bounded from above by $M_{p,d}$ which, for $V = (2\pi R_l)^d$ and $R_l^{-1} \simeq M_{\text{GUT}}$ in $d = 5$, yields $\mathcal{O}(10^{17})$GeV so the window for Λ is quite constraint.

As a final remark, recall that spontaneously broken supergravity yields a gravitino mass $m_{\frac{3}{2}} \simeq \frac{F_\Sigma}{\sqrt{3}M_p}$ (see definition in (2.41)). The relation between Λ and M_p is model dependent, however, as long as $\Lambda \ll M_\mathrm{p}$ the soft terms that correspond to gravity mediated interactions are sub-leading and thus can be neglected. Moreover, the gravitino mass generically appears as the lightest supersymmetric particle (LSP) and is a good dark matter candidate [152]. For a study on gravitino dark matter in gaugino mediation see [153]. One can estimate $m_{\frac{3}{2}}$ by using Λ as in the calculation of soft terms for $d = 5$ and $m_\mathrm{soft} \simeq M_0$. This yields

$$m_{\frac{3}{2}} \simeq \mathcal{O}(0.006 - 0.06)\, M_0. \tag{5.26}$$

Replacing M_0 as calculated in section 5.4 we find $m_{\frac{3}{2}} \simeq 10 - 100\,\mathrm{GeV}$ and thus the gravitino can be the LSP.

Chapter 6

High-Scale Supersymmetry

In this chapter we consider soft terms at arbitrary high supersymmetry breaking scales M_s. In particular we study high-scale supersymmetry in the NMSSM. This framework assumes that the low energy effective theory is the Standard model. More importantly, the matching conditions to the supersymmetric theory fix the parameters of the Standard Model and in particular the quartic coupling $\lambda(M_s)$. In turn, the Higgs mass is completely determined and, thus, consistency with the measured value can be used to constrain the size of M_s and the remaining parameters of the supersymmetric theory. Using the results in [22] we review the running of the SM parameters when defined at arbitrary high scales.[38] Futhermore, following [54] we compute the matching conditions to the NMSSM and present the results of the Higgs mass as a function of the NMSSM parameters and M_s.

6.1 The Standard Model as an effective theory

The Standard Model Higgs potential reads[39]

$$V_{SM} = \frac{1}{2}\lambda(|H|^2 - v^2)^2 \tag{6.1}$$

where $v = \sqrt{2} \cdot 174.1\text{GeV}$ is the Higgs vacuum expectation value.[40] Considering one loop threshold corrections, the value of λ determines the Higgs mass M_h via

$$M_h^2 = v^2(\lambda + \delta_\lambda)\,, \tag{6.2}$$

[38] In particular for the discussion we follow the short review [154]

[39] Note that λ also denotes the Yukawa coupling in the NMSSM. Historically, the quartic coupling is parametrized by λ. Therefore, to avoid confusion in this chapter we denote the Yukawa coupling by y_s.

[40] Often in the literature another convention for v is used, without the $\sqrt{2}$. In that case, $M_h^2 = 2\lambda v^2$. The result is of course independent of this definition.

and the top Yukawa coupling y_t is fixed by the mass of the top quark through

$$y_t = \frac{M_t}{v}\sqrt{2}(1+\delta_t)\,. \tag{6.3}$$

δ_λ and δ_t parametrize the threshold corrections at one loop and are defined at the renormalization scale which is conventionally taken at M_t. In particular, they were computed in [155] and [156] respectively. The state-of-the-art computations are given at the two loop level and dominant three loop correction for y_t in [19].

Notice that the experimental values of M_h and M_t completely fix λ and y_t respectively at the renormalization scale. Hence, by means of the RGEs of the Standard Model one can obtain their values at arbitrary high scales. The expressions for the RGEs of the Standard Model at three loops can be found in [19]. In Figure 6.1 (right) we show the evolution of the Standard Model parameters calculated at two loops for $M_h = 125$GeV and $M_t = 173.34$GeV. These were obtained using SPheno-3.3.6 [128, 129] created by SARAH-4.5.8 [130, 132, 157]. In particular, in Figure 6.1 (left) we show the evolution of the top Yukawa coupling together with the gauge couplings. One can recognize that gauge coupling unification is spoiled in the SM compared to the MSSM, but they become *close* at energy scales near 10^{14-15}GeV. The top Yukawa coupling decreases at high energy due to α_3 effects and eventually becomes smaller than the gauge couplings.

In Figure 6.1 (right) we show the evolution of the Higgs quartic coupling. It is small at the electroweak scale $\lambda(M_t) \simeq 0.25$ and decreases at higher scales.[41] The running of λ (β_λ) at low energies is dominated by the top Yukawa which contributes with a term that scales with y_t^4. This dependence explains the sensitivity of the running of λ to M_t and is illustrated by the dotted curves in Figure 6.1 corresponding to a 3σ interval of M_t. The larger (smaller) M_t the steeper (slower) the slope of λ. At larger energies the contribution from the top Yukawa competes with those from the gauge couplings which are comparable in size. Both add up approximately to zero, $\beta_\lambda \simeq 0$ and, thus, yield constant values of λ in this regime. Note that λ becomes negative at scales near $\mu_0 \simeq 10^{10}$GeV. As previously emphasized, the precise value of λ at high energy scales has large uncertainties coming from the measurement of the top mass. However, the possibility of λ staying in the positive regime, with the present bounds on M_t, is excluded. Let us conclude by pointing out that provided the Standard Model is embedded in a supersymmetric theory, the value of λ at the matching scale M_s should be precisely the corresponding value of $\lambda(\mu = M_s)$ in Figure 6.1 (right) to predict a Higgs mass consistent with the experimental measurement. Therefore, it is clear that $\lambda(M_s)$ should be negative if the SUSY-breaking scale lies beyond μ_0. Conversely, given λ at the matching scale, the Higgs mass is completely determined. As we will study in section 6.2, in the NMSSM $\lambda(M_s)$ is

[41] This value of λ and also in Figure 6.1 use the convention of defining λ with the factor $\frac{1}{2}$ in (6.1).

explicitly specified in (6.12) and, thus, particular sets of parameters can be probed.

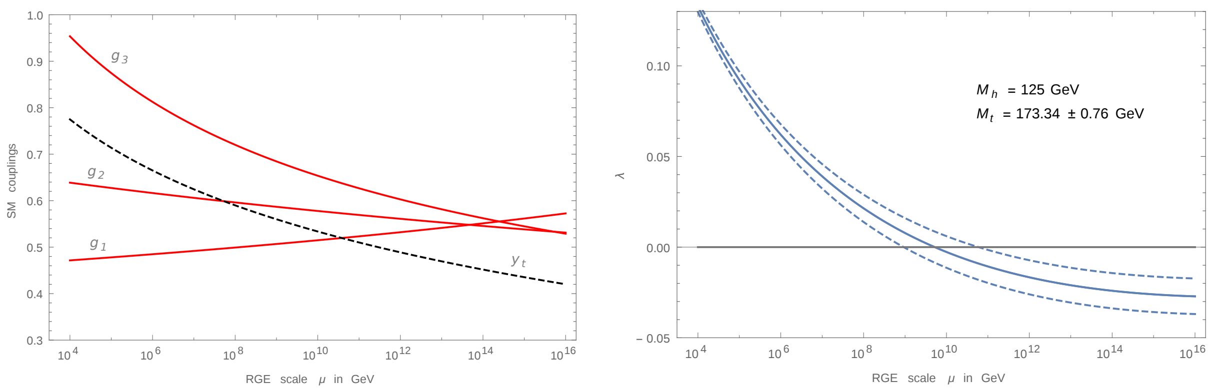

Figure 6.1: *(left) Evolution of g_1, g_2, g_3 and y_t in the SM as a function of the energy scale μ in GeV. (right) Evolution of λ in the SM as a function of the energy scale μ in GeV. The dashed curves are the corresponding evolutions when varying the top quark mass within the experimental bound.*

6.1.1 Vacuum instability

Recall the statement from the previous section that the quartic coupling λ becomes negative at energies above $\mu_0 \simeq 10^{10}$. At large Higgs field values h the potential is dominated by the quartic term, to a good approximation $V(\mu \gg M_t) \simeq \lambda(\mu \sim h)h^4$. For $\lambda < 0$ the latter implies that the potential becomes unbounded from below. One might assume (or hope) that some unspecified physics near the Planck scale could restore the boundedness of the Higgs potential. However, the fact that between $10^{10} - 10^{19}$GeV λ is negative means that the vacuum we live in is not the true vacuum and hence there is a non-zero probability of tunneling into the unstable direction. The state-of-the art computation of the tunneling probability is performed in detail in [19] (see references therein for earlier results). In particular, it was found that the average lifetime is longer than the age of the Universe and this property of the vacuum is defined as *meta-stability*. From the above one can conclude that the instability does not require the existence of new physics that stabilizes the potential and the extrapolation of the Standard Model up to arbitrary high scales is, in principle, consistent. A curious observation is that this conclusion would be radically different if the Higgs mass would be slightly different. In particular, a smaller value would lead to the instability regime and hence, the presence of new physics would be mandatory at an intermediate scale. The condition for stability parametrized in terms of the top mass is given by [19]

$$M_t < 171.36 \pm 0.46\text{GeV} \tag{6.4}$$

which compared to the experimental value of M_t manifests that the stability of the Standard Model up to the Planck scale is disfavored (and in a conservative approach excluded).

It is worth finishing this section by recalling that the above considerations hold under the assumption that no new physics exist between the electroweak and the GUT or Planck scale. When introducing supersymmetric embeddings at energy scales above the instability scale this statement might change [158–160]. In particular, the result is sensitive to higher order operators and these depend on the supersymmetric model considered at high energies. In this work we did not compute the tunneling probability for the regions in parameter space that predict $\lambda(M_s) \lesssim 0$, however, we remark that avoiding the unstable regime could introduce additional constraints on the parameter values.

6.2 High-Scale Supersymmetry in the NMSSM

6.2.1 Matching conditions

Let us consider the scenario in which above the (not necessarily low) supersymmetry breaking scale (M_s) the theory is described by the NMSSM with the superpotential in (3.1), where κ and ξ are set to vanish. After supersymmetry breaking the scalar potential of the Higgs sector develops soft terms given by (3.4) with vanishing A_κ, ξ_s.[42] The explicit computation of the scalar CP-even Higgs mass matrix squared one learns that at the scale M_s the following condition[43]

$$b^2 \simeq (m_{h_u}^2 + \mu^2)(m_{h_d}^2 + \mu^2) \tag{6.5}$$

generates a massless Higgs field given by the combination

$$H_{SM} = \sin\beta H_u - \cos\beta\epsilon H_d^* \tag{6.6}$$

where ϵ is the antisymmetric tensor and the angle β is determined by

$$\tan^2\beta = \frac{|m_{h_d}^2 + \mu^2|}{|m_{h_u}^2 + \mu^2|} \tag{6.7}$$

with m_{h_u}, m_{h_d} and μ evaluated at the scale M_s. On the other hand, the heavy Higgs is given by the following combination

$$H_A = \epsilon\sin\beta H_d^* + \cos\beta H_u \tag{6.8}$$

[42] It is worth pointing out that, unless they are forbidden by a symmetry, effective soft cubic and linear terms for the singlet can be generated radiatively. As long as y_s is small, the latter are suppressed and can be neglected. Throughout this chapter we will not consider them in the calculations.

[43] Here we used that the off-diagonal terms in the mass matrix (of the three CP-even scalars) that mix the MSSM Higgses with the singlet are of order $\simeq M_{\rm ew} M_s$ and thus can be neglected in the diagonalization.

with mass

$$m_A^2 \simeq m_{h_u}^2 + m_{h_d}^2 + 2\mu^2. \tag{6.9}$$

The solution of the singlet equation of motion can be easily computed and yields[44]

$$\langle s\rangle = \frac{\left(y_s\left(\mu_s + A_\lambda\right)\sin 2\beta/2 - y_s\,\mu_h\right)\langle H_{SM}\rangle^2}{m_s^2 + b_s + \mu_s^2 + y_s^2\langle H_{SM}\rangle^2}\,. \tag{6.10}$$

After replacing $\langle s\rangle$ back in the potential, it is straightforward to compute the mass of the singlet, which reads

$$m_{h_s}^2 \simeq m_s^2 + b_s + \mu_s^2 \tag{6.11}$$

and, as expected, it is $\mathcal{O}(M_s^2)$. From (6.10) one learns that $\langle s\rangle \ll M_{\rm ew}$ and thus the singlet contribution to μ is suppressed and can be neglected. Moreover, this implies effective quadratic terms are sub-leading and hence the fine-tuning conditions in (6.5) and (6.7) are identical to the MSSM case [15–17, 51–53].

It is worth pausing at this point and recall that the fine-tuning required in (6.5) restores the hierarchy problem. As an answer to this issue, it was suggested long ago that the origin of the weak scale can be understood in terms of environmental selection [18]. This proposal is built upon the idea that provided a large amount of vacua exist, the size of the electroweak scale could be the outcome of an anthropic choice. In the context of string theory, the amount of vacua was estimated to be of order of 10^{500} and is called the landscape [161]. Furthermore, the mechanism of selection of vacua is known as the anthropic principle and it was originally applied to justify the cosmological constant problem [162]. Although these arguments are highly speculative, we do not attempt to solve (or explain) the hierarchy problem and presume the condition (6.5) is satisfied.

Let us further assume the generic situation where the supersymmetric particles get masses of $\mathcal{O}(M_s)$ and can be integrated out, leaving an effective Standard Model description at energies below the cutoff scale M_s. In particular, the explicit computation of the effective Lagrangian provides the boundary condition for the quartic coupling in the Standard Model potential given in (6.1). These are the so called matching conditions and at tree level they read

$$\lambda^{\rm tree}(M_s) = \frac{1}{4}\left(g_2^2 + \frac{3}{5}g_1^2\right)\cos^2 2\beta + \frac{1}{2}y_s^2(1-\delta)\sin^2 2\beta \tag{6.12}$$

where

$$\delta = \frac{\left(2\mu_h/\sin 2\beta - A_\lambda - \mu_s\right)^2}{m_s^2 + b_s + \mu_s^2}\,. \tag{6.13}$$

In the limit of vanishing y_s one recovers the matching condition in the MSSM [15–17, 51–53],

[44] Here we used that $\langle H_A\rangle = 0$. Notice that $\langle H_{SM}\rangle = v$ and recall that here y_s stands for the Yukawa coupling of the NMSSM.

i.e.

$$\lambda_{\mathrm{MSSM}} = \tfrac{1}{4}\left(g_2^2 + \tfrac{3}{5}g_1^2\right)\cos^2 2\beta\,. \tag{6.14}$$

This is precisely the first term in (6.12) and, more importantly, note that it is semi-postive definite. On the other hand, the second term appears only in the singlet-extension. The latter is generated by two effects, an F-term contribution generated by the Yukawa interaction that couples the singlet to the Higgs, and an extra contribution (proportional to δ) originated from integrating out the singlet. Interestingly, the denominator in (6.13) corresponds to the mass of the (CP-even) scalar singlet as calculated in (6.11) and, thus, it is positive. In turn, δ can only take positive values and the correction to $\lambda(M_s)$ is always negative.

The matching given in (6.12) receives higher order threshold corrections ($\delta\lambda^{th}$) that for the MSSM were originally computed at one loop level in [15] (and recently reviewed in [53] with leading two loop effect). We follow [53] and parametrize the corrections as follows[45]

$$\delta\lambda^{th}(M_s) = \Delta\lambda^{1l,\mathrm{reg}} + \Delta\lambda^{1l,\phi} + \Delta\lambda^{1l,\chi^{1,2}}\,. \tag{6.15}$$

These originate from the change of renormalization schemes that relate the gauge couplings in the $\overline{\mathrm{DR}}$ scheme to the $\overline{\mathrm{MS}}$ scheme ($\Delta\lambda^{1l,\mathrm{reg}}$) and from integrating out the heavy scalars ($\Delta\lambda^{1l\phi}$) and fermionic superpartners ($\Delta\lambda^{1l\chi^{1,2}}$). The effect induced from stop mixing is also included, with the stop mixing parameter defined in (3.56).

To conclude, it is worth noticing that as long as one does not assume a special pattern of soft terms, the free parameters that determine $\lambda(M_s)$ in (6.12) are

$$y_s\,, \delta\,, \tan\beta \,\text{and}\, M_s\,. \tag{6.16}$$

However, if the soft terms are specified, the corresponding soft parameters at M_s completely determine $\tan\beta$ and δ via (6.7) and (6.13) respectively.

6.2.2 Calculation of the Higgs mass

Matching at M_s

We perform the numerical calculations using a modified version of SPheno-3.3.6 [128, 129] created by SARAH-4.5.8 [130, 132, 157]. Given $\lambda(M_s)$ the Renormalization Group Equations (RGEs) of the Standard Model are calculated at two-loop-level to yield the couplings at the weak scale. All couplings are renormalized at one loop at M_t in the $\overline{\mathrm{MS}}$ scheme and the corresponding Higgs mass is thus calculated at one loop level as in (6.2). For the top Yukawa coupling we include the two loop and dominant three loop QCD correction given in equa-

[45] In our setup these are not complete, the threshold corrections coming from integrating out the two scalar singlets and the singlino are not included.

tion (57) of [19]. For completeness we provide the values of the SM parameters used in the calculations

$$\begin{aligned} M_t &= 173.34 \pm 0.76\,\text{GeV}\,, \quad \alpha_3 = 0.1184\,, \\ M_z &= 91.18\,\text{GeV}\,, \quad G_F = 1.16637 \cdot 10^{-5}\,. \end{aligned} \tag{6.17}$$

A theoretical uncertainty on the Higgs mass of 3 GeV is generically applied to supersymmetric models. This was computed within the MSSM in [133], assuming low energy values of the SUSY-breaking scale. The computation was recently reviewed in [163], for arbitrary (large) values of the SUSY-breaking scale and yielded a 1 GeV uncertainty for the Higgs mass. In the following we use this result.

Matching at M_{GUT}

In this section we explain how to obtain $\lambda(M_s)$ from a set of universal soft terms at the GUT scale given by $m_0, M_0, A_0, \mu_{h_0}, \mu_{s_0}$ and b_{s_0}.[46] The procedure to calculate $\lambda(M_s)$ goes as follows. The values of the gauge and top Yukawa couplings in the NMSSM, $\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{y}_t$, are fixed by the corresponding g_1, g_2, g_3, y_t in the SM via the matching conditions at M_s.[47] The matching conditions are given at tree level by

$$\hat{g}_1 = \sqrt{\frac{5}{3}}g', \quad \hat{g}_2 = g_2, \quad \hat{g}_3 = g_3, \quad \hat{y}_t = \frac{y_t}{\sin\beta}\,, \tag{6.18}$$

and receive one loop threshold corrections which can be found in [53]. However, gauge couplings only enter in the calculation of the Higgs mass via the RGEs for the soft terms and thus, higher order corrections can be neglected.[48] With $\hat{y}_t, \hat{g}_1, \hat{g}_2, \hat{g}_3$ at hand we run from M_s up to M_{GUT} to calculate $\hat{y}_{t_0}, \hat{g}_{1_0}, \hat{g}_{2_0}, \hat{g}_{3_0}$. Using the latter boundary conditions for gauge and top Yukawa couplings together with the soft terms we implement the RGEs of the NMSSM again to obtain the corresponding soft terms at M_s. The RGEs are given in Appendix B and we neglect the contribution of all Yukawa couplings except $\hat{y}_t$.[49] The soft parameters at M_s determine the values of $\tan\beta$ and δ via the eqs. (6.7) and (6.13) respectively which yield $\lambda(M_s)$ through (6.12).[50] The one loop corrections given in (6.15), using the stop mixing in (3.56) are also included. With the value of $\lambda(M_s)$ at hand, we proceed as in 6.2.2.

[46] Notice that b_h is fixed by the fine-tuning condition (6.5).

[47] The corresponding values of $\hat{y}_t, \hat{g}_1, \hat{g}_2, \hat{g}_3$ used correspond to $M_h = 125$GeV. Variations of M_h between 50 and 150GeV yield variations of 10^{-6} in the gauge couplings and 10^{-3} in the top Yukawa.

[48] These could become important in precise estimations of gauge coupling unification.

[49] Neglecting the bottom Yukawa is a good approximation for low (or moderate) values of $\tan\beta$.

[50] Notice that the value of $\tan\beta$ already appears in the matching condition of the top Yukawa given in (6.18), the values should of course match.

6.2.3 The Higgs mass as a function of M_s

Let us proceed and present the results, in particular, we compute the Higgs mass as a function of M_s, varying the NMSSM parameters (6.16) that determine the value of $\lambda(M_s)$ in (6.12). For large $\tan\beta$ the situation resembles the MSSM case [15–17, 51–53]. In this regime the first term in (6.12) is maximized while the second is almost vanishing regardless of the value of y_s. The larger $\tan\beta$ the lower the value of M_s consistent with the Higgs mass, in the limiting case of $\tan\beta \gtrsim 10$ one recovers the condition of low energy susy of $M_s \simeq$ TeV. In the effective MSSM limit, i.e. for $y_s = 0$, $\lambda(M_s)$ is semi-positive definit and, in particular, vanishes for $\tan\beta = 1$. In turn, the M_s has an upper bound of 10^{10}GeV. In contrast, the contribution introduced by the singlet can take negative values. Specifically, the second term in (6.12) is maximized for $\tan\beta = 1$ and its size can be tuned by adjusting y_s and δ. As a result, in the NMSSM regime M_s has no upper bound. In Fig. 6.2 and 6.3 we plot the Higgs mass for various y_s, using $\delta = -2$ and $\tan\beta = 1, 2$ respectively. In these examples it can be clearly seen that M_s could take values up to M_{GUT}. The upper bound on the MSSM can be seen in Figure 6.2 for case with $y_s = 0$ and $\tan\beta = 1$.

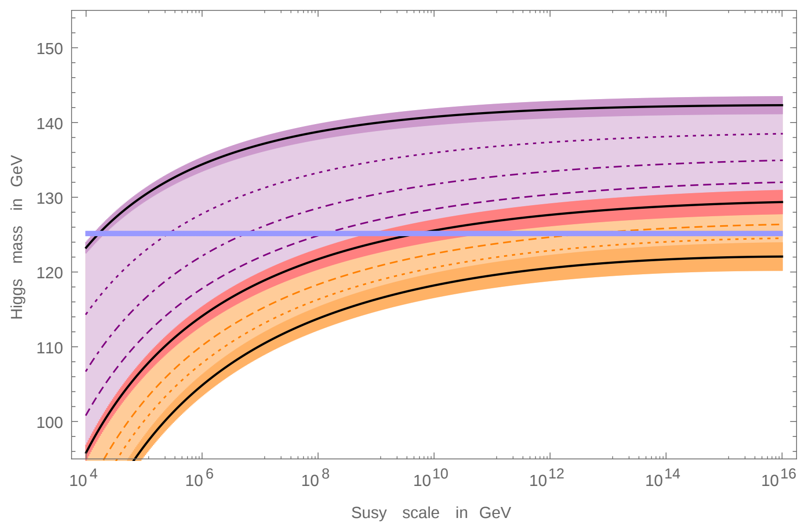

Figure 6.2: *Higgs mass as a function of M_s for $\tan\beta = 1$. The region shaded in violet (orange) corresponds to $\lambda > 0$ ($\lambda < 0$) and from bottom-up $y_s = 0.2, 0.3, 0.4, 0.5$ with $\delta = 0$ ($y_s = 0.3, 0.25, 0.2$ with $\delta = -2$). In red $\lambda = 0$, $y_s = 0$. We assumed $\tilde{X}_t = 0$ and degenerate superparticles at M_s. The bands display the uncertainty from varying $M_t = 173.34 \pm 0.76$GeV, we did not include them in all the curves to avoid confusion. The line in blue is the measured Higgs mass 125.15 ± 0.25GeV.*

UV dependece

We now analyze the Higgs mass as a function of M_s but using the values of $\tan\beta$ and δ in $\lambda(M_s)$ computed from universal patterns of soft terms at the GUT scale. Notice from (6.7) that

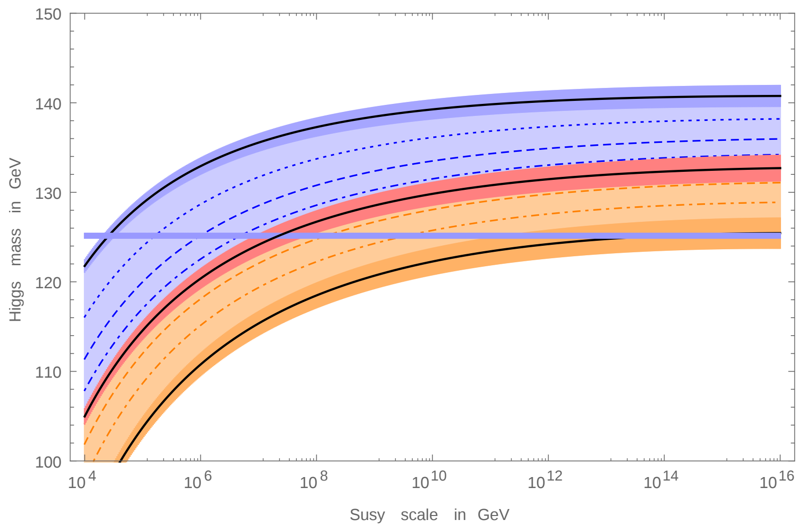

Figure 6.3: *Higgs mass as a function of M_s for $\tan\beta = 2$. The region shaded in blue (orange) corresponds to from bottom-up $y_s = 0.2, 0.3, 0.4, 0.5$ with $\delta = 0$ ($y_s = 0.4, 0.3, 0.2$ with $\delta = -2$). In red $y_s = 0$. We assumed $\tilde{X}_t = 0$ and degenerate superparticles at M_s. The bands display the uncertainty from varying $M_t = 173.34 \pm 0.76$GeV. The line in blue is the measured Higgs mass 125.15 ± 0.25GeV.*

$\tan\beta$ is equal to one at the GUT scale and as $m^2_{h_u}, m^2_{h_d}$ and μ_h run, $\tan\beta$ evolves accordingly. In particular, $m^2_{h_d}$ and μ stay stay almost constant while as $m^2_{h_u}$ decreases for lower values of M_s $\tan\beta$ increases. However, $\tan\beta$ remains close to one for large values of M_s. In other words, the unification (or universality) of soft masses predicts small values of $\tan\beta$.

On the other hand, m_s, b_s and μ_s appearing in δ do not run for small values of y_s and stay equal to their boundary conditions at the GUT scale. Thus, the running of δ is induced by μ_h and A_λ. As long as there are no hierarchies among the couplings, δ stays constant at large M_s and increases for low values of the SUSY-breaking scale.

In sum, for large values of M_s, $\tan\beta$ takes very small values and so enhances the NMSSM correction to $\lambda(M_s)$. In this regime, δ does not significantly vary and, thus, the Higgs mass dependence on M_s is (almost) constant. Furthermore, in this regime

$$10^9\text{GeV} \lesssim M_s \lesssim 10^{16}\text{GeV}\,, \tag{6.19}$$

by tuning the value of y_s near $\mathcal{O}(10^{-2})$, the Higgs mass can be easily accommodated in the experimental bound. For lower values of M_s, $\tan\beta$ starts to increase and thus $\lambda(M_s)$ becomes insensitive to the NMSSM correction. The latter competes with the MSSM contribution which grows with $\tan\beta$. The sum of these two terms leads to a slow decrease of the Higgs mass with M_s.

In Figure 6.4 we show the Higgs mass as a function of M_s for the following choice of soft masses at the GUT scale, i.e. $m_0 = M_0 = \mu_{s_0}$, $b_{s_0} = -\mu_s^2$ and $A_0 = \mu_{h_0} = -1.5M_0$, and various

y_s.[51] As y_s decreases the upper bound on M_s approaches the lower bound of $M_s \simeq 10^{10}$GeV, that corresponds to the limit of $\tan\beta = 1$ and $y_s = 0$. Whereas increasing y_s enhances the NMSSM negative contribution to $\lambda(M_s)$ and, thus, allows M_s to take larger values. Similarly, in Figure 6.5, by fixing y_s we show the effect of lowering $\lambda(M_s)$ by incrementing δ. This can be achieved by taking smaller values for the soft masses.

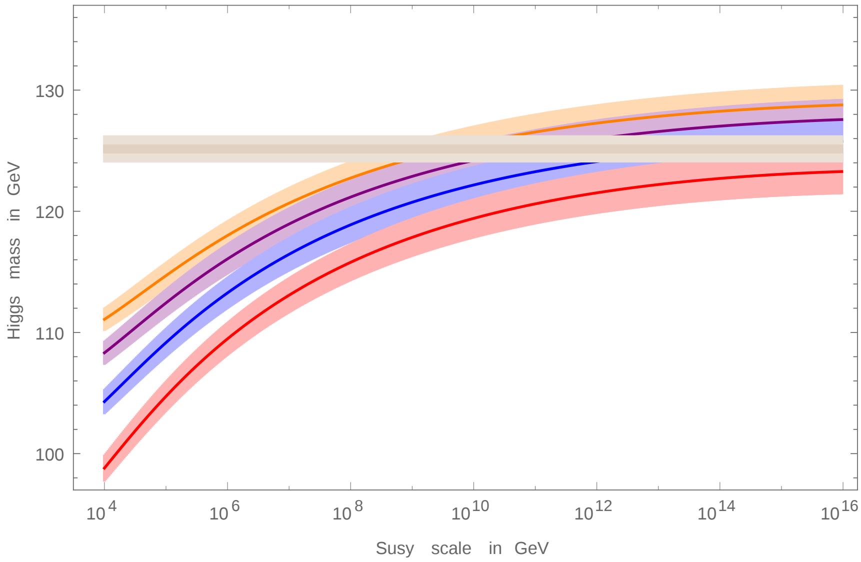

Figure 6.4: *Higgs mass as a function of M_s with soft terms at the GUT scale: $m_0 = M_0 = \mu_{s_0}$, $b_{s_0} = -\mu_{s_0}^2$ and $A_0 = \mu_{h_0} = -1.5M_0$ and $y_s = 0.05$ (orange), 0.075 (purple), 0.1 (blue), 0.125 (red). The bands correspond to the uncertainty bound in M_t and the region in lighter brown is $M_h = 125 \pm 1$GeV. In darker brown the experimental bound on M_h.*

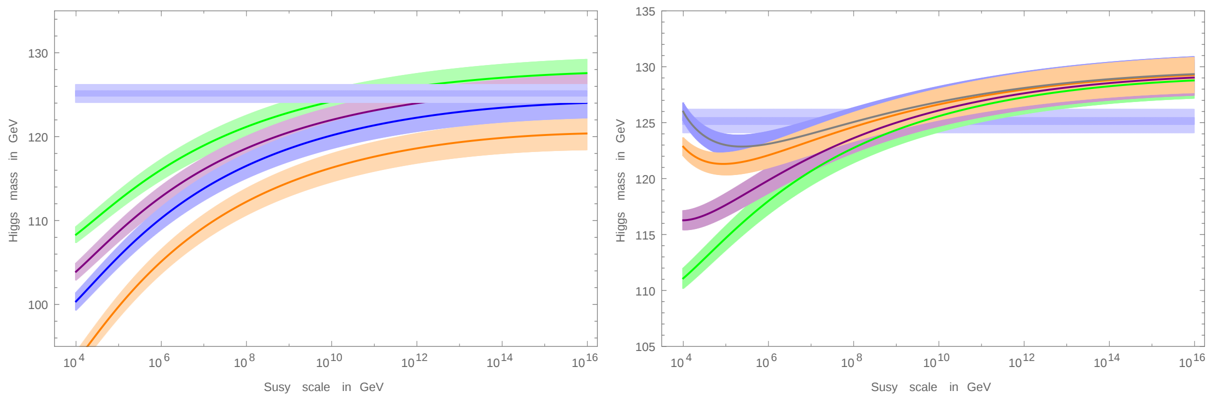

Figure 6.5: *Higgs mass as a function of M_s. The bands correspond to the uncertainty bound in M_t and the region in blue is $M_h = 125 \pm 1$GeV. In darker blue the experimental bound on M_h. (left) $y_s = 0.075$ and $m_0^2 = M_0^2$ (green), $0.6M_0^2$ (purple), $0.45M_0^2$ (blue), $0.3M_0^2$ (orange). (right) $y_s = 0.05$ and $\mu_{h_0} = -1.5M_0$ (green), $-1.25M_0$ (purple), $-M_0$ (orange), $-0.9M_0$ (blue).*

Notice that in the limit of vanishing y_s, the NMSSM contribution to $\lambda(M_s)$ is negligible.

[51] Notice that this choice of A_0 and μ_{h_0} minimizes the effect of stop mixing at large SUSY scales

Hence, larger $\tan\beta$ values raise the Higgs mass at low energies. This effect is manifest depending on the choice of soft terms, in particular of μ_{h_0}.[52] In Figure 6.5 (right) we show the Higgs mass as a function of M_s for fixed y_s and the same soft terms as before but varying μ_{h_0}. As seen in Figure 6.5, for lower μ_{h_0}, $\tan\beta$ becomes larger and boosts the Higgs mass. To conclude, for lower values of M_s consistency with the Higgs mass becomes more model dependent and requires small NMSSM contributions to $\lambda(M_s)$.[53] Furthermore, in this regime the soft masses spread out and, thus, one loop contributions to $\lambda(M_s)$ become large. Hence, the outcome of the Higgs mass relies on the details of the soft parameters.

[52]The choice of soft masses and A-terms, i.e. m_0 and A_0 have a milder effect on $\tan\beta$.

[53]In particular, $\tan\beta$ should take larger values and y_s should be negligible in order to suppress the singlet contribution to $\lambda(M_s)$.

Chapter 7

Conclusions

We stand at a turning point in high-energy physics. Present experimental bounds on supersymmetric extensions of the Standard Model severely threaten the theoretical principles of these theories. In particular, there is so far no evidence of superpartners up to a few TeV nor hints of deviations from Standard Model predictions. Furthermore, the regions already excluded are sufficient to induce a little hierarchy problem in most supersymmetric models. The requirement of naturaleness on the expected Higgs mass is an assumption that guides the formulation of supersymmetric frameworks but it might not be a theoretical necessity. Therefore, two possible approaches to resolve this tension follow

Hyp I: either pursue low energy supersymmetry and conceive possible explanations of the little hierarchy problem, or

Hyp II: drop the naturalness paradigm in the construction of supersymmetric theories.

The answer to this dilemma is conveyed by the soft terms.

As we explained in this thesis, soft interactions arise as effective terms in the low energy limit of non-renormalizable theories defined at the GUT scale. More specifically, they are induced by (non-renormalizable) couplings to hidden fields that break supersymmetry. In particular, the formulae for soft terms generated in supergravity embeddings are long known and were computed in [27, 28] while for models in (non-renormalizable) global supersymmetric theories they were presented in this thesis. In both cases, treating the couplings as free parameters induces a large arbitrariness of choices when studying phenomenological implications. The standard approach, driven by simplicity and biased by experimental constraints, is to assume a universal set of soft terms. Note that the absence of flavor mixing does not imply the universality of all scalar particles but only of the first two generations of sfermions. Therefore, the requirement of universality is generically an economical extrapolation. Behind this guess is the hope that the couplings of the high energy theory, and so the soft terms, will be eventually computed from the UV-completion and they will take precisely this structure. So far, the prime candidate for a UV-completion is string theory.

In this thesis we investigated the phenomenological implications within the Next-to-Minimal Supersymmetric Standard Model with special patterns of soft terms. We addressed different examples inspired by four dimensional effective theories in heterotic models. In chapter 4, we studied the dilaton dominated scenario as an explicit example that yields universal soft terms at the GUT scale. Following [36] we showed that this scheme is phenomenologically viable and very predictive. Firstly, we reviewed the dilaton domination in the MSSM. In particular, we found that the LSP is a bino neutralino and particular regions of parameter space could be tested at the LHC in the near future. As expected, higgsinos are generically heavy and, therefore, the model suffers from a little hierarchy problem. In the NMSSM the presence of additional parameters introduces new possibilities. Specifically, the tree level contribution to the Higgs mass can be larger than in the MSSM. As long as the mixing with the singlet is kept small the bound on the gravitino mass can be lower than in the MSSM case. In addition, in the regime of the Yukawa coupling $\lambda \to 0$, the singlino can be the LSP.

In chapter 5, we considered the structure of soft terms of gaugino mediated scenarios. These can be naturally embedded in higher dimensional orbifold GUTs, and realize particular examples of non-universal soft terms. Following the approach in (Hyp. I), we address the question of whether it is possible to devise a relation between the soft parameters that can solve the little hierarchy problem [29]. The sensitivity of the electroweak scale to the supersymmetry breaking scale can be displayed by parameterizing the Z-boson mass M_z in terms of the free soft parameters. Specifically, we looked for a condition that relates soft Higgs masses and soft gaugino masses which yields a suppression of M_z with respect to these. This special relation can be only explained with a UV setup, small deviations from this condition automatically restore the little hierarchy. However, we conjectured this possibility as an example that shows that the small hierarchy can be tied to our lack of knowledge and abuse of assumptions. In addition, we studied the phenomenological implications of this scenario. The Higgs mass is consistent with $M_s \simeq 3 - 5$TeV. Higgsinos have masses of a few hundred GeV, scalar masses are in the TeV range and in the limit of vanishing λ Yukawa coupling we found a light singlet sector $\mathcal{O}(100)$GeV.

Finally, in chapter 6 we adopted the path in (Hyp. II) and study the implications of universal soft terms with large supersymmetry breaking scales. As already pointed out, a hierarchy between the soft and Planck scales in string embeddings typically requires a large fine-tuning of the compactification data. Therefore, conceiving low energy supersymmetry frameworks in the context of string compactifications introduces an additional hierarchy problem which in practice is ignored. Without attempting to explain the hierarchy problem to any degree we study the implications on the Higgs mass. In this work, by computing the matching conditions of the Higgs quartic coupling to the NMSSM and using the experimentally measured Higgs mass, we derived bounds on the parameters of the supersymmetric theory [54]. In particular, we found that in the NMSSM the supersymmetry breaking scale can take values up to M_{GUT}. This is certainly different from the MSSM case where the latter is bounded from above by 10^{10}GeV. The Higgs mass was found in the experimental range by adjusting the Yukawa coupling $\lambda \lesssim 10^{-2}$.[54] Furthermore, the structure of universal soft terms at M_{GUT} favors values of M_s between $10^{9}\mathrm{GeV} \lesssim M_s \lesssim 10^{16}\mathrm{GeV}$.

To conclude, the supersymmetry breaking mechanism and, therefore, the UV-origin of soft terms play an essential role in the dynamics at low energies. Furthermore, it conveys the answer to the most pressing controversies of supersymmetric theories. A better understanding of high-energy theories would undoubtedly guide our intuition. However, experimental discoveries in the forthcoming future are necessary and will hopefully resolve the puzzle.

[54] This is not a strict bound but assumes there are no hierarchies between the soft terms.

Appendix A

Soft terms in non-renormalizable theories with global supersymmetry

In the spirit of section 2.2.3, we consider a supersymmetric $\mathcal{N}=1$ theory that consist of two sectors: the *observable sector*, which comprises the MSSM fields or the extensions considered, and the *hidden sector* that triggers supersymmetry breaking. The chiral superfields in the observable sector are denoted by A^I while the chiral fields in the hidden sector are called t^i.

Furthermore, we assume the theory provides an effective description with an intrinsic cutoff Λ. Thus, arbitrary non-renormalizable terms suppressed by Λ are included with the additional observation that couple the hidden to the observable sector. The Lagrangian can be completely specified in terms of the Kähler potential K, the superpotential W and the gauge kinetic function f. K is a real and gauge invariant and can be expanded in powers of the chiral fields $A^I, \bar{A}^{\bar{I}}$ and yields

$$K = \Lambda^2 \hat{K}(t,\bar{t}) + Z_{I\bar{J}}(t,\bar{t}) A^I \bar{A}^{\bar{J}} + (\frac{1}{2} H_{IJ}(t,\bar{t}) A^I A^J + c.c.) + \dots \ . \tag{A.1}$$

Analogously, the superpotential is also expanded in terms of the chiral observable fields as

$$W(t,A) = \hat{W}(t) + \frac{1}{2}\tilde{\mu}_{IJ}(t) A^I A^J + \frac{1}{3} Y_{IJK}(t) A^I A^J A^K + \dots \ . \tag{A.2}$$

The gauge kinetic function can depend on the hidden fields and defines the gauge couplings $g_a^{-2}(t,\bar{t})$ where a runs over different factors of the gauge group, i.e. $G = \prod_a G_a$. The g_a renormalize in field theory with an all order expression given by [82, 83]

$$\begin{aligned} g_a^{-2}(t,\bar{t},p) = \mathrm{Re} f_a(t) + \frac{b_a}{8\pi^2} \log\frac{\Lambda}{p} + \frac{T(G_a)}{8\pi^2} \log g_a^{-2}(t,\bar{t},p) \\ - \sum_r \frac{T_a(r)}{8\pi^2} \log\det Z^{(r)}(t,\bar{t},p) \ . \end{aligned} \tag{A.3}$$

Here $p < \Lambda$ is the renormalization scale and the numerical coefficients are given by $T_a(r) = \mathrm{Tr}_r(T_a^2)$, $T(G_a) = T_a(\text{adjoint})$ and $b_a = \sum_r n_r T_a(r) - 3T(G_a)$ where the summation is over representations r of the (observable) gauge group G. The first term corresponds to the tree level gauge couplings while the other are loop corrections.

We regard the dynamics of the hidden fields at energies at much higher energies than the electro-weak scale, i.e. $\langle t^i \rangle \gg m_{\mathrm{EW}}$. Hence, the effective scalar potential of the hidden fields can be approximated by replacing the scalar potential with $\langle A^I \rangle = 0$, which yields

$$V^{\mathrm{hid}} \simeq \Lambda^2 \hat{K}_{i\bar{j}} F^i \bar{F}^{\bar{j}} \ , \tag{A.4}$$

where

$$\bar{F}^{\bar{j}} = \Lambda^{-2} \hat{K}^{\bar{j}i} \partial_i \hat{W}. \tag{A.5}$$

Supersymmetry is spontaneously broken if $\langle F^i \rangle \neq 0$ which defines the scale of SUSY breaking via

$$m_{\mathrm{soft}} = \langle \hat{K}_{i\bar{j}} F^i \bar{F}^{\bar{j}} \rangle^{\frac{1}{2}}. \tag{A.6}$$

The calculation of the effective potential for the observable sector is analogous to the gravity dominated scenario and we follow the procedure in [27]. Specifically, we replace the hidden fields and their auxiliary partners by their VEVs and keep only the renormalizable couplings. We obtain

$$\begin{aligned} V(A, \bar{A}) = &\sum_a \frac{g_a^2}{4} (\bar{A}^{\bar{I}} Z_{\bar{I}J} T_a A^J)^2 + \partial_I W^{\mathrm{eff}} Z^{I\bar{J}} \bar{\partial}_{\bar{J}} \bar{W}^{\mathrm{eff}} \\ &+ m^2_{I\bar{J}} A^I \bar{A}^{\bar{J}} + (\frac{1}{3} A_{IJK} A^I A^J A^K + \frac{1}{2} b_{IJ} A^I A^J + c.c.) \ , \end{aligned} \tag{A.7}$$

where W^{eff} denotes an effective superpotential defined as follows

$$W^{\mathrm{eff}}(A) = \frac{1}{2} \mu_{IJ} A^I A^J + \frac{1}{3} Y_{IJK} A^I A^J A^K \ , \tag{A.8}$$

with

$$\mu_{IJ} = \tilde{\mu}_{IJ} - \bar{F}^{\bar{j}} \bar{\partial}_{\bar{j}} H_{IJ}. \tag{A.9}$$

From (A.7) we learn that the first line corresponds to the global supersymmetric scalar potential while the second line encode the soft supersymmetry breaking terms. The latter are given by

$$m^2_{I\bar{J}} = -F^i \bar{F}^{\bar{j}} R_{i\bar{j}I\bar{J}} \ , \tag{A.10}$$

$$A_{IJK} = F^i D_i Y_{IJK} \ , \tag{A.11}$$

$$b_{IJ} = F^i D_i \mu_{IJ} \ , \tag{A.12}$$

with

$$
\begin{aligned}
&R_{i\bar{j}I\bar{J}} = \partial_i \bar{\partial}_{\bar{j}} Z_{I\bar{J}} - \Gamma^N_{iI} Z_{N\bar{L}} \bar{\Gamma}^{\bar{L}}_{\bar{j}\bar{J}} \,, \quad \Gamma^N_{iI} = Z^{N\bar{J}} \partial_i Z_{\bar{J}I} \ , \\
&D_i Y_{IJK} = \partial_i Y_{IJK} - \Gamma^N_{i(I} Y_{JK)N} \ , \\
&D_i \mu_{IJ} = \partial_i \mu_{IJ} - \Gamma^N_{i(I} \mu_{J)N} \ .
\end{aligned}
\tag{A.13}
$$

Note from (A.10) that the sign of the scalar masses is model dependent. Furthermore, the soft masses $m_{I\bar{J}}$ are generically not universal, thus the potential appearance of FCNC is also a problem in the global susy setup. The kinetic term of the gauginos is given by

$$
g_a^{-2}(t,\bar{t})\, \bar{\lambda}_a \sigma^\mu D_\mu \lambda_a \,. \tag{A.14}
$$

After canonically normalizing the kinetic term of gauginos the soft gaugino masses read

$$
\frac{1}{2}(\, M_a \lambda^a \lambda^a + \ c.c.) \ , \qquad M_a = F^i \partial_i \log g_a^{-2}(t,\bar{t})\,. \tag{A.15}
$$

Appendix B

Renormalization group equations

In this appendix we provide the RGEs of the NMSSM to one loop, taken from [14]. The complete two loop expressions can be found in the same reference. The following expressions assume the first two generations of Yukawa couplings can be neglected, and use the standard normalization of gauge couplings at $M_{\rm GUT}$, $g_{\rm GUT} = g_2 = g_3 = \frac{5}{3} g_1$. Here $t(\mu) := \log \frac{\mu^2}{M_{\rm GUT}^2}$ with μ the renormalization scale.

Gauge couplings

$$16\pi^2 \frac{dg_a^2}{dt} = b_a\, g_a^4\,, \quad b_a = (11, 1, -3) \tag{B.1}$$

Yukawa couplings

$$16\pi^2 \frac{dy_t^2}{dt} = y_t^2 \left(6y_t^2 + y_b^2 + \lambda^2 - \frac{13}{9} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right) \tag{B.2}$$

$$16\pi^2 \frac{dy_b^2}{dt} = y_b^2 \left(6y_b^2 + y_t^2 + y_\tau^2 + \lambda^2 - \frac{7}{9} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right) \tag{B.3}$$

$$16\pi^2 \frac{d\lambda^2}{dt} = \lambda^2 \left(3y_t^2 + 3y_b^2 + y_\tau^2 + 4\lambda^2 + 2\kappa^2 - g_1^2 - 3g_2^2 \right) \tag{B.4}$$

$$16\pi^2 \frac{d\kappa^2}{dt} = \kappa^2 \left(6\lambda^2 + 6\kappa^2 \right) \tag{B.5}$$

Gaugino masses

$$16\pi^2 \frac{dM_a}{dt} = b_a\, g_a^2 M_a \tag{B.6}$$

Trilinear couplings

$$16\pi^2 \frac{dA_t}{dt} = 6y_t^2 A_t + y_b^2 A_b + \lambda^2 A_\lambda + \frac{13}{9} g_1^2 M_1 + 3g_2^2 M_2 + \frac{16}{3} g_3^2 M_3 \tag{B.7}$$

$$16\pi^2 \frac{dA_b}{dt} = 6y_b^2 A_b + y_t^2 A_t + y_\tau^2 A_\tau + \lambda^2 A_\lambda + \frac{7}{9} g_1^2 M_1 + 3g_2^2 M_2 + \frac{16}{3} g_3^2 M_3 \tag{B.8}$$

$$16\pi^2 \frac{dA_\tau}{dt} = 4y_\tau^2 A_\tau + 3y_b^2 A_b + \lambda^2 A_\lambda + 3g_1^2 M_1 + 3g_2^2 M_2 \tag{B.9}$$

$$16\pi^2 \frac{dA_\mu}{dt} = 3y_b^2 A_b + y_\tau^2 A_\tau + \lambda^2 A_\lambda + 3g_1^2 M_1 + 3g_2^2 M_2 \tag{B.10}$$

$$16\pi^2 \frac{dA_\lambda}{dt} = 4\lambda^2 A_\lambda + 3y_t^2 A_t + 3y_b^2 A_b + y_\tau^2 A_\tau + 2\kappa^2 A_\kappa + g_1^2 M_1 + 3g_2^2 M_2 \tag{B.11}$$

$$16\pi^2 \frac{dA_\kappa}{dt} = 6\kappa^2 A_\kappa + 6\lambda^2 A_\lambda \tag{B.12}$$

Squark and slepton masses

Let us define the following quantities:

$$M_t^2 = m_{q_3}^2 + m_{u_3}^2 + m_{h_u}^2 + A_t^2 \tag{B.13}$$

$$M_b^2 = m_{q_3}^2 + m_{d_3}^2 + m_{h_d}^2 + A_b^2 \tag{B.14}$$

$$M_\tau^2 = m_{l_3}^2 + m_{e_3}^2 + m_{h_d}^2 + A_\tau^2 \tag{B.15}$$

$$M_\lambda^2 = m_{h_u}^2 + m_{h_d}^2 + m_s^2 + A_\lambda^2 \tag{B.16}$$

$$M_\kappa^2 = 3m_s^2 + A_\kappa^2 \tag{B.17}$$

$$\xi = \mathrm{Tr}\left[\mathbf{m}_Q^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 - \mathbf{m}_L^2 + \mathbf{m}_E^2\right] + m_{h_u}^2 - m_{h_d}^2 \tag{B.18}$$

where $\mathbf{m}$ denote matrices in family space.

$$16\pi^2 \frac{dm_{q_a}^2}{dt} = \delta_{a3} y_t^2 M_t^2 + \delta_{a3} y_b^2 M_b^2 - \frac{1}{9} g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{6} g_1^2 \xi \tag{B.19}$$

$$16\pi^2 \frac{dm_{u_a}^2}{dt} = 2\delta_{a3} y_t^2 M_t^2 - \frac{16}{9} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 - \frac{2}{3} g_1^2 \xi \tag{B.20}$$

$$16\pi^2 \frac{dm_{d_a}^2}{dt} = 2\delta_{a3} y_b^2 M_b^2 - \frac{4}{9} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{3} g_1^2 \xi \tag{B.21}$$

$$16\pi^2 \frac{dm_{l_a}^2}{dt} = \delta_{a3} y_\tau^2 M_\tau^2 - g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{1}{2} g_1^2 \xi \tag{B.22}$$

$$16\pi^2 \frac{dm_{e_a}^2}{dt} = 2\delta_{a3} y_\tau^2 M_\tau^2 - 4g_1^2 M_1^2 + g_1^2 \xi \tag{B.23}$$

Higgs masses

$$16\pi^2\frac{dm_{h_u}^2}{dt} = 3y_t^2M_t^2 + \lambda^2M_\lambda^2 - g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{1}{2}g_1^2\xi \tag{B.24}$$

$$16\pi^2\frac{dm_{h_d}^2}{dt} = 3y_b^2M_b^2 + y_\tau^2M_\tau^2 + \lambda^2M_\lambda^2 - g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{1}{2}g_1^2\xi \tag{B.25}$$

$$16\pi^2\frac{dm_s^2}{dt} = 2\lambda^2M_\lambda^2 + 2\kappa^2M_\kappa^2 \tag{B.26}$$

Additional parameters of the general NMSSM

The one-loop RGEs for the SUSY conserving μ and μ_s terms are

$$32\pi^2\frac{d\mu_h}{dt} = \mu_h\big(3y_t^2 + 3y_b^2 + y_\tau^2 + 2\lambda^2 - g_1^2 - 3g_2^2\big) \tag{B.27}$$

$$16\pi^2\frac{d\mu_s}{dt} = \mu_s\big(2\lambda^2 + 2\kappa^2\big) \tag{B.28}$$

For the corresponding soft SUSY breaking terms b_h and b_s the RGEs read

$$\begin{aligned}32\pi^2\frac{db_h}{dt} &= 3y_t^2\big(b_h + 2\mu_hA_t\big) + 3y_b^2\big(b_h + 2\mu_hA_b\big) + y_\tau^2\big(b_h + 2\mu_hA_\tau\big) \\ &\quad + 2\lambda^2\big(3b_h + 2\mu_hA_\lambda\big) + 2\lambda\kappa b_s - g_1^2\big(b_h - 2\mu_hM_1\big) - 3g_2^2\big(b_h - 2\mu_hM_2\big)\end{aligned} \tag{B.29}$$

$$16\pi^2\frac{db_s}{dt} = 2\lambda^2\big(b_s + 2\mu_sA_\lambda\big) + 4\kappa^2\big(b_s + \mu_sA_\kappa\big) + 4\lambda\kappa b_h \tag{B.30}$$

Finally for the singlet tadpole terms, the RGEs read

$$16\pi^2\frac{d\xi}{dt} = \xi\big(\lambda^2 + \kappa^2\big) \tag{B.31}$$

$$\begin{aligned}16\pi^2\frac{d\xi_s}{dt} &= \lambda^2\big(\xi_s + 2A_\lambda\xi\big) + \kappa^2\big(\xi_s + 2A_\kappa\xi\big) \\ &\quad + 2\lambda\big(b_h(A_\lambda + \mu_s) + \mu(m_{h_u}^2 + m_{h_d}^2)\big) + \kappa\big(b_s(A_\kappa + \mu_s) + 2\mu_sm_s^2\big)\end{aligned} \tag{B.32}$$

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Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

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