SUPERSYMMETRIC ADS BACKGROUNDS IN GAUGED MAXIMAL SUPERGRAVITIES

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1 See also footnote 2 in chapter 6.

CONTENTS

1 INTRODUCTION

In this thesis we will examine maximal supergravities of different dimensions for their anti de-Sitter vacua. Supergravities are quantum field theories, which include gravity, i.e. a spin 2 field $g_{\mu\nu}$ [Tan98]. They build on supersymmetry - an extension of Poincaré symmetry, which is the usual spacetime symmetry in Physics.[1] Supersymmetry is so far merely a conjecture and has not yet been observed. It is however of great interest in theoretical and mathematical Physics, since the examination of supersymmetric theories sheds light on various areas of mathematics and quantum field theory.

Gauge theories with high degrees of supersymmetry can be solved by using new methods which build on intricate mathematics, which would not be possible in non-supersymmetric quantum field theories (cf. for example [Tes16]). Hence it is hoped to achieve a deeper understanding of perturbation theory as well as of non-perturbative features of quantum field theories in general.

Furthermore supergravities share the benefit of string theory, that they give a way to tackle questions of quantum gravity in a meaningful way. We will be concerned with maximally supersymmetric supergravities, which can be understood as massless limits of certain string theories in various dimensions. Our main point of interest will be *anti-de Sitter backgrounds* in these supergravities, which are maximally symmetric spaces with a negative cosmological constant Λ. These AdS backgrounds are of interest due to the *AdS/CFT conjecture*. The AdS/CFT conjecture was proposed by Maldacena in [Mal99] and was developed using string and M-theory. AdS/CFT is a duality between anti de-Sitter backgrounds in a D dimensional supergravity and a *superconformal field theory* (SCFT) in $D-1$ dimensions (for a review see [Aha+00]). The duality allows to relate observables in the different theories. Dualities like these are hoped to yield a deeper insight into quantum field theory, as they may relate yet unsolved problems in one theory to more tractable problems in the dual theory.

In our case of interest, we will find supersymmetric AdS backgrounds and their moduli in the maximal supergravities in four to seven dimensions. AdS backgrounds arise as vacuum states of gauged supergravities, but not all possible AdS solutions of a given maximal supergravity are compatible with all supercharges. The ones of interest in this thesis are only the maximally supersymmetric AdS backgrounds.

1 More precisely the Poincaré group usually refers to the symmetries of Minkowski spacetime.

In the cases considered in this thesis all such AdS solutions will be classified. Maximally symmetric spacetimes are classified by just one parameter, the cosmological constant Λ. Different solutions with the same $\Lambda < 0$ correspond to the same geometry. These solutions can sometimes be deformed by giving vacuum expectation values to some scalar fields. Giving such a vacuum expectation value to a scalar field does not break the symmetry of the underlying spacetime. Variations of the scalar fields which preserve the maximally supersymmetric AdS solution and thus in particular leave Λ constant and fixed are called *moduli* of the solution.

The AdS/CFT correspondence relates the moduli of a maximally supersymmetric AdS solution to the conformal manifold of the dual SCFT. The conformal manifold is a manifold spanned by *exactly marginal* deformations of the theory, i.e. those which preserve conformal invariance to all orders.

For the calculation an embedding tensor formalism is used. It describes all gauged supergravities of a given dimension in a covariant way i.e. in a unified description, which is the same for any chosen gauge group [DWST03][Sam08][DWNS08]. Using this formalism reduces the problems at hand to group theoretic calculations. Having a supersymmetric AdS background constrains the embedding tensor and with it the possible gauge groups. The resulting gauge group is also relevant for the AdS/CFT correspondence, since it corresponds to symmetries in the SCFT. In the maximally supersymmetric case the gauge group corresponds to the R-symmetry of the SCFT, which is the symmetry among the supercharges.

We will start the thesis with a preliminary part, which begins with a general introduction to supersymmetry and supergravity in chapter 2, which leads to the description of gauged supergravities in chapter 3.

The second part goes case-wise through the theories under consideration. We start with the seven dimensional case in chapter 4 and then lower the dimensions going through $D = 6$ in chapter 5, $D = 5$ in chapter 6 and finally $D = 4$ in chapter 7. Each case consists of a theory section, which provides the given theory, the calculations in which the AdS vacua, their moduli and the allowed gauge groups are examined, and a conclusion in which the obtained results are compared to results in the dual SCFT.

The thesis ends with the conclusion in chapter 8 in which we will give a short recapitulation of all obtained results and their correspondence to results in the dual theories.

2 MAXIMAL SUPERGRAVITY

2.1 RIGID SUPERSYMMETRY

Supersymmetry is an extension of the Poincaré algebra, which is the algebra of spacetime symmetries generated by infinitesimal rotations and boosts from the Lorentz algebra $\mathfrak{so}(1, D-1)$ together with infinitesimal translations P_μ in arbitrary directions.[1] Greek lowercase indices $\mu, \nu, ... = 0, ..., D-1$ are used to denote spacetime indices in D dimensions, if not stated otherwise. There is a no-go theorem by Coleman and Mandula concerning symmetries of the S-matrix, which states that there are no allowed symmetries besides the Poincaré symmetry and a finite number of operators belonging to a compact Lie group, which are Lorentz scalars [CM67]. One of the assumptions of the theorem by Coleman and Mandula is that all symmetries are realized as representations of a Lie algebra. There is a way to relax this assumption to circumvent the theorem.

For this reason the notion of a Lie algebra is extended to a *Lie superalgebra*. While ordinary Lie algebras are defined via commutation relations, Lie superalgebras also include anticommutation relations.[2] The resulting structure admits a $\mathbb{Z}_2$-grading, i.e. elements can be divided into *even* (or *bosonic*) elements of degree 0 and *odd* (or *fermionic*) elements of degree 1. The Lie (super-)bracket is compatible with the grading i.e. $\deg([x, y\}) = \deg(x) + \deg(y)$ for $x, y \in \mathfrak{g}$. In a $\mathbb{Z}_2$ grading the degree is taken modulo 2. Thus taking the bracket of two even elements gives an even one $(0 + 0 = 0)$, as does taking the bracket of two odd elements $(1 + 1 \bmod 2 = 0 \bmod 2)$. Any bracket $[\cdot, \cdot\}$ involving at least one even element is taken to be the commutator, denoted $[\cdot, \cdot]$. For two odd elements an anticommutator $\{\cdot, \cdot\}$ is used. Note that the even elements close into a Lie subalgebra - the *even* (or *bosonic*) *subalgebra* - since any combination of even elements is again even. This is not the case for the odd elements.

1 More precisely the Poincaré group can be written as a semidirect product $\mathbb{R}^{1,D-1} \rtimes SO(1, D-1)$.

2 More precisely a Lie algebra is defined via its Lie brackets, which are antisymmetric, bilinear maps $\mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g}$, which fulfill the Jacobi identity. To get to a Lie superalgebra, one includes symmetric, bilinear maps $\mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g}$, which fulfill an altered version of the Jacobi identity.

Connecting back to our initial motivation, there is an extension of the Poincaré algebra called *supersymmetry algebra*. In four dimensions it is[3]

$$\begin{aligned} \{Q^A_\alpha, \bar{Q}_{\dot{\alpha}B}\} &= 2\,\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A_B \\ \{Q^A_\alpha, Q^B_\beta\} &= 0 = \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} \\ [P_\mu, Q^A_\alpha] &= 0 = [P_\mu, \bar{Q}_{\dot{\alpha}A}] \\ [P_\mu, P_\nu] &= 0. \end{aligned} \tag{1}$$

The Q^A_α are the odd elements, called *supercharges*. They come in a spinor representation of $\mathfrak{so}(1,3)$, labeled by $\alpha, ... = 1,2$, with conjugate spinors labeled by $\dot{\alpha}, ... = 1,2$. $A, B = 1, ..., \mathcal{N}$ label the different supercharges. The algebra (1) is isomorphic under a complex rotation of supercharges into each other, i.e. under $U(\mathcal{N})$. This symmetry is called *R-symmetry*. It can be shown that supersymmetry leads to (*super*-)multiplets of particles which differ in helicity by $1/2$.[4] This symmetry is thus a symmetry between bosonic and fermionic fields. Each such multiplets contains $2^{\mathcal{N}}$ fields of $\mathcal{N}+1$ different helicities. For $\mathcal{N}=2$ for example there is the *vector multiplet* containing a vector field A_μ, two fermions λ_1, λ_2 and a scalar field ϕ (for a review of the $\mathcal{N}=2$ case see for example [Tac13]).

We can thus understand the notion of *maximal supersymmetry*: For a gauge theory without gravity, the highest helicity among fields should be $h = \pm 1$, corresponding to a vector field (or a p-form field in $D > 4$). In $D = 4$ the maximal supersymmetry for a gauge theory is thus $\mathcal{N} = 4$ since $1 - 4 \cdot (1/2) = -1$. Adding another supercharge gives a field of $h = \pm 3/2$.

The number $\mathcal{N}$ counts the number of spinorial supercharges Q. Any spinor in $D = 4$ contains four real entries. Therefore the $D = 4$, $\mathcal{N} = 4$ theory has 16 real supercharges. This is true for any gauge theory with maximal supersymmetry. In different dimensions these 16 real supercharges however fit into spinor representations of different dimensions. Going to $D = 3$ for example, real 2-component spinors are used instead of complex ones. The maximal supersymmetric gauge theory in $D = 3$ is thus labeled by $\mathcal{N} = 8$. To avoid confusion with different values of $\mathcal{N}$ for the same amount of supersymmetry it is customary to count the number of real supercharges. In the gauge theory context without gravity the maximal case is thus the one of 16 supercharges. For an overview of these theories, see [Sei98].

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3 While this chapter focuses on the $D = 4$ case, most of its arguments can be made for any other dimension. Appendix A gives an overview of the cases in different dimensions.

4 Labeling states by helicity assumes massless fields. In the massive case the spin s is used instead.

2.2 LOCAL SUPERSYMMETRY

Since the supercharges close into Poincaré transformations, gauging supersymmetry also gauges the Poincaré transformations. Gauged Poincaré symmetry however gives general coordinate transformations. Thus a theory with gauged supersymmetry is as a theory of gravity, as it includes a *graviton* field $g_{\mu\nu}$ as the gauge field of general coordinate transformations (cf. [Tan98]). Furthermore a spin $3/2$ field ψ_μ is introduced as the gauge field of supersymmetry. ψ_μ is called the *gravitino* and is the superpartner of the spin 2 graviton. It is customary to take the vielbein field e^a_μ instead of the $g_{\mu\nu}$ as the representative of gravity. They relate as

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} \tag{2}$$

with η_{ab} being the flat Lorentzian metric in D dimensions with $a, b, ... = 1, ...D-1$.

As done before let us take the $D = 4$, $\mathcal{N} = 1$ case as an example. The simplest theory to consider is pure supergravity, which field content consists solely of the *graviton multiplet*

$$(e^a_\mu; \psi_\mu). \tag{3}$$

The Lagrangian of this theory consists of the Einstein-Hilbert term and additional terms containing the gravitino

$$\mathcal{L} = -1/4 \det(e^a_\mu) R + \dots . \tag{4}$$

We can thus see, that classical gravity is reproduced with additional fermion terms, that would usually not appear. For more details on this case we refer to [Tan98], which also discusses various supergravities in different dimensions and with different amounts of supercharges.

We will however only be concerned with maximal supergravities. Our starting point for these is the unique supergravity in eleven dimensions.

2.3 THE D=11 SUPERGRAVITY

The $D = 11$ supergravity is a special case of interest, since eleven is the maximal dimension in which a supergravity exists. For $D > 11$ the spinor representations are at least 64-dimensional[5] and more than 32 real supercharges lead to higher spin fields with $s > 2$, which lead to inconsistent theories (see for example the review [Roe05]). It is also a unique theory without any possible deformations.

The field content of any maximal supergravity consists of a single multiplet - the *graviton multiplet*. For $D = 11$ it is

$$(e^a_\mu, C_{\mu\nu\rho}; \psi_\mu) \tag{5}$$

5 This is only true assuming Lorentzian spacetime signature.

with the vielbein field e^a_μ, a 3-form field $C_{\mu\nu\rho}$ and a Majorana gravitino ψ_μ. Defining the field strength $G = dC$ the bosonic Lagrangian is

$$\mathcal{L} = \sqrt{-g}\Big(R - \frac{1}{2}G \cdot G - \frac{1}{6}\star(G \wedge G \wedge C)\Big). \tag{6}$$

where the dot product is defined as contraction of all indices i.e. with $G = G_{\mu\nu\rho\sigma}dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma$, $G \cdot G = G_{\mu\nu\rho\sigma}G^{\mu\nu\rho\sigma}$. The Hodge star operator $\star$ denotes Hodge dualization and turns a p-form ω into a $D-p$-form using the Levi-Civita tensor $\epsilon_{\alpha_1,\ldots,\alpha_D}$.

The supergravity in $D = 11$ can be seen as being the fundamental theory from which the other cases can be obtained by dimensional reduction. In fact all maximal supergravities but one can be obtained by this procedure. Hence we will discuss dimensional reduction next and afterwards come back for the remaining case.

2.4 COMPACTIFICATIONS ON T^D

The starting point for dimensional reduction is the *Kaluza-Klein compactification* on a circle S^1. Starting from the theory in D dimensions we want to obtain a theory in $D-1$ dimensions, by exchanging the base manifold $\mathbb{R}^{D-1,1} \mapsto \mathbb{R}^{D-2,1} \times S^1$. We then call S^1 the *internal manifold* and want to truncate the theory to a theory on $\mathbb{R}^{D-2,1}$.

Writing the spacetime coordinate in D dimensions as $x^M = (x^\mu, y)$ this amounts to taking an equivalence relation $y \sim 2\pi R y$ where R is the radius of the circle $S^1 = S^1_R$. We can consider a massless scalar ϕ in D dimensions with Fourier decomposition in the y direction

$$\phi(x^\mu, y) = \int dk\, e^{iky}\phi_k(x^\mu). \tag{7}$$

Taking the equivalence relation $y \sim 2\pi R y$ and cyclic boundary conditions for the scalar $\phi(x^\mu, 0) = \phi(x^\mu, 2\pi R)$ turns the Fourier decomposition into a discrete spectrum

$$\phi(x^\mu, y) = \sum e^{iny/R}\phi_n(x^\mu) \tag{8}$$

with discrete modes $\phi_n(x^\mu)$ which have the momenta $k = n/R$ in the y-direction. The Klein-Gordon equation splits modewise as

$$\partial_\mu\partial^\mu\phi_n - k^2\phi_n = 0. \tag{9}$$

The momentum k in the compactified y-direction thus turns into a mass $m^2 = k^2 = (n/R)^2$. Hence all modes except the *zero mode* ϕ_0 are massive. One gets an *infinite tower* of massive scalar fields in the lower dimensional theory in $D-1$. The usual procedure from here is to truncate the field content to the zero mode ϕ_0. By choosing R to be very small, the masses n/R go towards infinity. This procedure can be repeated to get a $D-d$ dimensional theory from compactification on the d-dimensional torus $T^d = (S^1)^d$.

The torus compactification can be seen as a special case of more general compactifications since the torus is the only compact space in $d > 1$ which is flat. More general cases will be presented in chapter 3.

So far we have seen the reduction only for a scalar field. If we instead take a graviton g_{MN} on a circle S^1, we get a decomposition into fields of different spin

$$g_{MN} \mapsto \{g_{\mu\nu}, g_{\mu,D-1}, g_{D-1,D-1}\}. \tag{10}$$

$g_{\mu\nu}$ transforms as a spin-2-field, $g_{\mu,D-1}$ as a spin-1-field and $g_{D-1,D-1}$ as a scalar.[6] The diffeomorphism invariance of the theory with gravity in D dimensions turns into a diffeomorphism invariance, a local $U(1)$ gauge symmetry and a global scale symmetry in the lower dimensional theory. This was also the initial motivation by Kaluza and Klein: Gravity in five dimensions compactified on a circle gives gravity and a $U(1)$ gauge theory (e.g. electromagnetism) in four dimensions.[7]

The decomposition (10) can easily be generalized for the T^d case by iteration of the procedure. The resulting theory has a $U(1)^d$ gauge symmetry and a global $GL(n, \mathbb{R})$ symmetry, which can be decomposed into an $SL(n, \mathbb{R})$ and an $\mathbb{R}^+$ scaling symmetry.

As a final note, the other fields are dimensionally reduced in a similar way. For p-form fields the same procedure as in (10) is used. For fermion fields the spinor representation is decomposed and the spinors on the internal manifold are set to a constant value.

2.5 THE D=10 MAXIMAL SUPERGRAVITIES

In ten dimensions there are two different maximal supergravities. The Majorana-Weyl spinors used in $D = 10$ are 16-dimensional. Thus the 32 real supercharges form two spinors, which can be chosen to be of equal or opposite chirality. Taking spinors of opposite chirality gives the non-chiral theory with $\mathcal{N} = (1, 1)$, which is called *IIA supergravity* since it corresponds to the massless limit of the *IIA string theory*. Spinors of equal chirality lead to a chiral theory, labeled $\mathcal{N} = (2, 0)$, which is called *IIB supergravity* and corresponds to the massless limit of *IIB string theory*.

The IIA case can be obtained from the maximal $D = 11$ supergravity by compactification on a circle. This is not true for the IIB case. Nevertheless when going from ten to nine dimensions by S^1-compactification, both theories coincide in their zero modes. We thus get the same maximal supergravity in $D = 9$. Therefore for any supergravity in $D \leq 9$ there are two routes of compactifications that one can take. One route starts at $D = 11$ supergravity and then goes with iterations of compactifications on S^1 over the IIA supergravity to the supergravity in $D \leq 9$ The other route starts at the IIB supergravity in 10 dimensions and

6 This is the truncation to zero modes.
7 The additional scalar field $g_{D-1,D-1}$ however seemed to be unphysical at that time.

similarly uses S^1 compactifications to go to lower dimensions. For torus compactifications both routes coincide, but for more general internal manifolds different theories can be obtained.

2.6 GLOBAL SYMMETRIES

Maximal supergravities have certain global symmetry groups. In section 2.4 we have already seen, how the torus compactification gives rise to a $GL(n, \mathbb{R})$ symmetry in the lower dimensional theory. There is an even larger symmetry under which the resulting theory is invariant. The different fields obtained from the higher dimensional field content as done in (10) yield a larger symmetry group G, which is often called *hidden symmetry* of the theory.

For the resulting theory in $11 - d$ dimensions this symmetry group is $G = E_{d(d)}$. These groups are *normal real forms* of groups belonging to the *E-series*. The E-series consists of the exceptional groups E_8, E_7 and E_6 together with $E_5 \simeq D_5, E_4 \simeq A_4$ and further groups obtained from cutting the corresponding Dynkin diagrams. The normal real forms

D	9	8	7	6	5	4
G	$GL(2)$	$SL(2) \times SL(3)$	$SL(5)$	SO(5,5)	$E_{6(6)}$	$E_{7(7)}$

Table 1: Overview over the different global symmetry groups of maximal supergravities. Taken from [Sam08].

$E_{d(d)}$ can be seen as maximal non-compact versions of the groups E_d. A real form $\mathfrak{h}$ of a complex Lie algebra $\mathfrak{g}$ is a Lie algebra such that it complexifies to $\mathfrak{g}$: $\mathfrak{h}_\mathbb{C} \simeq \mathfrak{g}$. There are in general several inequivalent real forms for a given complex Lie algebra. Two special examples are the compact real form and the normal real form, the latter of which can be understood to be the least compact version of the algebra (cf. [FS03]).

2.7 SCALAR COSETS

The scalars in maximal supergravities form a sigma model with the coset space G/H as its target space, where G is the global symmetry group, we just discussed, and H is its maximal compact subgroup. This scalar coset is usually described by a matrix $\mathcal{V} \in G$ which transforms under rigid G and local H transformations as

$$\mathcal{V} \mapsto g\mathcal{V}h(x) \tag{11}$$

with $g \in G$ and $h(x) \in H$. Gauge fixing the action of H in this description is equivalent to picking a representative of the coset G/H.

The fermion fields also transform under local H transformations. Thus $\mathcal{V}$ is also used in fermion interactions. One important application for us, will be to map tensors of G to tensors of H and vice versa, which is possible, since $\mathcal{V}$ carries a representation of both of them. We will see this in more detail in section 3.3.

To obtain kinetic terms for these scalar matrices $\mathcal{V}$ different routes can be taken. One route starts from the current [Sam08]

$$J_\mu = \mathcal{V}^{-1}\partial_\mu\mathcal{V} \in \mathfrak{g} = \operatorname{Lie} G, \tag{12}$$

which can be split into

$$J_\mu = Q_\mu + P_\mu \tag{13}$$

with $Q_\mu \in \mathfrak{h} = \operatorname{Lie} H$ and $P_\mu \in \mathfrak{h}^\perp$ where $\mathfrak{h}^\perp$ is the orthogonal complement such that $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{h}^\perp$.

The scalar Lagrangian is then [Sam08]

$$e^{-1}\mathcal{L}_{\text{scalar}} = -\frac{1}{2}\operatorname{Tr}\Big(P_\mu P^\mu\Big). \tag{14}$$

A different option is to define a positive definite symmetric scalar matrix $\mathcal{M}$ by

$$\mathcal{M} = \mathcal{V}\Delta\mathcal{V}^T \tag{15}$$

where Δ is an H-invariant positive definite matrix, e.g. $\Delta = \mathbb{1}$. The Lagrangian is then [Sam08]

$$e^{-1}\mathcal{L}_{\text{scalar}} = \frac{1}{8}\operatorname{Tr}\Big(\partial_\mu\mathcal{M}\partial^\mu\mathcal{M}^{-1}\Big). \tag{16}$$

3 GAUGED SUPERGRAVITY

3.1 THE EMBEDDING TENSOR

When gauging a maximal supergravity, the choice of gauge group G_0 is restricted to be a subgroup of the global symmetry group G. The reason for this is the unique field content, which can not be altered. The vector fields form a representation of G. For a gauging, a subset of them is chosen to be in the adjoint representation of $\mathfrak{g}_0 = \mathrm{Lie}\, G_0$. Thus G_0 is a subset of G and its dimensions is restricted by the number of vector fields as $\dim R_{ad}(\mathfrak{g}_0) = \dim G_0$.

There is a general formalism to describe all possible gauge theories in maximal supergravity using an *embedding tensor* Θ (cf. [Sam08] and [DWNS08]). The embedding tensor can be understood as projecting $\mathfrak{g}$ onto $\mathfrak{g}_0$. This then defines a map from (the universal cover of) G_0 into (the universal cover of) G. While in a given gauge theory Θ would be set to a constant value, yielding a specific G_0, it is instead assigned a proper transformation behavior under G to ensure that the theory formulated via Θ stays invariant under G. *"Freezing"* Θ then breaks the symmetry to give the desired gauge theory. The terminology for this procedure is that Θ is a *spurionic* object.

The vector fields are denoted by A_μ^M, where M refers to a representation of $\mathfrak{g}$ to which A_μ^M belongs. Labeling this representation $R_{A_\mu}(\mathfrak{g})$, $\Theta = \Theta_M{}^\alpha$ is an object in $\overline{R_{A_\mu}(\mathfrak{g})} \otimes R_{ad}(\mathfrak{g})$.[1] Taking the generators $t_\alpha = (t_\alpha)_M{}^N$ in the adjoint representation of $\mathfrak{g}$, one defines the generators of G_0 as

$$X_M = \Theta_M{}^\alpha t_\alpha. \tag{17}$$

These can be coupled to A_μ^M. The gauge covariant derivative for example is

$$D_\mu = \partial_\mu - g\, A_\mu^M X_M, \tag{18}$$

where the flat spacetime derivative ∂_μ is used and $g \in \mathbb{R}$ is a *gauge coupling parameter*. g can be thought of as parametrizing the deformation of the theory into a gauge theory. The limit $g \mapsto 0$ restores the ungauged theory. We can conversely interpret Θ as a map sending A_μ^M to a field $\tilde{A}_\mu^\alpha = \Theta_M{}^\alpha t_\alpha$ in the adjoint representation of $\mathfrak{g}_0 \subset \mathfrak{g}$.

A generic feature arising in gauged supergravities is the existence of a potential term in the Lagrangian, which can be brought to the form

$$V = g^2\, V^{MN}{}_{\alpha\beta} \Theta_M{}^\alpha \Theta_N{}^\beta, \tag{19}$$

1 The indices run to the dimension of the respective representation, i.e. $M = 1, ..., \dim R_{A_\mu}$ and $\alpha = 1, ..., \dim R_{ad}$.

where ${V^{MN}}_{\alpha\beta}$ is a scalar dependent matrix (cf. [Sam08]). This potential is usually not positive definite. It thus supports *anti de-Sitter vacuum states* with a negative cosmological constant, which are the main point of interest of this thesis.

So far we did not ensure, that the image of Θ actually forms a Lie algebra. For this a *closure constraint* is imposed on Θ, which can be shown to follow from the invariance of the embedding tensor under G_0. Starting from the variation of Θ under G_0

$$\delta_P {\Theta_M}^\alpha \equiv {\Theta_P}^\beta {t_{\beta M}}^N {\Theta_N}^\alpha + {\Theta_P}^\beta {f_{\beta\gamma}}^\alpha {\Theta_M}^\gamma = 0 \tag{20}$$

one can contract it with t_α to get the closure constraint

$$[X_M, X_N] = -{X_{MN}}^P X_p = -{X_{[MN]}}^P X_P = {f_{MN}}^P. \tag{21}$$

This gives a relation between ${X_{MN}}^P$ and the structure constants ${f_{MN}}^P$. Unlike ${f_{MN}}^P$, ${X_{MN}}^P$ has a symmetric part ${X_{(MN)}}^P = {Z_{MN}}^P$. The generators are only antisymmetric in the projected subspace of Θ. Therefore one demands

$${\Theta_P}^\alpha {Z_{MN}}^P = 0. \tag{22}$$

As a further remark, the ${X_{MN}}^P$ also only satisfy the Jacobi identity in the subspace projected by Θ and the violation is again proportional to Z. Besides the quadratic closure constraint there is a linear constraint on Θ coming from supersymmetry. It is called the *representation constraint* and can be examined in a case by case basis. [DWNS08] gives an overview of the possible representations and their restrictions from the representation constraint.

3.2 TENSOR HIERARCHIES

The failure of ${X_{MN}}^P$ to satisfy the Jacobi identity also reflects in problems with the *field strength* defined as usual

$$[D_\mu, D_\nu] = -g\, \mathcal{F}^M_{\mu\nu} X_M, \tag{23}$$

which is not covariant. To define a covariant field strength, the transformation behavior of A^M_μ is altered, including a 1-*form gauge parameter* Ξ^{NP}_μ

$$\delta A^M_\mu = D_\mu \Lambda^M - g\, {Z_{NP}}^M \Xi^{NP}_\mu. \tag{24}$$

Ξ^{NP}_μ is used to gauge away the vector fields in the sector of ${X_{MN}}^P$ which do not satisfy the Jacobi identity. The resulting transformation behavior of $\mathcal{F}^M_{\mu\nu}$ is not yet covariant. One defines a modified field strength

$$\mathcal{H}^M_{\mu\nu} = \mathcal{F}^M_{\mu\nu} + g\, {Z_{NP}}^M B^{NP}_{\mu\nu} \tag{25}$$

using 2-*form fields* $B^{MN}_{\mu\nu} = B^{(MN)}_{\mu\nu}$,[2] which are assigned a transformation behavior that leads to

$$\delta \mathcal{H}^M_{\mu\nu} = -g\,\Lambda^P X_{PN}{}^M \mathcal{H}^N_{\mu\nu}. \tag{26}$$

To obtain the desired transformation behavior of $\mathcal{H}^M_{\mu\nu}$, $B^{MN}_{\mu\nu}$ is gauge transformed as

$$\delta B^{MN}_{\mu\nu} = 2\,D_{[\mu}\Xi^{MN}_{\nu]} - 2\,\Lambda^{(M}\mathcal{H}^{N)}_{\mu\nu} + 2\,A^{(M}_{[\mu}\,\delta A^{N)}_{\nu]} + ... \tag{27}$$

where the dots refer to yet unspecified parts, i.e. those that vanish under contraction with $Z_{MN}{}^P$ and therefore do not contribute to the transformation of $\mathcal{H}^M_{\mu\nu}$.

So to get a covariant transformation of the field strength of A^M_μ we had to introduce a 2-form field $B^{MN}_{\mu\nu}$ which also is gauge transformed. For these 2-form fields one can define a field strength $\mathcal{F}^{MN}_{\mu\nu\rho}$. To ensure the covariant transformation behavior of these, further 3-form fields have to be introduced. Continuing this procedure gives a *tensor hierarchy* (cf. [DWNS08]) of gauge fields, which can be worked out in a case by case study for every spacetime dimension D.

One might wonder, if adding new fields is in conflict with supersymmetry, since supersymmetry fixes the number of degrees of freedom. The procedure outlined here in fact adds fields, but does not add degrees of freedom. The additional fields are just part of a redundant description of the theory. For any given gauge theory with constant Θ, the number of degrees of freedom is just the value needed for it to be maximally supersymmetric. The embedding tensor distributes the degrees of freedom on the different p-form fields needed for the gauging. The remaining fields decouple from the theory.

3.3 THE T-TENSOR

An important object derived from the embedding tensor is the *T-tensor*, which is a tensor of the maximal compact subgroup $H \subset G$. The T-tensor occurs in couplings of fermions and also plays an important role in our calculation. It is defined as the embedding tensor dressed with the scalar matrix $\mathcal{V}$ i.e.

$$T_{\underline{N}}{}^{\underline{\beta}} = \Theta_M{}^\alpha \mathcal{V}^M{}_{\underline{N}} \mathcal{V}^{\underline{\beta}}{}_\alpha \tag{28}$$

where underlined indices are used to denote the indices of the subgroup H and α again denotes the adjoint, while M denotes the representation of A^M_μ. $\mathcal{V}^M{}_{\underline{N}}$ and $\mathcal{V}^{\underline{\beta}}{}_\alpha$ refer to the scalar matrix in the respective representations i.e. to an appropriate product of $\mathcal{V}$ in the fundamental representation (cf. [Sam08]). Since T is defined via the scalar matrices, it is field dependent $T = T(\phi)$. Also note that T inherits the linear

2 $B^{MN}_{\mu\nu}$ is actually just in a subset of the symmetric tensor product. For details see [DWNS08] p.7 following.

representation constraint from Θ. In practice one starts with Θ in the representation(s) that are allowed by the linear constraint and then decomposes these under $\mathfrak{h}$ to get components of T.

Supersymmetric AdS vacua impose constraints on T. These can be translated back to constraints on Θ and therefore restrictions on the choice of $G_0 \subset G$. Furthermore as T is field dependent, solutions to its constraints can be varied along the scalar fields ϕ.

3.4 GAUGED SUPERGRAVITY AND COMPACTIFICATIONS

A different way to construct gauged supergravities is by dimensional reduction. In section 2.4 we outlined the procedure of torus compactifications. This procedure can be altered, for example by using a different internal manifold than the torus. Another way to alter the procedure is by including a *twist*, where the cyclic boundary conditions imposed on the fields are modified to include a transformation of a symmetry G of the theory (cf. [Roe05]). Furthermore in the higher dimensional theory, a p-form field can be given a background flux (cf. [Sam08]), or the internal manifold can be supplied with torsion (a *geometric flux*, cf. [Sam08]).

In any case, the resulting lower dimensional theory usually is a gauged supergravity. It is expected that all these theories are included in the embedding tensor formalism [Sam08]. One case of interest are coset reductions, where T^d is replaced by a coset G/K, with G being a symmetry group of the theory and $K \subset G$ being any subgroup of G. The resulting gauge symmetry is the isometry group of the metric taken on G/K. The most symmetric metric is called *round metric* and has G as its isometry group.

As an example consider the d-sphere S^d. S^d is a coset $S^d = SO(d+1)/SO(d)$ and the standard metric on S^d is indeed round, with isometry given by $SO(d+1)$. One can deform this metric to obtain smaller isometry groups, corresponding to *squashed spheres*.

4

ADS VACUA IN D=7 SUPERGRAVITY

4.1 GAUGED MAXIMAL D=7 SUPERGRAVITIES

The ungauged maximal $D = 7$ supergravity was constructed by Sezgin and Salam in [SS82]. [SW05] gives a description of its gaugings via the embedding tensor and is thus used as the main reference for this section.

The field content of the ungauged $D = 7$ theory is given by the graviton multiplet

$$(e_\mu^r, A_\mu^{MN}, B_{\mu\nu M}, \mathcal{V}_M^{ab}; \psi_\mu^a, \chi^{abc}). \tag{29}$$

The bosonic fields are the vielbein e_μ^r, with flat spacetime indices $r, s, ... = 0, ..., 6$, vector fields A_μ^{MN} and 2-form fields $B_{\mu\nu M}$ with $M, N, ... = 1, ..., 5$ and the scalar matrix $\mathcal{V}_M^{ab}$ with $a, b, ... = 1, ..., 4$. $M, N, ...$ label the fundamental representation of $G = SL(5)$, which is the global symmetry group of the $D = 7$ theory. Its maximal compact subgroup is $H = SO(5) \subset SL(5)$. By $\mathfrak{so}(5) \simeq \mathfrak{usp}(4)$ we use indices $a, b, ... = 1, ..., 4$ to denote the representations of H as symplectic Majorana spinors.[1] The fermions come in such representations, namely the gravitino ψ_μ^a and the graviphotino χ^{abc}.

The generators of G in the adjoint representation are $(t_\alpha)_M{}^N$ with $\alpha = 1, ..., 24$. The vector fields transform in the $\overline{\mathbf{10}}$ of $SL(5)$, which is the second exterior power of the fundamental representation i.e. $A_\mu^{MN} = A_\mu^{[MN]}$. The embedding tensor $\Theta_{MN,P}{}^Q$ thus is in the $\mathbf{10} \otimes \mathbf{24}$. Using the generators of G and the embedding tensor, the generators X_{MN} of the gauge group $G_0 \subset G$ are defined to be

$$X_{MN} = \Theta_{MN,P}{}^Q t_P{}^Q. \tag{30}$$

The gauge covariant derivative is

$$D_\mu = \partial_\mu - g\, A_\mu^{MN} X_{MN} = \partial_\mu - g\, A_\mu^{MN} \Theta_{MN,P}{}^Q t_P{}^Q. \tag{31}$$

The tensor product $\mathbf{10} \otimes \mathbf{24}$ of Θ can be decomposed into

$$\mathbf{10} \otimes \mathbf{24} = \mathbf{10} + \mathbf{15} + \overline{\mathbf{40}} + \mathbf{175}. \tag{32}$$

Supersymmetry constrains the possible representations of Θ to the $\mathbf{15}$ and $\overline{\mathbf{40}}$ [SW05]. The respective components are denoted by $Y_{MN} = Y_{(MN)}$ and $Z^{MN,P} = Z^{[MN],P}$ with $Z^{[MN,P]} = 0$. Explicitly

$$\Theta_{MN,P}{}^Q = \delta^Q_{[M} Y_{N]P} - 2\epsilon_{MNPRS} Z^{RS,Q}. \tag{33}$$

1 The definition of symplectic Majorana spinors is given in Appendix A.

As described in section 3.2 one can define covariant field strengths $\mathcal{H}^{(2)MN}_{\mu\nu}$ for the 1-form field A^{MN}_μ and $\mathcal{H}^{(3)}_{\mu\nu\rho M}$ for the 2-form field $B_{\mu\nu M}$. For $\mathcal{H}^{(2)MN}_{\mu\nu}$ a term proportional to $B_{\mu\nu M}$ is added to the usual field strength $\mathcal{F}^{(2)MN}_{\mu\nu}$. Similarly for $\mathcal{H}^{(3)}_{\mu\nu\rho M}$ one needs to add a 3-form field $S^M_{\mu\nu\rho}$. This 3-form field does not add degrees of freedom. In the ungauged theory, it can be dualized via Hodge duality to $B_{\mu\nu M}$. In gauge theories there can be obstructions to these dualizations, coming from the necessity of the fields carrying certain representations (cf. [SW05]). Therefore $S^M_{\mu\nu\rho}$ is needed in the general description of gauge theories. Θ then distributes the degrees of freedom among the different p-form fields.

The scalar sector of the theory is described by a matrix $\mathcal{V}_M^{ab}$. The pair ab denotes the **5** of $USp(4)$, which is a vector representation. Vector representations of $USp(4)$ can be expressed using antisymmetric, symplectic traceless spinor index pairs.[2] $\mathcal{V}_M^{ab}$ thus satisfies $\mathcal{V}_M^{ab} = \mathcal{V}_M^{[ab]}$ and $\Omega_{ab}\mathcal{V}_M^{ab} = 0$, where Ω_{ab} denotes the symplectic matrix preserved by $USp(4)$. Ω_{ab} is also used to raise and lower indices, e.g. $\mathcal{V}_{Mab} = \Omega_{ac}\Omega_{bd}\mathcal{V}_M^{cd}$. The used representations are *pseudoreal*, i.e. $\left(\mathcal{V}_M^{ab}\right)^* = \mathcal{V}_{Mab}$.[3]

For the kinetic terms in the bosonic Lagrangian the matrix $\mathcal{M}_{MN}$ is defined from $\mathcal{V}_M^{ab}$ as

$$\mathcal{M}_{MN} = \mathcal{V}_M^{ab}\mathcal{V}_N^{cd}\Omega_{ac}\Omega_{bd}. \tag{34}$$

This matrix is unimodular and positive definite (cf. [DWST03]). The bosonic Lagrangian is then

$$\begin{aligned} e^{-1}\mathcal{L} = &-\frac{1}{2}R - \mathcal{M}_{MP}\mathcal{M}_{NQ}\mathcal{H}^{(2)MN}_{\mu\nu}\mathcal{H}^{(2)\mu\nu PQ} \\ &-\frac{1}{6}\mathcal{M}^{MN}\mathcal{H}^{(3)}_{\mu\nu\rho M}\mathcal{H}^{(3)\mu\nu\rho}_N \\ &+\frac{1}{8}(\partial_\mu\mathcal{M}_{MN})(\partial^\mu\mathcal{M}^{MN}) - g^2\,V + e^{-1}\mathcal{L}_{VT} \end{aligned} \tag{35}$$

where V is the potential, which is defined at a later point in (39) and $\mathcal{L}_{VT}$ is a collection of topological terms. These topological terms couple the different vector and tensor fields and in particular include a kinetic term for the 3-form field $S^M_{\mu\nu\rho}$. They are needed to ensure supersymmetry invariance of the gauged theory [SW05].

$\mathcal{V}_M^{ab}$ is also used to define the T-tensor, which is the $USp(4)$ analogue of the embedding tensor. It is given by

$$T_{(ef)[ab]}{}^{[cd]} = \sqrt{2}\,\mathcal{V}^M_{eg}\mathcal{V}^N_{fh}\Omega^{gh}\mathcal{V}^P_{ab}\Theta_{MN,P}{}^Q\mathcal{V}^{cd}_Q. \tag{36}$$

2 With *vector representations* we mean any actual tensor representation of $SO(5)$ as opposed to spinor representations which can only be defined by lifting to the universal cover $USp(4)$. (The vectors have an even number of spinor indices.)

3 Pseudoreality can only be demanded for the vector representations.

The T-tensor is used in couplings of fermions and is the main object needed for the calculation in section 4.2. It inherits the linear representation constraint on Θ. The decomposition of Θ into **15** and $\overline{\mathbf{40}}$ branches under $\mathfrak{usp}(4)$ as

$$\mathbf{15} + \overline{\mathbf{40}} \mapsto (\mathbf{1} + \mathbf{14}) + (\mathbf{5} + \mathbf{35}) \tag{37}$$

with the representations

$$\begin{aligned}
\mathbf{1} &: B \in \mathbb{R} \\
\mathbf{14} &: B^{ab}{}_{cd} = B^{[ab]}{}_{[cd]},\ \Omega_{ab} B^{ab}{}_{cd} = 0 = \Omega^{cd} B^{ab}{}_{cd} \\
\mathbf{5} &: C^{ab} = C^{[ab]},\ \Omega_{ab} C^{ab} = 0 \\
\mathbf{35} &: C^{ab}{}_{cd} = C^{[ab]}{}_{(cd)},\ \Omega_{ab} C^{ab}{}_{cd} = 0.
\end{aligned} \tag{38}$$

All these representations are pseudoreal. These T-tensor components contribute to the potential

$$V = -\frac{1}{128}\Big(15\, B^2 + 2\, C^{ab} C_{ab} - 2\, B^{ab}{}_{cd} B^{cd}{}_{ab} - 2\, C^{ab}{}_{cd} C_{ab}{}^{cd}\Big). \tag{39}$$

This potential can take negative values. Thus there can be anti-de Sitter backgrounds. For a supersymmetric AdS background, further constraints have to be fulfilled, which will be examined in the next section.

4.2 FINDING THE ADS VACUA

The supersymmetric AdS_7 backgrounds can be found, by setting all fields that break Lorentz invariance and all variations under supersymmetry transformations to zero (or more precisely their expectation values). The fermion variations are the ones that impose constraints. These are given by the gravitino variation[4]

$$\delta\psi^a_\mu = D_\mu \epsilon^a - g\, \Gamma_\mu A_1^{ab} \Omega_{bc} \epsilon^c + ... \tag{40}$$

and the graviphotino variation

$$\delta\chi^{abc} = g\, A_2^{d,abc} \Omega_{de} \epsilon^e + ... \tag{41}$$

with the fermion shift matrices A_1 and A_2 given by

$$\begin{aligned}
A_1^{ab} &= -\frac{1}{4\sqrt{2}}\Big(\frac{1}{4} B\, \Omega^{ab} + \frac{1}{5} C^{ab}\Big) \\
A_2^{d,abc} &= \frac{1}{2\sqrt{2}}\Big[C^{abcd} - B^{abcd} \\
&\qquad + \frac{1}{4}\Big(C^{ab}\Omega^{cd} + \frac{1}{5}\Omega^{ab} C^{cd} + \frac{4}{5}\Omega^{c[a} C^{b]d}\Big)\Big].
\end{aligned} \tag{42}$$

4 Trivially vanishing terms are denoted by "...". The full variations can be found in [SW05].

$\delta\chi = 0$ is easily solved as[5]

$$\begin{aligned} 0 =& A_2^{d,abc} \\ =& \frac{1}{2\sqrt{2}}\Big[C^{abcd} - B^{abcd} + \frac{1}{4}\Big(C^{ab}\Omega^{cd} + \frac{1}{5}\Omega^{ab}C^{cd} + \frac{4}{5}\Omega^{c[a}C^{b]d}\Big)\Big] \\ =& \frac{1}{2\sqrt{2}}\Big[C^{[ab](cd)} - B^{[ab][cd]} \\ &+ \frac{1}{4}\Big(C^{[ab]}\Omega^{[cd]} + \frac{1}{5}\Omega^{[ab]}C^{[cd]} + \frac{4}{5}\Omega^{c[a}C^{b]d}\Big)\Big]. \end{aligned} \tag{43}$$

Collecting the tensors in the same representation gives

$$C^{[ab](cd)} = -\frac{1}{5}\Omega^{(c[a}C^{b]d)} \tag{44}$$

and

$$B^{[ab][cd]} = \frac{1}{4}C^{[ab]}\Omega^{[cd]} + \frac{1}{20}\Omega^{[ab]}C^{[cd]} + \frac{1}{5}\Omega^{[c[a}C^{b]d]}, \tag{45}$$

where the last term in (43) was decomposed into symmetric and anti-symmetric parts.[6]

$\delta\psi = 0$ can be reformulated by using that ϵ^a is a Killing spinor on anti-de Sitter spacetime. The resulting equation is (cf. [SW05])

$$2\,A_{1\,ab}\epsilon^b = \pm\sqrt{-V/15}\,\Omega_{ab}\epsilon^b \tag{46}$$

leading to

$$\frac{-1}{2\sqrt{2}}\Big(\frac{1}{4}B\,\Omega_{ab} + \frac{1}{5}C_{ab}\Big) = \pm\sqrt{-V/15}\,\Omega_{ab}. \tag{47}$$

Since the right-hand side is proportional to Ω_{ab}, C_{ab} is restricted to be of the form $C_{ab} = C\,\Omega_{ab}$ for some $C \in \mathbb{R}$.[7] Furthermore $\Omega^{ab}C_{ab} = 0 = 4\,C$. Therefore

$$C^{ab} = 0 \tag{48}$$

and

$$B = \mp 8\sqrt{\frac{-2}{15}V}. \tag{49}$$

Inserting (48) into (44) and (45) gives

$$C^{ab}{}_{cd} = 0 = B^{ab}{}_{cd}. \tag{50}$$

So the only nonzero component of the T-tensor is B as given in (49).

We should check, if this T-tensor satisfies the closure constraint, which is (cf. [SW05])

$$Y_{MQ}Z^{QN,P} + 2\,\epsilon_{MRSTU}Z^{RS,N}Z^{TU,P} = 0, \tag{51}$$

5 ϵ^e is an arbitrary symplectic spinor and Ω_{de} is nondegenerate.
6 Nested brackets are interpreted as $T^{(a[bc]d)} = T^{(a|[bc]|d)} = \frac{1}{2}\big(T^{a[bc]d} + T^{d[bc]a}\big)$.
7 From $(C^{ab})^* = C_{ab}$ it can be followed, that C is indeed real, since $(\Omega^{ab})^* = \Omega_{ab}$.

with the Θ components $Z^{MN,P}$ and Y_{MN} as in (33). $Z^{MN,P}$ can be obtained from the T-tensor via its $USp(4)$ analogue Z^{abcd}

$$Z^{MN,P} = \sqrt{2}\,\mathcal{V}^M_{ab}\mathcal{V}^N_{cd}\mathcal{V}^P_{ef}\Omega^{bd}Z^{(ac)[ef]}. \tag{52}$$

In terms of the components of the T-tensor, Z^{abcd} reads

$$Z^{abcd} = \frac{1}{16}\,\Omega^{a[c}C^{d]b} + \frac{1}{16}\,\Omega^{b[c}C^{d]a} - \frac{1}{8}\,\Omega^{ae}\Omega^{bf}C^{cd}{}_{ef}. \tag{53}$$

We thus see that $Z^{MN,P}$ vanishes by (48) and (50). Therefore (51) is trivially fulfilled and imposes no further constraints.

4.3 MODULI OF THE ADS VACUA

In the previous section we found AdS vacua parametrized by the T-tensor component B. The T-tensor is field dependent, since it is defined using the scalar matrix $\mathcal{V}^{ab}_M$. Therefore the solutions can be varied along the scalar manifold. We want to find moduli of the solutions, i.e. variations of the scalars, along which solutions vary into new solutions, while Λ is left constant and fixed.

A way to describe scalar variations is by right multiplication of $\mathcal{V}$ with an element $L_{ab}{}^{cd}(x)$ of the adjoint of $\mathfrak{sl}(5)$. One can use that the adjoint of $\mathfrak{sl}(5)$ splits under $\mathfrak{usp}(4)$ as $\mathbf{24} \mapsto \mathbf{10} + \mathbf{14}$ and therefore decompose

$$L_{ab}{}^{cd} = 2\,\Lambda_{[a}{}^{[c}\delta^{d]}_{b]} + \Sigma^{cd}{}_{ab}, \tag{54}$$

where $\Lambda_a{}^c$ is in the $\mathbf{10}$ and $\Sigma^{cd}{}_{ab}$ is in the $\mathbf{14}$. Then $\mathcal{V}$ varies as

$$\delta\mathcal{V}^{ab}_M = \mathcal{V}^{cd}_M\Sigma^{ab}{}_{cd}(x) - 2\mathcal{V}^{c[a}_m\Lambda_c{}^{b]}(x). \tag{55}$$

The $\mathbf{10}$ denoted by $\Lambda_a{}^c$ is the adjoint representation of $USp(4)$. These are the $USp(4) \simeq SO(5)$ transformations which are divided out to get the scalar coset $SL(5)/SO(5)$. The $\mathbf{14}$ denoted by $\Sigma^{cd}{}_{ab}$ then parametrizes the 14 physical degrees of freedom of the coset space. As we are interested in physical moduli, we thus need to consider

$$\delta_\Sigma\mathcal{V}^{ab}_M = \mathcal{V}^{cd}_M\Sigma^{ab}{}_{cd(x)}. \tag{56}$$

Under this variation there are corresponding variations of B, $B^{ab}{}_{cd}$, C^{ab} and $C^{ab}{}_{cd}$, since these are defined via $\mathcal{V}$ and therefore depend on the scalars. These variations are (cf. [SW05])

$$\begin{aligned}
\delta_\Sigma B &= -\frac{2}{5}\,\Sigma^{ab}{}_{cd}B^{cd}{}_{ab}\\
\delta_\Sigma B^{ab}{}_{cd} &= -2\,B\,\Sigma^{ab}{}_{cd} - \Sigma^{ab}{}_{ef}B^{ef}{}_{cd} - \Sigma^{ef}{}_{cd}B^{ab}{}_{ef}\\
&\qquad + \frac{2}{5}\Big(\delta^{ab}_{cd} - \frac{1}{4}\Omega^{ab}\Omega_{cd}\Big)\Sigma^{ef}{}_{gh}B^{gh}{}_{ef}
\end{aligned} \tag{57}$$

and

$$\begin{aligned}
\delta_\Sigma C^{ab} &= \frac{1}{2}\,\Sigma^{ab}{}_{cd}C^{cd} + 2\Omega^{e[a}\Sigma^{b]f}{}_{cd}C^{cd}{}_{ef} \\
\delta_\Sigma C^{ab}{}_{cd} &= 4\,\Omega^{e[a}\Sigma^{b]f}{}_{e(c}C_{d)f} + \Omega^{e[a}\delta^{b]}_{(c}\Sigma^{gf}{}_{d)e}C_{gf} \\
&\quad + \Omega^{eg}\delta^{[a}_{(c}\Sigma^{b]f}{}_{d)e}C_{gf} \\
&\quad + \Sigma^{ab}{}_{ef}C^{ef}{}_{cd} + \Sigma^{g[a}{}_{ef}\delta^{b]}_{(c}C^{ef}{}_{d)g} \\
&\quad + 4\,\Sigma^{eg}{}_{f(c}\Omega_{d)e}\Omega^{h[a}C^{b]f}{}_{gh} \\
&\quad - \delta^{[a}_{(c}\Omega_{d)f}\Omega^{b]k}\Sigma^{fh}{}_{ge}C^{eg}{}_{hk},
\end{aligned} \tag{58}$$

where $\delta^{ab}_{cd} = \delta^{[a}_{c}\delta^{b]}_{d} = \delta^{a}_{[c}\delta^{b}_{d]}$.

Along the moduli, solutions are varied into new solutions therefore we need to solve $\delta_\Sigma B^{ab}{}_{cd} = \delta_\Sigma C^{ab} = \delta_\Sigma C^{ab}{}_{cd} = 0$. Furthermore V needs to stay constant. Thus we also impose $\delta_\Sigma B = 0$. Inserting the solutions $C^{ab} = C^{ab}{}_{cd} = B^{ab}{}_{cd} = 0$ leads to

$$\begin{aligned}
\delta_\Sigma B &= 0, \\
\delta_\Sigma B^{ab}{}_{cd} &= -\,2B\,\Sigma^{ab}{}_{cd}, \\
\delta_\Sigma C^{ab} &= 0, \\
\delta_\Sigma C^{ab}{}_{cd} &= 0.
\end{aligned} \tag{59}$$

The only nontrivial constraint is $0 = -2B\,\Sigma^{ab}{}_{cd}$. For an AdS background $B \propto \sqrt{-\Lambda} \neq 0$. Therefore $\Sigma^{ab}{}_{cd} = 0$, i.e. there are no physical moduli to the solution. Note, that in a Minkowski background $B \propto \sqrt{-\Lambda} = 0$. This makes all constraints trivial and therefore all directions of the scalar manifold are moduli of the Minkowski solution.

4.4 ALLOWED GAUGE GROUPS

An AdS background imposes constraints on the T-tensor. These can be translated into constraints on the embedding tensor Θ and thus into constraints on the possible gauge groups. In (53) it was shown that the Θ component $Z^{MN,P}$ vanishes in an AdS solution. The remaining component in the decomposition (33) is thus Y_{MN}.

In analogy to the way we defined $Z^{(ab)[cd]}$ from $Z^{MN,P}$ in (52), one defines $Y_{ab,cd}$ such that

$$Y_{MN} = \mathcal{V}_M^{ab}\mathcal{V}_N^{cd}Y_{ab,cd}. \tag{60}$$

With only the **1** component of T being nonzero $Y_{ab,cd}$ is of the form[8]

$$Y_{ab,cd} = \frac{1}{\sqrt{2}}\Big(\Omega_{ac}\Omega_{bd} - \frac{1}{4}\Omega_{ab}\Omega_{cd}\Big)B. \tag{61}$$

8 The general formula can be found in [SW05].

Inserting (61) into (60) gives

$$Y_{MN} = \frac{1}{\sqrt{2}} \mathcal{V}_M^{ab} \mathcal{V}_N^{cd} (\Omega_{ac}\Omega_{bd} - \frac{1}{4}\Omega_{ab}\Omega_{cd}) B. \tag{62}$$

Since $\mathcal{V}_M^{ab}$ belongs to the **5** of $USP(4)$, $\Omega_{ab}\mathcal{V}_M^{ab} = 0$ and (62) reduces to

$$Y_{MN} = \frac{1}{\sqrt{2}} \mathcal{V}_M^{ab} \mathcal{V}_N^{cd} \Omega_{ac}\Omega_{bd} B. \tag{63}$$

One can insert the definition of $\mathcal{M}_{MN}$ (34) to get

$$Y_{MN} = \frac{B}{\sqrt{2}} \mathcal{M}_{MN}. \tag{64}$$

[SW05] discusses the case of $Z^{MN,P} = 0 = Z^{(ab)[cd]}$ and specifies the resulting gauge group. First they diagonalize Y_{MN}:

$$Y_{MN} = \text{diag}(\underbrace{1, ..., 1}_{p}, \underbrace{-1, ...-, 1}_{q}, \underbrace{0, ..., 0}_{r}), \tag{65}$$

with $p + q + r = 5$. The resulting gauge group is then

$$G_0^{(p,q,r)} = CSO(p, q, r) \equiv SO(p, q) \ltimes \mathbb{R}^{(p+q)\cdot r}. \tag{66}$$

Since $Y_{MN} \propto \pm\sqrt{-\Lambda}\,\mathcal{M}_{MN}$ and $\mathcal{M}_{MN}$ is positive definite, Y_{MN} is either positive or negative definite. The only possible gauge group is thus $SO(5) = CSO(5, 0, 0) \simeq CSO(0, 5, 0)$. Again we can make a consistency check with the Minkowski case $Y_{MN} = 0$, in which (66) gives no gauge group.

There is thus a single possible gauge group consistent with a maximally supersymmetric anti-de Sitter background. This gauge group is $G_0 = SO(5)$.

4.5 COMPARISON TO THE DUAL SCFT

In section 4.2 we found a one parameter family of AdS vacua parametrized by the T-tensor component $B \propto \sqrt{-\Lambda}$. Note that there are two solutions $\pm B$ for any given value of $\Lambda < 0$.[9] There are however no moduli of the solution. The only allowed gauge group is $G_0 = SO(5)$.

On the SCFT side, AdS backgrounds in the $D = 7$ maximal case correspond to the $\mathcal{N} = (2, 0)$ SCFT in six dimensions. The moduli of the AdS backgrounds correspond to the conformal manifold, spanned by marginal operators in the SCFT. [LL15] examines the existence of marginal operators for the $\mathcal{N} = (1, 0)$ case and concludes that there are no marginal operators. Hence if there are no marginal operators for $\mathcal{N} = (1, 0)$ there can not be any for $\mathcal{N} = (2, 0)$ either. The result we

9 From pseudoreality of the $USP(4)$ representations, $B^* = B \in \mathbb{R}$ and therefore $\Lambda \leq 0$. For $\Lambda = 0$ there is just one solution $B = 0$.

obtained is thus in accordance with AdS/CFT as the nonexistence of moduli corresponds to the nonexistence of marginal operators and vice versa.

Furthermore the gauge group on the AdS side corresponds to the R-symmetry of the SCFT. This is true for our result of $G_0 = SO(5) \simeq USp(4)$ [CDI16].

5 ADS VACUA IN D=6 SUPERGRAVITY

5.1 GAUGED MAXIMAL D=6 SUPERGRAVITIES

The ungauged maximal $D = 6$ supergravity was constructed by Tanii in [Tan84]. [BSS08] gives a description of its gaugings via the embedding tensor and is thus used as the main reference for this section.

The field content of the ungauged $D = 6$ theory is given by the graviton multiplet

$$(e_\mu^r, A_\mu^A, B_{\mu\nu M}, \mathcal{V}_A^{\alpha\dot{\alpha}}; \psi_{\mu+}^\alpha, \psi_{\mu-}^{\dot{\alpha}}, \chi_{\dot{\alpha}+}^a, \chi_{\alpha-}^{\dot{b}}). \tag{67}$$

The bosonic fields are the vielbein e_μ^r, with flat spacetime indices $r, s, ...$ $= 0, ..., 5$, vector fields A_μ^A with $A, B, ... = 1, ..., 16$, 2-form fields $B_{\mu\nu M}$ with $M, N, ... = 1, ..., 10$ and the scalar matrix $\mathcal{V}_M^{\alpha\dot{\alpha}}$ with $\alpha, \beta, ...; \dot{\alpha}, \dot{\beta}, ...$ $= 1, ..., 4$. $M, N, ...$ label the fundamental representation of $G =$ $SO(5, 5)$, which is the global symmetry group of the $D = 6$ theory.

A generic feature in even dimensions is that the global symmetry group is not manifest in the Lagrangian and in fact not realized off shell. In even dimensions field strengths can be selfdual under Hodge duality. This leads to the division of $(D/2-1)$-forms into electric and magnetic degrees of freedom. Both kinds of degrees of freedom are needed for invariance under G and since the equations of motion involve both, these are indeed invariant under G. The Lagrangian however contains just the electric degrees of freedom and is thus only invariant under a subgroup $GL(5) \subset SO(5, 5)$. The 2-forms therefore split into $GL(5)$ tensors $B_{\mu\nu m}$ and $B_{\mu\nu}^m$ with $m, n, ... = 1, ..., 5$ which are the electric and magnetic 2-forms respectively. The used notation for $B_{\mu\nu M}$ is then $B_{\mu\nu M} = (B_{\mu\nu m}, B_{\mu\nu}^m)$.

The maximal compact subgroup of G is $H = SO(5) \times SO(5)$. As in seven dimensions indices $\alpha, \beta, ... = 1, ..., 4$ and $\dot{\alpha}\dot{\beta}, ... = 1, .., 4$ are used to denote the representations of H as $USp(4)$ representations. The dotted indices correspond to the right copy of $SO(5)$ in the product, while the undotted ones correspond to the left copy.

The fermions in $D = 6$ come in chiral representations. In the maximal case there are two pairs of supersymmetry generators, with opposite chirality. Hence it is usually labeled $\mathcal{N} = (2, 2)$ supergravity. The gravitini and graviphotini also come in chiral representations as $\psi_\pm$ and $\chi_\pm$. For the graviphotini $\chi_{\dot{\alpha}+}^a, \chi_{\alpha-}^{\dot{b}}$ the vector representation of $SO(5)$ is also used and denoted by $a, b, ...; \dot{a}, \dot{b}, ... = 1, ..., 5$. Again dotted and undotted indices correspond to the right- and left copy of $SO(5) \times SO(5)$. Vectors of $SO(5) \times SO(5)$ are then denoted by underlined indices $\underline{A} = (a, \dot{a})$.

The vector field A_μ^A is in the $\overline{\mathbf{16}}$, which is the (Majorana-Weyl) spinor representation of $G = SO(5,5)$. Thus, since the adjoint of $SO(5,5)$ is the $\mathbf{45}$, ${\Theta_{A,P}}^Q$ is in the $\mathbf{16} \otimes \mathbf{45}$. Using the generators ${t_P}^Q$ of G and the embedding tensor, the generators X_A of the gauge group $G_0 \subset G$ are defined to be

$$X_A = {\Theta_{A,P}}^Q {t_P}^Q. \tag{68}$$

The gauge covariant derivative is

$$D_\mu = \partial_\mu - g\, A_\mu^A X_A = \partial_\mu - g\, A_\mu^A {\Theta_{A,P}}^Q {t_P}^Q, \tag{69}$$

where g is again the gauge coupling parameter. The tensor product $\mathbf{16} \otimes \mathbf{45}$ of Θ can be decomposed into (cf. [DWNS08])

$$\mathbf{16} \otimes \mathbf{45} = \mathbf{16} + \mathbf{144} + \mathbf{560}. \tag{70}$$

Supersymmetry constrains the possible representations of Θ to the $\mathbf{144}_c$ [BSS08]. With $\theta \in \mathbf{144}_c$, Θ is given by

$$\Theta_A^{MN} = -\theta^{B[M}\gamma_{BA}^{N]} \tag{71}$$

where γ_{AB}^N is the gamma matrix of $SO(5,5)$.

To describe the scalar coset of the theory, two different matrices $\mathcal{V}_M^A$ and $V_A^{\alpha\dot\alpha}$ are used. $\mathcal{V}_M^A$ is a 10×10 matrix, which can be written as a block matrix

$$\mathcal{V}_M^A = \begin{pmatrix} \mathcal{V}_m^a & \mathcal{V}_m^{\dot a} \\ \mathcal{V}^{ma} & \mathcal{V}^{m\dot a} \end{pmatrix}. \tag{72}$$

it fulfills the relations

$$\begin{aligned} &\mathcal{V}^{Ma}\mathcal{V}_M^b = \delta^{ab}, & &\mathcal{V}^{M\dot a}\mathcal{V}_M^{\dot b} = \delta^{\dot a\dot b}, \\ &\mathcal{V}^{Ma}\mathcal{V}_M^{\dot a} = 0, & &\mathcal{V}_M^a\mathcal{V}^{Na} - \mathcal{V}_M^{\dot a}\mathcal{V}^{N\dot a} = \delta_M^N. \end{aligned} \tag{73}$$

The other object needed is the 16×16 $V_A^{\alpha\dot\alpha}$ with inverses $V_A^{\alpha\dot\alpha}V_{\alpha\dot\alpha}^B = \delta_A^B$ and $V_A^{\alpha\dot\alpha}V_{\beta\dot\beta}^A = \delta_\beta^\alpha\delta_{\dot\beta}^{\dot\alpha}$. It can be understood as being the spinorial version of $\mathcal{V}_M^A$ as it is in the spinor representation of both G and H and is related to $\mathcal{V}_M^A$ by

$$\begin{aligned} \mathcal{V}_M^a &= \frac{1}{16}\, V^{A\alpha\dot\alpha}\gamma_{MAB}{\gamma^a}_\alpha{}^\beta V_{\beta\dot\alpha}^B, \\ \mathcal{V}_M^{\dot a} &= \frac{1}{16}\, V^{A\alpha\dot\alpha}\gamma_{MAB}{\gamma^{\dot a}}_{\dot\alpha}{}^{\dot\beta} V_{\alpha\dot\beta}^B. \end{aligned} \tag{74}$$

From $V_A^{\alpha\dot\alpha}$ one defines the matrix

$$M_{AB} = V_A^{\alpha\dot\alpha}V_{B\alpha\dot\alpha}. \tag{75}$$

Using covariant field strengths $\mathcal{H}^{(2)A}_{\mu\nu}$ for A^A_μ and $\mathcal{H}^{(3)}_{\mu\nu\rho m}$ for $B_{\mu\nu m}$ the bosonic Lagrangian is then given by

$$\begin{aligned} e^{-1}\mathcal{L}_B = &\frac{1}{4}R - \frac{1}{4}M_{AB}\mathcal{H}^{(2)A}_{\mu\nu}\mathcal{H}^{(2)\mu\nu B} \\ &- \frac{1}{12}K^{mn}\mathcal{H}^{(3)}_{\mu\nu\rho m}\mathcal{H}^{(3)\mu\nu\rho}_{n} - \frac{1}{16}P^{a\dot{a}}_{\mu}P^{\mu}_{a\dot{a}} \\ &- g^2 V + e^{-1}\mathcal{L}_{VT} \end{aligned} \tag{76}$$

where K^{mn} is a matrix built from the scalar fields to ensure duality invariance of the Lagrangian, i.e. invariance under exchange of electric into magnetic degrees of freedom (cf. [BSS08]). The potential V is given in (80) and $\mathcal{L}_{VT}$ is a collection of topological terms, needed to ensure supersymmetry invariance of the gauged theory.

The scalar matrices $\mathcal{V}$ and V are also used to map the embedding tensor to the T-tensor

$$\begin{aligned} T^{a}_{\alpha\dot{\alpha}} &= \mathcal{V}^{a}_{M}\theta^{AM}V_{A\alpha\dot{\alpha}}, \\ T^{\dot{a}}_{\alpha\dot{\alpha}} &= -\mathcal{V}^{\dot{a}}_{M}\theta^{AM}V_{A\alpha\dot{\alpha}}. \end{aligned} \tag{77}$$

$T^{\underline{A}}$ denotes the full T-tensor. Furthermore one can define

$$T^{ab} = \gamma^{[a}T^{b]},\ T^{\dot{a}\dot{b}} = -T^{[\dot{a}}\gamma^{\dot{b}]} \tag{78}$$

and

$$T = \gamma^{a}T^{a} = -T^{\dot{a}}\gamma^{\dot{a}} \tag{79}$$

where spinor indices are suppressed in both definitions.[1] The T-tensor contributes to the potential

$$V = -T^{a}_{\alpha\dot{\alpha}}T^{a\alpha\dot{\alpha}} + \frac{1}{2}T_{\alpha\dot{\alpha}}T^{\alpha\dot{\alpha}}. \tag{80}$$

As in seven dimensions this potential can take negative values and thus support AdS vacua. In the upcoming section the existence of supersymmetric AdS vacua will be examined.

5.2 FINDING THE ADS VACUA

For supersymmetric AdS vacua the supersymmetry variations of the fermions need to vanish. We get the constraints

$$0 = \delta\psi_{\mu\pm} = D_\mu\epsilon_\pm \pm \frac{1}{4}g\gamma_\mu T\epsilon_\mp + \ldots \tag{81}$$

for the gravitini with different chirality $\pm$, where trivially vanishing terms have been omitted and spinor indices are suppressed. Furthermore for the graviphotini we get the constraints

$$\begin{aligned} 0 = \delta\chi^{a}_{\dot{\alpha}+} &= 2g\,T^{a}_{\dot{\alpha}\alpha}\epsilon^{\alpha} - \frac{g}{2}\,T_{\dot{\alpha}\beta}\gamma^{a}{}_{\alpha}{}^{\beta}\epsilon^{\alpha} + \ldots, \\ 0 = \delta\chi^{\dot{a}}_{\alpha-} &= 2g\,T^{\dot{a}}_{\alpha\dot{\alpha}}\epsilon^{\dot{\alpha}} + \frac{g}{2}\,T_{\alpha\dot{\beta}}\gamma^{\dot{a}}{}_{\dot{\alpha}}{}^{\dot{\beta}}\epsilon^{\dot{\alpha}} + \ldots. \end{aligned} \tag{82}$$

1 $T^{ab} = T^{ab}_{\alpha\dot{\alpha}}$ and $T = T_{\alpha\dot{\alpha}}$.

The first equation of (82) can be reformulated by omitting the arbitrary spinor ϵ^α to get

$$T^a_{\dot{\alpha}\alpha} = -\frac{1}{4} T_{\dot{\alpha}\beta} \gamma^a{}_\alpha{}^\beta. \tag{83}$$

One can multiply (83) by $\gamma^a{}_\rho{}^\alpha$ to get

$$T_{\dot{\alpha}\rho} = -\frac{1}{4} T_{\dot{\alpha}\beta} \gamma^a{}_\alpha{}^\beta \gamma^a{}_\rho{}^\alpha. \tag{84}$$

One can show $\gamma^a{}_\alpha{}^\beta \gamma^a{}_\rho{}^\alpha = 5\,\delta^\beta_\rho$ by using the defining anticommutation relation of gamma matrices. We get

$$T_{\dot{\alpha}\rho} = -\frac{5}{4} T_{\dot{\alpha}\rho}. \tag{85}$$

This is only fulfilled for $T_{\dot{\alpha}\rho} = 0$. We can insert this result back into (83) to find that $T^a_{\dot{\alpha}\alpha} = 0$. The same argument can be made for $T^{\dot{\alpha}}$. Hence the T-tensor vanishes and with it the embedding tensor[2]. The potential (80) vanishes for $T = 0$ and thus vanishes for supersymmetric backgrounds. There are no supersymmetric AdS vacua for the $D = 6$ maximal supergravity.

5.3 COMPARISON TO THE DUAL SCFT

In section 5.2 it was shown that no maximally supersymmetric AdS vacua exist for the $D = 6$ theory. This is in accordance with the fact that there is no $\mathcal{N} = 2$ SCFT in five dimensions [Min98]. The dual SCFT thus does not exist.

2 Note that the assignment of T to θ in (77) is invertible, since the scalar matrices are invertible.

6 ADS VACUA IN D=5 SUPERGRAVITY

6.1 GAUGED MAXIMAL D=5 SUPERGRAVITIES

The ungauged maximal $D = 5$ supergravity was constructed by Cremmer, Scherk and Schwarz in [CSS79]. [DWST05] gives a description of its gaugings via the embedding tensor and is thus used as the main reference for this section.

The field content of the ungauged $D = 5$ theory is given by the graviton multiplet

$$(e^r_\mu, A^M_\mu, \mathcal{V}^{ab}_M; \psi^a_\mu, \chi^{abc}). \tag{86}$$

The bosonic fields are the vielbein e^r_μ, with flat spacetime indices $r, s, ... = 0, ..., 4$, vector fields A^M_μ with $M, N, ... = 1, ..., 27$ and the scalar matrix $\mathcal{V}^{ab}_M$ with $a, b, ... = 1, ..., 8$. $M, N, ...$ label the fundamental representation of $G = E_{6(6)}$, which is the global symmetry group of the $D = 5$ theory. Its maximal compact subgroup is $H = USp(8)$. $USp(8)$ is represented by symplectic Majorana spinors with indices $a, b, ... = 1, ..., 8$. The fermions come in these representations, namely the gravitino ψ^a_μ and the graviphotino χ^{abc}. The symplectic matrix preserved by $USp(8)$ is denoted as Ω^{ab}. As in seven and six dimensions, the vector representations are pseudoreal, i.e. given a $V^{ab}{}_{cd}$, the conjugate is given by $(V^{ab}{}_{cd})^* = V_{ab}{}^{cd} = \Omega_{ai}\Omega_{bj}\Omega^{kc}\Omega^{ld}V^{ij}{}_{kl}$. A single vector index can be mapped to a pair of antisymmetric, symplectic traceless indices $[ab]$.

The generators of G in the adjoint representation are $(t_\alpha)_M{}^N$ with $\alpha = 1, ..., 78$. The vector fields A^M_μ are in the $\overline{\mathbf{27}}$ of $E_{6(6)}$. Hence, since the adjoint representation is the **78**, the embedding tensor $\Theta_M{}^\alpha$ is in the $\mathbf{27} \otimes \mathbf{78}$. Using the generators of G and the embedding tensor, the generators X_M of the gauge group $G_0 \subset G$ are defined to be

$$X_M = \Theta_M{}^\alpha t_\alpha. \tag{87}$$

The gauge covariant derivative is

$$D_\mu = \partial_\mu - g\, A^M_\mu X_M = \partial_\mu - g\, A^M_\mu \Theta_M{}^\alpha t_\alpha, \tag{88}$$

where g is the gauge coupling parameter. The tensor product $\mathbf{27} \otimes \mathbf{78}$ of Θ can be decomposed into

$$\mathbf{27} \otimes \mathbf{78} = \mathbf{27} + \mathbf{351} + \overline{\mathbf{1728}}. \tag{89}$$

Supersymmetry constrains the possible representations of Θ to the **351** [DWST05]. This implies the conditions

$$(t_\alpha)_M{}^N \Theta_N{}^\alpha = 0,\ \ (t_\beta t^\alpha)_M{}^N \Theta_N{}^\beta = -\frac{2}{3}\Theta_M{}^\alpha, \tag{90}$$

where $\alpha, \beta, ...$ are raised and lowered by the $E_{6(6)}$ Killing form $\eta_{\alpha\beta} = tr(t_\alpha t_\beta)$.

As in seven dimensions, higher p-form fields need to be added for consistent gauge theories. Therefore the 2-form fields $B_{\mu\nu M}$ are introduced. In the ungauged theory they can be obtained as the Hodge dual of the 1-forms A_μ^M. These 2-form fields do not add degrees of freedom. The embedding tensor encodes which fields become actual degrees of freedom and which decouple (cf. [DWST05]).

The scalar coset space is represented by the matrix $\mathcal{V}_M^{ab} = \mathcal{V}_M^{[ab]}$ with $\Omega_{ab}\mathcal{V}_M^{ab} = 0$. Its inverse is given by $\mathcal{V}_{ab}^M \mathcal{V}_N^{ab} = \delta_N^M$. Furthermore $\mathcal{V}_{ab}^M \mathcal{V}_M^{cd} = \delta_{ab}^{cd} - \frac{1}{8}\,\Omega^{cd}\Omega_{ab}$. As in previous cases we define

$$\mathcal{M}_{MN} = \mathcal{V}_M^{ij}\mathcal{V}_N^{kl}\Omega_{ik}\Omega_{jl} \tag{91}$$

which is used in the bosonic Lagrangian

$$\begin{aligned} e^{-1}\mathcal{L}_B = &- \frac{1}{2}\,R - \frac{1}{16}\,\mathcal{M}_{MN}\mathcal{H}_{\mu\nu}^M\mathcal{H}^{\mu\nu N} \\ &- \frac{1}{12}\,|P_\mu^{ijl}|^2 - g^2\,V - e^{-1}\mathcal{L}_{VT}. \end{aligned} \tag{92}$$

$\mathcal{H}_{\mu\nu}^M$ is the covariant field strength of A_μ^M and $\mathcal{L}_{VT}$ is a collection of topological terms, needed for consistency of the gauged theory.

$\mathcal{V}_M^{ab}$ is used to define the T-tensor from Θ. The **351** of $E_{6(6)}$ branches under $\mathfrak{usp}(8)$ as

$$\mathbf{351} \mapsto \mathbf{315} + \mathbf{36}. \tag{93}$$

The **36** is denoted by $A_1^{ij} = A_1^{(ij)}$ and the **315** by $A_2^{i,jkl} = A_2^{i,[jkl]}$ with $A_2^{[i,jkl]} = 0$. The T-tensor is then expressed by the components $T^{klmn}{}_{ij}$ in the **315** and $T^i{}_{jkl}$ which belongs to both the **315** and the **36**. Explicitly

$$\begin{aligned} T^{klmn}{}_{ij} &= 4\,A_2^{q,[klm}\delta_{[i}^{n]}\Omega_{j]q} + 3\,A_2^{p,q[kl}\Omega^{mn]}\Omega_{p[i}\Omega_{j]q}, \\ T^i{}_{jkl} &= -\,\Omega_{im}\Big(\Omega_{m[k}A_{1l]j} + \Omega_{j[k}A_{1l]m} + \frac{1}{4}\,\Omega_{kl}A_{1mj}\Big) \\ &\quad -\,\Omega^{im}A_{2(m,j)kl}. \end{aligned} \tag{94}$$

The T-tensor components contribute to the potential

$$V = -3\,|A_1^{ij}|^2 + \frac{1}{3}|A_2^{i,jkl}|^2. \tag{95}$$

As in previous cases this potential can take negative values, leading to AdS Vacua. In the upcoming section the existence of supersymmetric vacua will be examined.

6.2 FINDING THE ADS VACUA

For supersymmetric AdS vacua we have to solve the constraints

$$\delta\chi^{ijk} = 0 = g\,A_2^{l,ijk}\Omega_{lm}\epsilon^m + ... \tag{96}$$

and

$$\delta\psi^i_\mu = 0 = D_\mu\epsilon^i - ig\,\gamma_\mu A_1^{ij}\Omega_{jk}\epsilon^k + ... \tag{97}$$

where in both cases terms which vanish trivially have been omitted. From the first constraint (96) we read off $A_2^{l,ijk} = 0$.[1]

The second condition can be reformulated [DWST05] to yield

$$A_1^{im}A_{1jm}\epsilon^j = \frac{1}{8}\Big(|A_1|^2 - \frac{1}{9}\,|A_2|^2\Big)\epsilon^i, \tag{98}$$

where $|A_1|^2 = A_{1ij}A_1^{ij}$ and $|A_2|^2 = A_2^{l,ijk}A_{2l,ijk}$. Hence

$$A_1^{im}A_{1jm} = \frac{|A_1|^2}{8}\delta^i_j. \tag{99}$$

Note that this equation coincides with the closure constraint for $A_2 = 0$ (cf. [DWST05] eq. (4.30)). Furthermore note that $|A| \propto \sqrt{-\Lambda}$ by the potential (95). For $A_2 = 0$ it reads

$$V = -3g^2\,|A_1|^2. \tag{100}$$

For our further calculations it is convenient to express $A = A_1$ in an $\mathfrak{su}(4)$ basis.[2] In (99) one can use $A_1^{im}A_{1jm} = -A^i_{1m}A^m_{1j} = -\frac{|A_1|^2}{8}\delta^i_j$ and thus see that the constraint can be brought to the form $A^2 \propto -\mathbb{1}$. Hence A acts as a complex structure and can be diagonalized, with four eigenvalues $+i\lambda$ and four eigenvalues $-i\lambda$ (with constant $\lambda \in \mathbb{R}$). One can take an orthonormal basis of eigenvectors e^i_α, $e^j_{\bar\alpha}$ such that

$$\begin{aligned} A^i_j e^j_\alpha &= i\lambda\, e^i_\alpha, \\ A^i_j e^j_{\bar\alpha} &= -i\lambda\, e^i_{\bar\alpha}. \end{aligned} \tag{101}$$

Then

$$A^\alpha_\beta = e^\alpha_i A^i_j e^j_\beta = i\lambda\, e^\alpha_i e^i_\beta = i\lambda\,\delta^\alpha_\beta. \tag{102}$$

This implies the normalization $\lambda = |A|/\sqrt{8}$. In a similar fashion we can define $\Omega_{\alpha\bar\beta} = e^i_\alpha\Omega_{ij}e^j_{\bar\beta}$. Using the symmetry properties of A one finds that it is consistent to have

$$\begin{aligned} A_{\alpha\bar\beta} &= \frac{i|A|}{\sqrt{8}}\,\delta_{\alpha\bar\beta} & A_{\bar\alpha\beta} &= \frac{i|A|}{\sqrt{8}}\,\delta_{\bar\alpha\beta}, \\ \Omega_{\alpha\bar\beta} &= \delta_{\alpha\bar\beta} & \Omega_{\bar\alpha\beta} &= -\,\delta_{\bar\alpha\beta}. \end{aligned} \tag{103}$$

All other components i.e. the $\alpha\beta$ and $\bar\alpha\bar\beta$ components are zero.

1 Recall: ϵ^m is an arbitrary symplectic spinor and Ω_{lm} is nondegenerate.

2 The upcoming change of basis is based on the help of Severin Lüst, who not only had the initial idea to change to an $SU(4)$ basis, but also developed most of the details used to determine the moduli in $D = 5$.

6.3 MODULI OF THE ADS VACUA

As seen in the previous chapter, supersymmetric AdS backgrounds impose constraints on the T-tensor components A_1 and A_2. To find moduli we will have to consider the variations of the A's under variations of the scalar fields. In a similar manner to the $D = 7$ case, variations of $\mathcal{V}$ can be parametrized by an element of the adjoint of $E_{6(6)}$. Under $\mathfrak{usp}(8)$ it splits as $\mathbf{78} \mapsto \mathbf{42} + \mathbf{36}$. The $\mathbf{36}$ corresponds to the adjoint of $USp(8)$. These are the directions that are divided out to get the G/H coset. The physical variations are given by the $\mathbf{42}$, denoted by $\Sigma^{ijkl} = \Sigma^{[ijkl]}$, which is symplectic traceless $\Sigma^{ijkl}\Omega_{ij} = 0$.

The corresponding variations of the T-tensor components are given by [DWST05]

$$\begin{aligned} \delta A_1^{ij} &= \frac{4}{9}\,\Omega^{p(i}\Sigma^{j)klm}A_{2p,klm}, \\ \delta A_2^{i,jkl} &= \frac{3}{2}\Big(\Omega^{mi}\Sigma^{jkln} + \Omega^{m[j}\Sigma^{kl]in}\Big)A_{1mn} \\ &\quad - \Big(\Omega^{i[j}\Omega^{k|m|}\Sigma^{l]npq} - 3\,\Omega^{ni}\Omega^{m[j}\Sigma^{kl]pq} \\ &\quad - \frac{1}{6}\,\Omega^{im}\Omega^{[kl}\Sigma^{j]npq} + \frac{1}{6}\,\Omega^{m[j}\Omega^{kl]}\Sigma^{inpq}\Big)A_{2m,npq}. \end{aligned} \tag{104}$$

These have to vanish along the moduli. It is helpful to use the $\mathfrak{su}(4)$ basis as defined above. Under $\mathfrak{su}(4)$ Σ^{ijkl} decomposes as[3]

$$\mathbf{42} \mapsto \mathbf{1} + \overline{\mathbf{1}} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{20}. \tag{105}$$

Σ is completely antisymmetric, hence the $\mathbf{1}$ can be expressed as

$$\Sigma^{\alpha\beta\gamma\delta} = \sigma\,\epsilon^{\alpha\beta\gamma\delta} \tag{106}$$

where $\sigma \in \mathbb{C}$ s.t. $\Sigma^{\bar\alpha\bar\beta\bar\gamma\bar\delta} = \bar\sigma\,\epsilon^{\bar\alpha\bar\beta\bar\gamma\bar\delta}$ as can be inferred from pseudoreality of Σ^{ijkl}. Given $A_2 = 0$ the first constraint from (104) is trivial. The other reads schematically

$$\delta A_2^{\alpha,\beta\gamma\delta} = 0 = \Omega^{m\alpha}\Sigma^{\beta\gamma\delta n}A_{mn} + \Omega^{m[\beta}\Sigma^{\gamma\delta]\alpha n}A_{mn} \tag{107}$$

where the bars have to be picked consistently for $\alpha, \beta, \gamma, \delta$. m is used in (107) to represent either an unbarred index ρ or a barred one $\bar\rho$. The summation extends over both ρ and $\bar\rho$. The same is true for n which represents either σ or $\bar\sigma$. In this summation either the $m = \rho$, $n = \bar\sigma$ or the $m = \bar\rho$, $n = \sigma$ term remains as $A_{\rho\sigma} = 0 = A_{\bar\rho\bar\sigma}$. Furthermore $\Omega^{m\alpha}$ in the first term is zero for the terms with α and $m = \rho$ or $\bar\alpha$ and $m = \bar\rho$. Similarly only one of the possibilities in the second term is nonzero for a given choice of bars on $\alpha, \beta, \gamma, \delta$.

Another helpful observation is that for each choice of bars on $\alpha, \beta, \gamma, \delta$, Σ inherits this index structure. For example checking $\delta A_2^{\alpha,\beta\gamma\delta} = 0$ will

3 This decomposition is demonstrated in appendix B.

involve only the **1** of Σ. Conversely this is the only constraining equation for the **1**. Checking the constraints componentwise one can use the antisymmetry of Σ to find that the **1** and $\overline{\mathbf{1}}$ are unconstrained, while all other components of Σ have to vanish. The full calculation can be found in appendix B.

Therefore the **1** and $\overline{\mathbf{1}}$ are the moduli of the solution. As one consistency check one can consider Minkowski backgrounds. By (100) $\Lambda = 0$ implies $A_1 = 0$. In that case (107) is trivial, i.e. all directions of the scalar coset are moduli.

6.4 ALLOWED GAUGE GROUPS

A in the $\mathfrak{su}(4)$ basis can be inserted into (94) to get the components of the T-tensor in this basis. One finds that most of the components are zero. The remaining cases[4] are captured by

$$T^{\alpha}{}_{\beta\bar{\gamma}\delta} = \frac{i|A|}{\sqrt{8}}\Big(-\delta^{\alpha}_{\delta}\delta_{\bar{\gamma}\beta} + \frac{1}{4}\,\delta^{\alpha}_{\beta}\delta_{\bar{\gamma}\delta}\Big). \tag{108}$$

With $T_{\bar{\gamma}\delta} = (T_{\bar{\gamma}\delta})_{\beta}{}^{\alpha} = T^{\alpha}{}_{\beta\bar{\gamma}\delta}$ the commutator reads

$$[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = T^{\sigma}{}_{\beta\bar{\gamma}\delta}T^{\alpha}{}_{\sigma\bar{\mu}\nu} - \begin{pmatrix}\bar{\gamma} \leftrightarrow \bar{\mu}\\ \delta \leftrightarrow \nu\end{pmatrix}. \tag{109}$$

Inserting (108) into (109) yields

$$\begin{aligned}[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = &- \frac{|A|^2}{8}\Big(\delta_{\bar{\mu}\delta}\delta_{\bar{\gamma}\beta}\delta^{\alpha}_{\nu} - \frac{1}{4}\,\delta^{\alpha}_{\delta}\delta_{\bar{\gamma}\beta}\delta_{\bar{\mu}\nu} - \frac{1}{4}\,\delta^{\alpha}_{\nu}\delta_{\bar{\gamma}\delta}\delta_{\bar{\mu}\beta}\\ &+ \frac{1}{16}\,\delta^{\alpha}_{\beta}\delta_{\bar{\mu}\nu}\delta_{\bar{\gamma}\delta}\Big) - \begin{pmatrix}\bar{\gamma} \leftrightarrow \bar{\mu}\\ \delta \leftrightarrow \nu\end{pmatrix}.\end{aligned} \tag{110}$$

The second and third term together are symmetric under $\left(\begin{smallmatrix}\bar{\gamma}\leftrightarrow\bar{\mu}\\ \delta\leftrightarrow\nu\end{smallmatrix}\right)$ and thus vanish in the commutator. The same holds true for the last term. We thus find

$$[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = -\frac{|A|^2}{8}\delta_{\bar{\mu}\delta}\delta^{\alpha}_{\nu}\delta_{\bar{\gamma}\beta} - \begin{pmatrix}\bar{\gamma} \leftrightarrow \bar{\mu}\\ \delta \leftrightarrow \nu\end{pmatrix}. \tag{111}$$

This can be reformulated to give the defining relation of $\mathfrak{su}(N)$

$$[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = \delta_{\bar{\mu}\delta}T_{\bar{\gamma}\nu} - \delta_{\bar{\gamma}\nu}T_{\bar{\mu}\delta}. \tag{112}$$

Using that $\frac{1}{4}\,\delta^{\alpha}_{\beta}\delta_{\bar{\gamma}\nu}\delta_{\bar{\mu}\delta}$ is symmetric under $\left(\begin{smallmatrix}\bar{\gamma}\leftrightarrow\bar{\mu}\\ \delta\leftrightarrow\nu\end{smallmatrix}\right)$ one obtains

$$[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = \frac{|A|^2}{8}\delta_{\bar{\mu}\delta}\Big(-\delta^{\alpha}_{\nu}\delta_{\bar{\gamma}\beta} + \frac{1}{4}\,\delta^{\alpha}_{\beta}\delta_{\bar{\gamma}\nu}\Big) - \begin{pmatrix}\bar{\gamma} \leftrightarrow \bar{\mu}\\ \delta \leftrightarrow \nu\end{pmatrix}. \tag{113}$$

4 Three additional nonzero components can be produced by symmetry arguments. T is antisymmetric in its last two indices ($T^{\alpha}{}_{\beta\gamma\bar{\delta}} = -T^{\alpha}{}_{\beta\delta\bar{\gamma}}$). Furthermore flipping all indices from barred to unbarred and vice versa leaves T invariant.

The bracketed term is just the definition of ${T^\alpha}_{\beta\bar{\gamma}\nu}$. Thus

$$[T_{\bar{\gamma}\delta}, T_{\bar{\mu}\nu}] = \frac{-i|A|}{\sqrt{8}}\delta_{\bar{\mu}\delta}T_{\bar{\gamma}\nu} - \begin{pmatrix} \bar{\gamma} \leftrightarrow \bar{\mu} \\ \delta \leftrightarrow \nu \end{pmatrix} \tag{114}$$

which is the relation (112) up to a constant $\frac{-i|A|}{\sqrt{8}}$ which can be absorbed into the definition of the Lie bracket. Having 16 4×4 matrices that fulfill (112) as generators, we conclude that the allowed gauge group is $SU(4)$.

6.5 COMPARISON TO THE DUAL SCFT

In section 6.2 AdS vacua were found which are parametrized by $|A| \propto \sqrt{-\Lambda}$. To obtain the parametrization by $|A|$, the 36-dimensional T-tensor component A_1^{ij} was diagonalized. It thus comes from an infinite number of solutions, spanning a 35-dimensional solution space for any $\Lambda < 0$.[5] The moduli of the solutions where found to be parametrized by a complex coordinate $\sigma \in \mathbb{C}$. This result is in accordance with AdS/CFT in so far as the dual $\mathcal{N} = 4$ SYM in four dimensions has a conformal manifold of complex dimension one [BNP15]. We were however not able to determine the metric of the resulting moduli space.

The gauge group was found to be $G_0 = SU(4)$. This indeed corresponds to the R-symmetry of the dual theory [CDI16].

5 There are no solutions for $\Lambda > 0$ as $\sqrt{-\Lambda} \propto |A| \in \mathbb{R}$. For $\Lambda = 0$ there is just one solution $A = 0$.

7 ADS VACUA IN D=4 SUPERGRAVITY

7.1 GAUGED MAXIMAL D=4 SUPERGRAVITIES

The ungauged maximal $D = 4$ supergravity was constructed by [CJ78] and in more detail by de Wit and Nicolai in [DWN82]. [DWST07] gives a description of its gaugings via the embedding tensor and is thus used as the main reference for this section.

The field content of the ungauged $D = 4$ theory is given by the graviton multiplet

$$(e^r_\mu, A^M_\mu, \mathcal{V}^{ab}_M; \psi^a_\mu, \chi^{abc}). \tag{115}$$

The bosonic fields are the vielbein e^r_μ, with flat spacetime indices $r, s, ... = 0, ..., 3$, vector fields A^M_μ with $M, N, ... = 1, ..., 56$ and the scalar matrix $\mathcal{V}^{ab}_M$ with $a, b, ... = 1, ..., 8$. $M, N, ...$ label the fundamental representation of $G = E_{7(7)}$, which is the global symmetry group of the $D = 4$ theory. Its maximal compact subgroup is $H = SU(8)$. The fundamental representation of H is denoted by indices $a, b, ... = 1, ..., 8$. Complex conjugation for these indices is affected by raising respectively lowering all indices. The fermions come in $SU(8)$ representations, namely the gravitino ψ^a_μ and the graviphotino χ^{abc}.

The generators of G in the adjoint representation are $(t_\alpha)_M{}^N$ with $\alpha = 1, ..., 133$. The vector fields A^M_μ are in the $\overline{\mathbf{56}}$ of G. Hence, since the adjoint representation is the $\mathbf{133}$, the embedding tensor $\Theta_M{}^\alpha$ is in the $\mathbf{56} \otimes \mathbf{133}$. Using the generators of G and the embedding tensor, the generators X_M of the gauge group $G_0 \subset G$ are defined to be

$$X_{MN} = \Theta_M{}^\alpha t_\alpha. \tag{116}$$

The gauge covariant derivative is

$$D_\mu = \partial_\mu - g\, A^M_\mu X_M = \partial_\mu - g\, A^M_\mu \Theta_M{}^\alpha t_\alpha, \tag{117}$$

where g is the gauge coupling parameter. The tensor product $\mathbf{56} \otimes \mathbf{133}$ of Θ can be decomposed into

$$\mathbf{56} \otimes \mathbf{133} = \mathbf{56} + \mathbf{912} + \mathbf{6480}. \tag{118}$$

Supersymmetry constrains the possible representations of Θ to the $\mathbf{912}$ [DWST07]. This implies the constraints

$$(t_\alpha)_M{}^N \Theta_N{}^\alpha = 0,\ (t_\beta t^\alpha)_M{}^N \Theta_N{}^\beta = -\frac{1}{2}\Theta_M{}^\alpha \tag{119}$$

where $\alpha, \beta, ...$ are raised and lowered by the $E_{7(7)}$ Killing form $\eta_{\alpha\beta} = tr(t_\alpha t_\beta)$.

Similar to what we have seen in six dimensions, the 1-forms A_μ^M are split into electric and magnetic degrees of freedom. The electric 1-forms are denoted by A_μ^Λ and the magnetic ones by $A_{\mu\Lambda}$. $\Lambda, \Sigma, ... = 1, ..., 28$ denote the representations which are obtained when splitting the $\overline{\mathbf{56}}$ of G under $SL(8)$. The splitting is $\overline{\mathbf{56}} \to \mathbf{28} + \mathbf{28}'$. The $\mathbf{28}$ of $SL(8)$ is the second exterior power of the fundamental representation. The Lagrangian contains only the electric degrees of freedom and thus is invariant under the off-shell subgroup $SL(8) \subset G$. The full G symmetry is only realized on shell.

The scalar coset space is represented by a 56×56 matrix $V_M{}^{\underline{N}}$ which splits under $\mathfrak{su}(8)$ and $\mathfrak{sl}(8)$ into blocks

$$V_M{}^{\underline{N}} = \begin{pmatrix} \mathcal{V}_\Lambda^{ij} & \mathcal{V}_{\Lambda kl} \\ \mathcal{V}^{\Sigma ij} & \mathcal{V}_{kl}^{\Sigma} \end{pmatrix}. \tag{120}$$

The blocks $\mathcal{V}$ are antisymmetric in i and j and fulfill the relations

$$\begin{aligned} \mathcal{V}_M^{ij}\mathcal{V}_{Nij} - \mathcal{V}_{Mij}\mathcal{V}_N^{ij} &= i\Omega_{MN}, \\ \Omega^{MN}\mathcal{V}_M^{ij}\mathcal{V}_{Nkl} &= i\delta_{kl}^{ij}, \\ \Omega^{MN}\mathcal{V}_M^{ij}\mathcal{V}^{Nkl} &= 0, \end{aligned} \tag{121}$$

where Ω^{MN} is an $E_{7(7)}$ invariant which is antisymmetric and can be written as a block matrix with $\mathbb{1}$ and $-\mathbb{1}$ as off-diagonal blocks. Furthermore $\Omega^{MN}\Omega_{NP} = -\delta_P^M$.

The bosonic Lagrangian is given by

$$\begin{aligned} e^{-1}\mathcal{L}_B = &-\frac{1}{2}R - \frac{1}{4}i\Big(\mathcal{N}_{\Lambda\Sigma}\mathcal{H}_{\mu\nu}^{+\Lambda}\mathcal{H}^{+\mu\nu\Sigma} - \bar{\mathcal{N}}_{\Lambda\Sigma}\mathcal{H}_{\mu\nu}^{-\Lambda}\mathcal{H}^{-\mu\nu\Sigma}\Big) \\ &-\frac{1}{12}|P_\mu^{ijkl}|^2 - g^2 V + e^{-1}\mathcal{L}_{VT}, \end{aligned} \tag{122}$$

where $\mathcal{H}^\pm$ denote the self and anti-selfdual parts of $\mathcal{H}_{\mu\nu}$. The matrix $\mathcal{N}_{\Lambda\Sigma}$ is determined by the relation (cf. [DWST07])

$$\mathcal{V}^{\Sigma ij}\mathcal{N}_{\Lambda\Sigma} = -\mathcal{V}_\Lambda^{ij}. \tag{123}$$

$\mathcal{L}_{VT}$ is a collection of topological terms needed for consistency of the theory (cf.[DWST07]) and the potential V is given in (126).

Using $V_M{}^{\underline{N}}$, the T-tensor can be defined. It can be decomposed according to the split

$$\mathbf{912} \mapsto \mathbf{420} + \overline{\mathbf{420}} + \mathbf{36} + \overline{\mathbf{36}}. \tag{124}$$

The $\mathbf{36}$ is denoted by $A_1^{ij} = A_1^{(ij)}$ and the $\mathbf{420}$ by $A_{2i}{}^{jkl} = A_{2i}{}^{[jkl]}$ with $A_{2i}{}^{ikl} = 0$. The barred representations are then related via complex conjugation. We define

$$\begin{aligned} T_i^{jkl} &= -\frac{3}{4}A_{2i}{}^{jkl} - \frac{3}{2}A_1^{j[k}\delta_i^{l]} \\ T^{klmn}{}_{ij} &= -\frac{4}{3}\delta_{[i}^{[k}T_{j]}{}^{lmn]}. \end{aligned} \tag{125}$$

The T-tensor components contribute to the potential

$$V = \frac{1}{24}\,|A_{2i}^{jkl}|^2 - \frac{3}{4}\,|A_1^{ij}|^2 \tag{126}$$

which can take negative values and thus supports AdS vacua.

7.2 FINDING THE ADS VACUA

To find the supersymmetric AdS vacua we have to set the fermion variations to zero. The graviphotino variation yields

$$0 = \delta\chi^{ijk} = -2g\,A_{2l}{}^{ijk}\epsilon^l \tag{127}$$

from which we read off $A_2 = 0$. The gravitino variation reads

$$0 = \delta\psi_\mu^i = D_\mu\epsilon^i - \frac{g}{\sqrt{2}}\,A_1^{ij}\gamma_\mu\epsilon_j. \tag{128}$$

The constraint coming from $\delta\psi = 0$ can be obtained by acting on (128) with D_ν and antisymmetrizing over μ and ν. Alternatively the closure constraint on A_1 and A_2 can be examined to get the same result. With $A_2 = 0$ there is only one nontrivial closure constraint (cf. [DWST07])

$$-2\,\delta_l^m A_{1ni}A_1^{ki} + 2\delta_n^k A_{1li}A_1^{mi} = 0. \tag{129}$$

Acting on (129) with δ_k^n gives

$$A_{1li}A_1^{im} = \frac{|A_1|^2}{8}\delta_l^m. \tag{130}$$

Since A_1 is symmetric it can be diagonalized. Combined with (130) this leaves A_1 to be proportional to $\mathrm{diag}(+1, ..., +1, -1, ..., -1)$ with p positive and q negative eigenvalues. One can insert this result into (125) to get

$$\begin{aligned} T_i^{jkl} &= -\frac{3}{2}\,A_1^{j[k}\delta_i^{l]} \\ T^{klmn}{}_{ij} &= 0. \end{aligned} \tag{131}$$

Furthermore one can relate $|A_1|$ to $\sqrt{-\Lambda}$ via the potential given in (126). For $A_2 = 0$ one gets

$$V = \frac{3}{4}\,|A_1|^2. \tag{132}$$

For $V = \Lambda$ this gives the relation $|A_1| = 2\sqrt{-\Lambda/3}$.

7.3 MODULI OF THE ADS VACUA

As done in the previous cases the constraints on T can be varied along the scalar manifold. The scalar variations can be expressed by an element of the adjoint of $E_{7(7)}$, which is the **133**. Under $\mathfrak{su}(8)$ it splits as

$\mathbf{133} \to \mathbf{63} + \mathbf{70}$. The $\mathbf{63}$ is the adjoint of $SU(8)$ which is divided out to obtain the scalar coset $E_{7(7)}/SU(8)$. The remaining $\mathbf{70}$ parametrizes the physical variations denoted by $\Sigma^{ijkl} = \Sigma^{[ijkl]}$.

The resulting variations of the $T-$tensor are (cf. [DWST07])

$$\begin{aligned} \delta T_i^{jkl} &= 2\,\Sigma^{jmnp} T_{imnp}{}^{kl} - \frac{1}{4}\,\delta_i^j \Sigma^{mnpq} T_{mnpq}{}^{kl} + \Sigma^{klmn} T^j{}_{imn}, \\ \delta T_{ijkl}{}^{mn} &= -\frac{4}{3} \Sigma_{p[ijk} T_{l]}{}^{pmn} - \frac{1}{24} \epsilon_{ijklpqrs} \Sigma^{mntu} T^{pqrs}{}_{tu}, \end{aligned} \tag{133}$$

where $\epsilon_{ijklpqrs}$ is the fully antisymmetric tensor, which is an invariant of $SU(8)$.

Along the moduli these variations have to vanish. Using the form of T given in (131) the first constraint reads

$$0 = \Sigma^{klmn} A_{1i[m} \delta^j_{n]}. \tag{134}$$

Multiplication with A_1^{ip} gives

$$0 = \frac{|A|^2}{8} \Sigma^{klmn} \delta^p_m \delta^j_n = \frac{|A|^2}{8} \Sigma^{klpj}. \tag{135}$$

Thus either $\Sigma^{klpj} = 0$ or $|A| = 0$. For an AdS background $|A| \propto \sqrt{-\Lambda}$ can not be zero and hence there can be no moduli. For Minkowski space however Σ^{ijkl} stays unconstrained. In either case the second condition coming from (133) is trivially fulfilled.

7.4 ALLOWED GAUGE GROUPS

The generators X_{MN}^P of the gauge group G_0 are just T_i^{jkl} dressed with the scalar matrices. Hence it is enough to examine the form of T_i^{jkl}. With $A_2 = 0$ one gets

$$T_i^{jkl} = -\frac{3}{2}\, A_1^{j[k} \delta_i^{l]}. \tag{136}$$

These are the generators of the $\mathfrak{cso}$ algebra. Diagonalizing A_1 to read

$$A_1^{ij} \propto \mathrm{diag}(\underbrace{1, ..., 1}_{p}, \underbrace{-1, ...-, 1}_{q}, \underbrace{0, ..., 0}_{r}) \tag{137}$$

then gives the gauge group $CSO(p, q, r)$ with $p + q + r = 8$ (cf. [Roe05]).

We note from (130) that A_1 is of full rank and therefore $r = 0$. This leaves us with $SO(p, 8 - p)$ as possible gauge groups. As done before we can also have a look at the Minkowski limit. $\Lambda = 0$ gives $A_1 = 0$ and thus leaves no non-zero components of the T-tensor. No gauging is compatible with the supersymmetric Minkowski vacuum.

7.5 COMPARISON TO THE DUAL SCFT

Similar to the case in five dimensions we found moduli parametrized by a 36-dimensional A_1^{ij}, which can be diagonalized with $|A_1| \propto \sqrt{-\Lambda}$, leaving a 35-dimensional solution space. It was found that there are no moduli to the solution. This result is in accordance with the dual SCFT in three dimensions which indeed has no conformal manifold [CDI16].

The allowed gauge groups are $G_0 = SO(8, 8-p)$ for $0 \leq p \leq 8$. The R-symmetry of the $D = 3$ SCFT rotates eight real supercharges and is thus $SO(8)$. It is therefore expected that another constraint on A_1 has been overlooked, since only a positive or negative definite A_1 gives $SO(8)$ as the gauge group.

8 CONCLUSION

In the chapters 4 to 7 the maximally supergravities in dimension four to seven were examined for their maximally supersymmetric AdS vacua. In each case these vacua were found by setting the gravitino and graviphotino variations under supersymmetry to zero. The resulting constraints were varied along the scalar manifold to examine the existence of moduli. Furthermore the gauge groups which are consistent with supersymmetric AdS vacua were determined in every case.

For the maximal $D = 7$ supergravity two solutions for any given value of $\Lambda < 0$ were found. It was found that there are no moduli. This result is in accordance with the dual $D = 6$, $\mathcal{N} = (2, 0)$ SCFT having no marginal deformations. The resulting gauge group $G_0 = SO(5)$ corresponds to the R-symmetry $USp(4)$ of the dual $D = 6$ theory.

For the maximal $D = 6$ supergravity it was found that there are no supersymmetric AdS vacua and hence no gauge group. This result is in accordance with the nonexistence of the dual theory in five dimensions.

For the maximal $D = 5$ supergravity a 35-dimensional solution space was found for any given value of $\Lambda < 0$. These solutions have moduli parametrized by a complex coordinate $\sigma \in \mathbb{C}$. This is in accordance with the fact that the conformal manifold of the dual SCFT has one complex dimension. The metric of the moduli space was however not determined and can thus not be compared to the SCFT result. The gauge group was determined to be $G_0 = SU(4)$ which corresponds to the $SU(4)$ R-symmetry of the dual $\mathcal{N} = 4$ super Yang Mills theory.

For the maximal $D = 4$ supergravity a 35-dimensional solution space was found for any given value of $\Lambda < 0$. These solutions do not have any moduli. The corresponding SCFT in three dimensions has no conformal manifold. The result is thus in agreement with AdS/CFT. The gauge group was narrowed down to be $G_0 = SO(p, 8 - p)$. From the R-symmetry of the dual $D = 3$ SCFT the expected gauge group is $SO(8)$. Thus the result is only partly in accordance with AdS/CFT, as only solutions with $p = 0$ and $p = 8$ give the predicted gauge group.

There are thus two loose ends to be tied. In $D = 5$ the precise form of the moduli space was not determined and in $D = 4$ it is expected from AdS/CFT that additional constraints on the gauge group exist. All other results were found to be in agreement with predictions from the AdS/CFT conjecture.

A

SPINOR REPRESENTATIONS AND SUPERSYMMETRY IN DIFFERENT DIMENSIONS

A.1 SPINOR REPRESENTATIONS OF SO(P,Q)

Spinor representations are representations of $\mathfrak{so}(p,q)$ which can not be obtained by tensoring the fundamental representation [FS03]. Spinor representations arise for $SO(p,q)$ since these groups are not simply connected. They are defined using the universal covering group, which is denoted $Spin(p,q)$ for $SO(p,q)$. For four dimensional spacetime the corresponding symmetry group is the *Lorentz group* $SO(1,3) \simeq SO(3,1)$. The universal cover is $SL(2,\mathbb{C})$ [Wal10]. A *spinor* of $SO(1,3)$ can thus be represented by a vector with two complex components. The action of $\Lambda \in SO(1,3)$ on these spinors is constructed by assigning an element $\lambda \in SL(2,\mathbb{C})$ to Λ.

In a more general setting the group $SO(p,q)$ keeps a (pseudo-)metric η^{ab} invariant which has p positive and q negative eigenvalues i.e.

$$\eta^{ab} = \text{diag}(\underbrace{+1,...,+1}_{p},\underbrace{-1,...,-1}_{q}) \tag{138}$$

where $a,b,... = 1,...,d$ with $d = p+q$. Using η^{ab} one can define *gamma matrices* by the anticommutation relation [Tan98]

$$\{\gamma^a,\gamma^b\} = 2\,\eta^{ab}. \tag{139}$$

The smallest realization of such matrices are $2^{[d/2]} \times 2^{[d/2]}$ matrices, where the brackets $[d/2]$ denote the integer part of $d/2$ i.e. $d/2$ for even d and $(d-1)/2$ for odd d. The gamma matrices act on *Dirac spinors* ψ which are vectors with $2^{[d/2]}$ complex components. Given a generator $\Lambda^{\mu\nu}$ of $SO(p,q)$, its action on ψ is given by

$$\delta\psi = -\frac{1}{4}\Lambda^{\mu\nu}\gamma_{[\mu}\gamma_{\nu]}\psi. \tag{140}$$

The Dirac spinors in general are reducible representations of $SO(p,q)$. There are different ways to reduce the representation by Dirac spinors depending on the dimension and the signature (p,q). The Weyl condition can only be defined in even dimensions and uses the matrix

$$\bar{\gamma} = (-1)^{\frac{1}{4}(p-q)}\gamma^1\gamma^2...\gamma^d \tag{141}$$

which squares to 1 and anticommutes with all other gamma matrices. It is a generalization of the matrix γ^5 in four dimensions and can be used to define *Weyl spinors* $\psi_\pm$ of positive and negative chirality by

$$\bar{\gamma}\psi_\pm = \pm\psi_\pm. \tag{142}$$

A Dirac spinor ψ can then be reduced to its Weyl components, using the projector $P_{\pm}$ with

$$P_{\pm}\psi \equiv \frac{1 \pm \bar{\gamma}}{2}\psi = \psi_{\pm}. \tag{143}$$

The other reducibility condition is the *Majorana condition.* We start with the observation that $\pm(\gamma^a)^*$ satisfy the same anticommutation relations as γ^a [Tan98].[1] Then there are matrices $B_{\pm}$ which fulfill

$$\pm(\gamma^a)^* = B_{\pm}\gamma^a B_{\pm}^{-1}. \tag{144}$$

which are used to define *charge conjugation.* In even dimensions it is defined as either

$$\psi^c = B_+^{-1}\psi^* \text{ or } \psi^c = B_-^{-1}\psi^* \tag{145}$$

such that the relation $(\psi^c)^c = \psi$ holds. The Majorana condition is then

$$\psi = \psi^c. \tag{146}$$

In odd dimensions charge conjugation as well as the Majorana condition are defined similarly, but the condition (144) holds only for $a = 1, ..., d-1$. For the d-th gamma matrix a different condition is used, which is

$$B_{\pm}\gamma^d B_{\pm}^{-1} = (-1)^{\frac{1}{2}(p-q+1)}(\gamma^d)^*. \tag{147}$$

The Majorana condition can not always be imposed, since the relation $(\psi^c)^c = \psi$ can not be achieved in general.

We thus get up to two different reducibility conditions, depending on the dimension and the signature (p, q), each of which reduces the dimension of the minimal spinor representation by the factor of a half. In some cases either the Majorana or the Weyl condition can be used to get a irreducible representation. This is true for example for spacetime spinors in four dimensions. There are also special cases in which both conditions can be used together to get a *Majorana-Weyl* spinor. Table 2 gives an overview over which conditions can be used for spacetime spinors i.e. $p = 1$ or $q = 1$.

In the cases where the Majorana condition can not be imposed, one gets $(\psi^c)^c = -\psi$. This allows to define a similar condition. Using an even number of spinors ψ^i with $i = 1, ..., 2n$ one can impose

$$\psi^i = \Omega^{ij}(\psi^j)^c, \tag{148}$$

with $\Omega^{ij} = -\Omega^{ji}$. Spinors satisfying (148) are *symplectic Majorana spinors.* $2n$ such spinors are equivalent to n Dirac spinors. This representation is useful, if there is a symplectic symmetry, as is indeed the case for the $D = 5, 6, 7$ maximal supergravities in which the R-symmetry is given by the unitary symplectic group $USp(8)$, $USp(4) \times USp(4)$ and $USp(4)$ respectively.

1 $(\cdot)^*$ denotes complex conjugation

d	Majorana	Weyl	Majorana-Weyl	Minimimal Dimension
2	yes	yes	yes	1
3	yes	-	-	2
4	yes	yes	-	4
5	-	-	-	8
6	-	yes	-	8
7	-	-	-	16
8	yes	yes	-	16
9	yes	-	-	16
10	yes	yes	yes	16
11	yes	-	-	32

Table 2: Overview over spinor conditions that can or can not be imposed on spinors of $SO(d-1,1)$. Taken from [Pol98].

A.2 SUPERSYMMETRY ALGEBRA IN DIFFERENT DIMENSIONS

In different dimensions the real supercharges fit differently into the Weyl-, Majorana- or Majorana-Weyl spinors as outlined in the previous section. The supersymmetry algebra thus looks slightly different in any case.

In $d = 4, 8 \bmod 8$ the supercharges form Weyl spinors Q^i_+ with positive chirality with $i, j, ... = 1, ..., \mathcal{N}$ [Tan98]. The charge conjugate supercharges have the opposite chirality $(Q^i_+)^c = Q_{-i}$.[2] The nonzero anticommutators of the supercharges are then[3]

$$\{Q^i_+, Q^T_{-j}\} = P_+\gamma^\mu C P_\mu \delta^i_j, \tag{149}$$

where P_μ are the generators of translations and C is a charge conjugation matrix which is derived from $B_\pm$ as $C_\pm = B_\pm^{-1}\gamma^{0T}$. The $d = 4 \bmod 8$ case uses $C = C_-$ and the $d = 8 \bmod 8$ case uses $C = C_+$. The R-symmetry is $U(\mathcal{N})$.

For $d = 10 \bmod 8$ the supercharges form Majorana-Weyl spinors Q^i_+ and $Q^{i'}_-$ with positive and negative chirality with $i, j, ... = 1, ..., \mathcal{N}_+$ and $i', j', ... = 1, ..., \mathcal{N}_-$. The nonzero anticommutators of the supercharges are then

$$\begin{aligned} \{Q^i_+, Q^{jT}_+\} &= P_+\gamma^\mu C_- P_\mu \delta^{ij} \\ \{Q^{i'}_-, Q^{j'T}_-\} &= P_-\gamma^\mu C_- P_\mu \delta^{i'j'}. \end{aligned} \tag{150}$$

The R-symmetry is $SO(\mathcal{N}_+) \times SO(\mathcal{N}_-)$.

2 The notation with $\pm$ is equivalent to the notation with Q and $\bar{Q}$ used in the second chapter.

3 For this supersymmetry algebra as well as all other cases the possibility of additional *central charges* is not considered. These would show up in the anticommutators between Q's of the same chirality.

For $d = 6 \bmod 8$ the supercharges form symplectic Majorana-Weyl spinors Q^i_+ and $Q^{i'}_-$ with positive and negative chirality with $i, j, ... = 1, ..., \mathcal{N}_+$ and $i', j', ... = 1, ..., \mathcal{N}_-$. The symplectic Majorana conditions read $\Omega^{ij}_+(Q^j_+)^c = Q^i_+$ and $\Omega^{i'j'}_-(Q^{j'}_-)^c = Q^{i'}_-$, where $\Omega_\pm$ are antisymmetric as discussed above. $\mathcal{N}_+$ as well as $\mathcal{N}_-$ can only be even numbers. The nonzero anticommutators of the supercharges are then

$$\begin{aligned} \{Q^i_+, Q^{jT}_+\} &= P_+\gamma^\mu C_- P_\mu \Omega^{ij}_+ \\ \{Q^{i'}_-, Q^{j'T}_-\} &= P_-\gamma^\mu C_- P_\mu \Omega^{i'j'}_-. \end{aligned} \tag{151}$$

The R-symmetry is $USp(\mathcal{N}_+) \times USp(\mathcal{N}_-)$.

For $d = 9, 11 \bmod 8$ the supercharges form Majorana spinors Q^i with $i, j, ... = 1, ..., \mathcal{N}$. The nonzero anticommutators of the supercharges are then

$$\{Q^i, Q^{jT}\} = \gamma^\mu C P_\mu \delta^{ij}, \tag{152}$$

where $C = C_+$ for $d = 9 \bmod 8$ and $C = C_-$ for $d = 11 \bmod 8$. The R-symmetry is $SO(\mathcal{N})$.

For $d = 5, 7 \bmod 8$ the supercharges form symplectic Majorana spinors Q^i with $i, j, ... = 1, ..., \mathcal{N}$. The symplectic Majorana condition is $\Omega^{ij}(Q^j)^c = Q^i$. $\mathcal{N}$ can only be an even number. The nonzero anticommutators of the supercharges are then

$$\{Q^i, Q^{jT}\} = \gamma^\mu C P_\mu \Omega^{ij}, \tag{153}$$

where $C = C_+$ for $d = 5 \bmod 8$ and $C = C_-$ for $d = 7 \bmod 8$. The R-symmetry is $USp(\mathcal{N})$.

B CONSTRAINTS ON MODULI IN D=5

B.1 VARIATIONS AND CONSTRAINTS

As described in section 6.3 the physical variations of the scalar coset are described by $\Sigma^{ijkl} = \Sigma^{[ijkl]}$ which is symplectic traceless. In the $\mathfrak{su}(4)$ basis this leads to the following components

$$\begin{aligned}
\mathbf{1}: \Sigma^{\alpha\beta\gamma\delta} &= e_i^{\alpha} e_j^{\beta} e_k^{\gamma} e_l^{\delta} \Sigma^{ijkl}, \\
\overline{\mathbf{1}}: \Sigma^{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}} &= e_i^{\bar{\alpha}} e_j^{\bar{\beta}} e_k^{\bar{\gamma}} e_l^{\bar{\delta}} \Sigma^{ijkl}, \\
\mathbf{10}: \Sigma^{\alpha\beta\gamma\bar{\delta}} &= e_i^{\alpha} e_j^{\beta} e_k^{\gamma} e_l^{\bar{\delta}} \Sigma^{ijkl}, \\
\overline{\mathbf{10}}: \Sigma^{\bar{\alpha}\bar{\beta}\bar{\gamma}\delta} &= e_i^{\bar{\alpha}} e_j^{\bar{\beta}} e_k^{\bar{\gamma}} e_l^{\delta} \Sigma^{ijkl}, \\
\overline{\mathbf{20}}: \Sigma^{\alpha\beta\bar{\gamma}\bar{\delta}} &= e_i^{\alpha} e_j^{\beta} e_k^{\bar{\gamma}} e_l^{\bar{\delta}} \Sigma^{ijkl}.
\end{aligned} \tag{154}$$

These components inherit the antisymmetry of Σ^{ijkl}. $\Sigma^{\alpha\beta\bar{\gamma}\delta}$ for example can be obtained as $\Sigma^{\alpha\beta\bar{\gamma}\delta} = -\Sigma^{\alpha\beta\delta\bar{\gamma}}$.

To find the moduli one can start from the schematic constraint given in (107)

$$\delta A_2^{\alpha,\beta\gamma\delta} = 0 = \Omega^{m\alpha}\Sigma^{\beta\gamma\delta n} A_{mn} + \Omega^{m[\beta}\Sigma^{\gamma\delta]\alpha n} A_{mn} \tag{155}$$

where α stands schematically for α or $\bar{\alpha}$, β for β or $\bar{\beta}$ and so on. For m and n the different possibilities $m = \rho, \bar{\rho}$ and $n = \sigma, \bar{\sigma}$ are summed up. Conveniently only one term in this summation is nonzero as the other terms vanish by either $A^{\alpha\beta} = 0 = A^{\bar{\alpha}\bar{\beta}}$ or $\Omega^{\alpha\beta} = 0 = \Omega^{\bar{\alpha}\bar{\beta}}$.

In the resulting equations $\delta A_2^{\alpha,\beta\gamma\delta} = 0$ with different choice of bars on $\alpha, \beta, \gamma, \delta$, Σ inherits this choice of bars, i.e. $\delta A_2^{\alpha,\beta\gamma\delta} = 0$ involves only the $\mathbf{1}$, $\delta A_2^{\alpha,\beta\gamma\bar{\delta}} = 0$ involves only the $\mathbf{10}$ and so on.

B.2 SOLVING THE CONSTRAINTS

For the $\mathbf{1}$ the resulting constraint from (155) is

$$\begin{aligned}
0 &= \Omega^{\bar{\rho}\alpha}\Sigma^{\beta\gamma\delta\sigma} A_{\bar{\rho}\sigma} + \Omega^{\bar{\rho}[\beta}\Sigma^{\gamma\delta]\alpha\sigma} A_{\bar{\rho}\sigma} \\
&= -\delta_{\sigma}^{\alpha}\Sigma^{\beta\gamma\delta\sigma} - \delta_{\sigma}^{[\beta}\Sigma^{\gamma\delta]\alpha\sigma} \\
&= -\Sigma^{\beta\gamma\delta\alpha} - \Sigma^{[\gamma\delta|\alpha|\beta]} \\
&= -\Sigma^{\beta\gamma\delta\alpha} + \Sigma^{[\gamma\delta\beta]\alpha} \\
&= 0
\end{aligned} \tag{156}$$

which is thus trivially fulfilled for any choice of $\Sigma^{\alpha\beta\gamma\delta}$. Since the constraints are homogeneous with respect to the involved representations, there is no other constraint involving the **1**. Still Σ can be brought to a simpler form by using the fully antisymmetric invariant tensor $\epsilon^{\alpha\beta\gamma\delta}$ of $\mathfrak{su}(4)$. Then

$$\Sigma^{\alpha\beta\gamma\delta} = \sigma\epsilon^{\alpha\beta\gamma\delta}. \tag{157}$$

The same arguments hold for the $\overline{\mathbf{1}}$, where $\Sigma^{\bar\alpha\bar\beta\bar\gamma\bar\delta} = \bar\sigma\epsilon^{\bar\alpha\bar\beta\bar\gamma\bar\delta}$ is taken to fulfill pseudoreality.

For the **10** we will start with $\delta A_2^{\bar\alpha,\beta\gamma\delta} = 0$. This constraint is a bit easier to manage than the others, since one does not need to resolve the antisymmetrization brackets in the second term. The resulting equation is

$$\begin{aligned}
0 &= \Omega^{\rho\bar\alpha}\Sigma^{\beta\gamma\delta\bar\sigma}A_{\rho\bar\sigma} + \Omega^{\bar\rho[\beta}\Sigma^{\gamma\delta]\bar\alpha\sigma}A_{\bar\rho\sigma} \\
&= \delta^{\bar\alpha}_{\bar\sigma}\Sigma^{\beta\gamma\delta\bar\sigma} - \delta^{[\beta}_{\sigma}\Sigma^{\gamma\delta]\bar\alpha\sigma} \\
&= \Sigma^{\beta\gamma\delta\bar\alpha} + \Sigma^{[\gamma\delta\beta]\bar\alpha} \\
&= 2\,\Sigma^{\beta\gamma\delta\bar\alpha}.
\end{aligned} \tag{158}$$

Thus $\Sigma^{\beta\gamma\delta\bar\alpha}$ vanishes, as do all components obtained by permutation of indices. The same arguments hold for the $\overline{\mathbf{10}}$ which thus also vanishes.

For the **20** examine $\delta A_2^{\bar\alpha,\bar\beta\gamma\delta} = 0$ to get

$$\begin{aligned}
0 &= \Omega^{\rho\bar\alpha}\Sigma^{\bar\beta\gamma\delta\bar\sigma}A_{\rho\bar\sigma} \\
&\quad + \frac{1}{3}\Big(\Omega^{\rho\bar\beta}\Sigma^{\gamma\delta\bar\alpha\bar\delta}A_{\rho\bar\sigma} + \Omega^{\bar\rho\gamma}\Sigma^{\delta\bar\beta\bar\alpha\sigma}A_{\bar\rho\sigma} + \Omega^{\bar\rho\delta}\Sigma^{\bar\beta\gamma\bar\alpha\sigma}A_{\bar\rho\sigma}\Big) \\
&= \Sigma^{\bar\beta\gamma\delta\bar\alpha} + \frac{1}{3}\Big(\Sigma^{\gamma\delta\bar\alpha\bar\beta} - \Sigma^{\delta\bar\beta\bar\alpha\gamma} - \Sigma^{\bar\beta\gamma\bar\alpha\delta}\Big) \\
&= -\,\Sigma^{\bar\alpha\bar\beta\gamma\delta} + \frac{1}{3}\Big(\Sigma^{\bar\alpha\bar\beta\gamma\delta} - \Sigma^{\bar\alpha\bar\beta\gamma\delta} - \Sigma^{\bar\alpha\bar\beta\gamma\delta}\Big) \\
&= -\,\frac{4}{3}\,\Sigma^{\bar\alpha\bar\beta\gamma\delta}.
\end{aligned} \tag{159}$$

The **20** thus vanishes.

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DECLARATION

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