Refs:
Phys.Rev.Lett.94:181102,2005
Phys.Rev.D72:084011,2005
arXiv:0705.1301 [astro-ph]
arXiv:0706.0288 [hep-th]

MASSIVE GRAVITY

Preliminary remarks:

- Subject is not yet settled, many unanswered questions

- notations:

$g_{\mu\nu} = \mathrm{diag}\,(1\ -1\ -1\ -1)$

$\hbar = c = 1$

$2 = 3 = \pi = 1$

Outline

I. Motivation
II. Overview of existing approaches
III. Generic problems
IV. The model
V. Particle content (perturbations around Minkowski)
VI. Phenomenology
- cosmological solutions
- structure formation
- BH's.

VII. Conclusions

I. Motivation

The alternatives to GR were looked for ever since it has been created.

1939 – Fierz-Pauli model of massive gravity

1961 – Brans & Dicke scalar-tensor theory

...

There is a revival of interest recently, for the following reasons:

1). Recent cosmological data strongly suggest that the Universe is filled with a dark stuff:

5% - B – only!
25% – DM
70% – DE

The dark compounds have only been detected gravitationally. Since the need for the dark compounds occurs at galactic scales and larger, an alternative would be to assume that gravity is modified at these distances.

It is important to note that gravity has not been tested directly at these scales.

Note also that the status of DM and DE is very different. While there are many different pieces of evidence in favor of DM, very little (if anything) is known about DE.
⇒ every new insight is very important.

2) Alternative models are needed to test GR. One has to have an idea what kind of deviations may be expected in order to interpret the experimental data.

Ex.: – PPN phenomenological parameters
– Brans-Dicke model.

→ Very concrete and important motivation

3) Theoretical challenge. The symmetries of GR fix the theory completely (apart from the cosmological constant term).
⇒ gravity is very difficult to change at large distances, the number of consistent modifications is very limited. Basically, one can

– add extra fields
– make the graviton a massive particle or a resonance.

The work presented in these lectures is motivated by this third point. The logic is to construct any meaningful (free from pathologies) model and then see what kind of phenomenological consequences it implies.

II. OVERVIEW OF EXISTING APPROACHES (incomplete and superficial).

"Experiment - motivated":

– MOND & TeVeS
– f(R)

"Theory - motivated":

- Fierz - Pauli model
- Brans - Dicke model
- GRS (extra dims.)
- DGP
- ghost condensate & massive gravity

this is what we will concentrate on afterwards (in the second lecture).

$\boxed{\text{Mond}}$ Milgrom 1983

Astron. Astrophys. 1977, 54, 661
Tully - Fisher 1977

Aaronson 1982
Ap. J. Suppl. 50 (1982) 241

key observation:

$$F = \frac{GM}{r^2} \cdot f(r/r_0) \Rightarrow \text{bad idea!}$$

This would lead at $r \gg r_0$ to $v^2 = GM/r_0$ which is not supported by observations

it would imply that big galaxies have bigger discrepancy, which is contrary to observations.

Instead one should use $a_0 \simeq 10^{-8}\ cm/s^2$ $\sim 1/7\ cH_0$

$$ma \cdot \mu(a/a_0) = F$$

$$\mu(x) = \begin{cases} x & \text{at } x \ll 1 \\ 1 & \text{at } x \gg 1. \end{cases}$$

$$\cancel{m} \frac{v^4}{\cancel{r^2}} \frac{1}{a_0} = \frac{GM \cancel{m}}{\cancel{r^2}}$$

$$v^4 \propto GM a_0. \quad \Rightarrow \text{Tully-Fisher!}$$

($M \propto L$)

- problems with clusters of galaxies - ?
- bullet cluster ?
- relativistic formulation (TeVeS) Bekenstein 2004.

$\boxed{\text{TeVeS}}$:

$g_{\mu\nu}$, u_μ (timelike) vector field

$$g^{\alpha\beta} u_\alpha u_\beta = -1 \qquad (-+++)$$

scalar ϕ

field content:

$g_{\mu\nu}$ - metric
u_μ - vector
ϕ - scalar

σ, λ - non-dynamical fields (scalar).

The (technical) problem with TeVeS is that the expansion about flat space is singular. This is a technical complication, but this is rather uncomfortable. In particular, it means that particle content of the theory may depend on the background curvature.

Note: Lorentz symmetry is broken (spontaneously).

Feix, Fedeli, Bartelmann 0707.0790 $\Rightarrow$ TeVeS cannot explain bullet clusters

Clayton, (2001) gr-qc/0104103 $\Rightarrow$ Hamiltonian of TeVeS is unbounded & theory is unstable.

Expansion is apparently OK about FRW background
Skordis et al 2005

$\boxed{f(R)}$

Carrol, Duvvuri, Trodden, Turner PRD 70 (2004) 043528

modification

$R - 2\Lambda \to f(R)$

$f(R) \Leftrightarrow$ Jordan frame

$R + F(\phi) \Leftrightarrow$ Einstein frame

ϕ couples to matter as gravity.

Example:

$R - \frac{\mu^4}{R}$; $\mu \sim H_0$

→ does not pass solar system tests.

→ is not consistent with cosmo. evolution in the presence of non-relativistic matter.

The problem with $f(R)$ is that in terms of scalar-tensor theory the coupling of scalar to matter is fixed.

$$S_{JF} = \int \sqrt{g} \left\{ M_p^2 f(R) + S_m(\ldots) \right\}$$

$$\to \int \sqrt{\tilde g} \left(M^2 \tilde R - \frac{1}{2} \tilde g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - S_M \right)$$

$\tilde g = e^{\phi/M_{pe}} g$

depends on $f(R)$

contains $g = e^{-\phi/M} \tilde g$.

This fixes coupling to matter and induces detectable effects in the solar system which are ruled out.

Eöt-Wash experiment?

adding functions of other invariants may help? Navarro, Alcoleyen PL B622 (2005) 1

Fierz - Pauli massive gravity (1939)

$$S = M_p^2 \int d^4x \left\{ L_E^{(2)}(h_{\mu\nu}) - \right.$$

$$\left. - m^2 \left(h_{\mu\nu}^2 - (h_\mu{}^\mu)^2 \right) \right\}$$

schematically can be written as $h \Box h$. Comes from $\sqrt{g}\, R$ term.

graviton mass

mass term. Note a special combination. This comes from the requirement that there are no ghosts (see below)

This is a ghost-free model, but it has other problems (to be discussed later).

Brans - Dicke model (1961)

$\omega > 40\,000$ from Cassini

$$S = \int d^4x \sqrt{g} \left\{ M_p^2 \phi R - \omega \frac{\partial_a \phi \partial^a \phi}{\phi} M_{pl}^2 + L_M \right\}$$

field eqs:

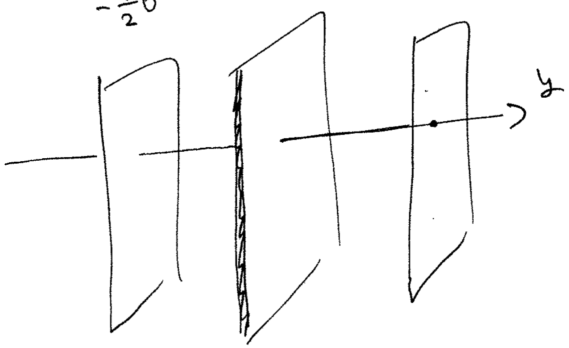
$$\begin{cases} M_p^2 \Box \phi = \dfrac{8\pi}{3+2\omega} T \\[2ex] G_{ab} = \dfrac{8\pi}{M_p^2 \phi} T_{ab} + \dfrac{\omega}{\phi^2} \left(\partial_a \phi \partial_b \phi - \dfrac{1}{2} g_{ab} (\partial\phi)^2 \right) \\ \qquad + \dfrac{1}{\phi} \left(\nabla_a \nabla_b \phi - g_{ab} \Box \phi \right) \end{cases}$$

Constraints

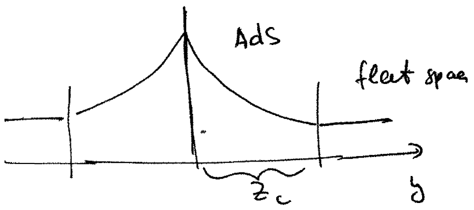
$\omega > 40\,000$ (Cassini satelite)

Gregory, Rubakov & Sibiryakov (2000).

$\boxed{\text{GRS}}$ Large extra dimensions



$$ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2$$



$\Rightarrow$ Calculate gravitational potential between two sources on the brane. Find

$\frac{1}{r}$ at $k^{-1} < r < r_c = \frac{1}{k}\, e^{3kz_c}$

$\frac{1}{r^2}$ at $r > r_c$

However: ghost is present $\Rightarrow$ bad!

$\boxed{\text{DGP}}$ Dvali, Gabadadze, Porrati (2000)

Another model in the context of extra dimensions:

$$S = M^3 \int d^5x \sqrt{G}\, R^{(5)} + M_p^2 \int d^4x \sqrt{g}\, R^{(4)}$$

$$G_{\mu\nu}(x, y=0) = g_{\mu\nu}$$

One finds in this model:

$$V(r) = \begin{cases} -\dfrac{1}{M_p^2}\dfrac{1}{r} & \text{at } r \ll r_0 \\[2ex] -\dfrac{1}{M^3}\dfrac{1}{r^2} & \text{at } r \gg r_0 \end{cases} \qquad r_0 = \frac{M_p^2}{M^3}$$

to have $r_0 \sim H^{-1}$ one sets $M \sim 100\,\text{MeV}$

- The model leads to cosmologically interesting self-accelerating solutions

However, it is still debated whether it is consistent as a model (ghost?)

Ghost condensate

Arkani-Hamed, Cheng, Luty, Mukohyama (2004).

$$S = M^2 \int \sqrt{g}\, R + \int \sqrt{g}\, \Lambda^4 P(X)$$

$$X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

vacuum solution which is time-dependent

$$\phi = \Lambda^2 t$$

- graviton is massless
- Lorentz symmetry broken
- there is a scalar degree of freedom with the dispersion relation of unusual form $\omega^2 \propto k^4$

This is very similar to the model we will consider in detail later. Leave the detailed discussion till then.

III. Generic problems of $m \neq 0$ gravity.

1 Ghosts & instabilities

Terminology: Generic quadratic (not necessarily Lorentz-invariant) Lagrangian.

$$\alpha \dot{\phi}^2 - \beta (\partial_i \phi)^2 - m^2 \phi^2$$

Eqs. of motion:

$$-\alpha \ddot{\phi} - \beta k^2 \phi - m^2 \phi = 0$$

$$\ddot{\phi} + \omega^2 \phi = 0$$

$$\omega^2 \to \frac{\beta k^2 + m^2}{\alpha}$$

$\omega^2 = \dfrac{\beta k^2 + m^2}{\alpha} < 0 \Rightarrow$ instability (exponentially growing solution)

$\alpha, \beta, m^2 < 0 \Rightarrow$ ghost – a field with the wrong sign of the action. Bad if couples to other fields.

* Consider the graviton mass term and show that the theory is not consistent unless a particular combination of terms (Fiertz-Pauli mass term) is choosen.

General Lorentz-invariant quadratic action:

$$S = M_{pl}^2 \int d^4x \left\{ \underbrace{L^{(2)}(h_{\mu\nu})}_{\text{quadratic part of the Einstein action } \sqrt{g}R\text{, schematically } h\Box h} + \alpha h_{\mu\nu}^2 + \beta (h_\mu{}^\mu)^2 \right\}$$

Even in this apparently simple case the analysis is quite involved. But there is a simplifying trick: consider high-momentum case. Note the following:

$L^{(2)}$ is invariant under the (linearized) coordinate transformations, which imply

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

four parameters.

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

↳ transverse, $\partial^\mu \bar{h}_{\mu\nu} = 0$.

$L^{(2)}$ does not depend on ξ_μ, all the dependence is in the mass term. At high momenta the term quadratic in ξ_μ dominates as it comes with two derivatives. So we may forget about $\bar{h}_{\mu\nu}$ (this is the point where the simplification is achieved).

So we get

$$\alpha\left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right)^2 + \beta \cdot 4 \cdot \left(\partial_\mu \xi^\mu\right)^2 =$$

$$= 2\alpha \left(\partial_\mu \xi_\nu\right)^2 + (2\alpha + 4\beta)\left(\partial_\mu \xi^\mu\right)^2$$

This is the action for the vector field ξ_μ! We know that it has to have the form

$$-\#\left(\partial_\mu \xi_\nu - \partial_\nu \xi_\mu\right)^2$$

$$\Rightarrow \boxed{\alpha = -\beta < 0.} \longleftarrow \text{consistency condition.}$$

$\Rightarrow$ Fierz-Pauli mass term $\alpha = -m^2$

$$\boxed{L_{FP} = -m^2\left(h_{\mu\nu}^2 - h_\mu{}^{\mu\,2}\right)}$$

[Note the trick: at high momenta only the symmetry-breaking terms play the role]

②. vDVZ discontinuity.

We have escaped ghosts but have run into another problem which is quite generic: the absence of the smooth limit $m_g \to 0$. This problem is known as the vDVZ (van Dam – Veltman – Zakharov) discontinuity.

The problem is that Einstein gravity and Fierz-Pauli model in the limit $m_g \to 0$ predict different bending of light by massive bodies.

This can be understood easily in the field theory language. In this language the coupling is characterized by the combination

$$G \cdot T^{\mu\nu} \cdot P_{\mu\nu\lambda\rho} \cdot T'^{\lambda\rho} \qquad \left[\text{cf. } \lambda \cdot JDJ\right)$$

coupling

sources (energy-momenta of two bodies)

propagator

We do not need to know how $\frac{1}{r}$ potential arises from this expression; we only need to compare the massless and massive cases.

In general,

$$P_{\mu\nu\lambda\rho} = \frac{\sum_i e^i_{\mu\nu} e^i_{\lambda\rho}}{p^2 - m^2}$$

Here is where the difference between massive and massless cases enters: the number of polarization vectors is different

$m \neq 0$: 5 polarizations

$$\tilde{P}_{\mu\nu\lambda\rho} = \frac{1}{p^2 - m^2}\Big\{ \frac{1}{2}\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\lambda} - \underline{\underline{\frac{1}{3}}}\eta_{\mu\nu}\eta_{\lambda\rho} + \ldots + (p\text{-dependent terms}).\Big\}$$

$m = 0$: 2 polarizations

$$P_{\mu\nu\lambda\rho} = \frac{1}{p^2}\Big\{ \frac{1}{2}\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\lambda} - \underset{\approx}{\frac{1}{2}}\eta_{\mu\nu}\eta_{\lambda\rho} + \ldots\Big\}$$

Note: different coefficients in front of $\eta_{\mu\nu}\eta_{\lambda\rho}$ terms (even in the limit $m^2 \to 0$!) This is the origin of discontinuity.

Take 2 massive bodies (only T_{00} is non-negligible)

$m=0$ $\quad G\, T_{\mu\nu} P_{\mu\nu\lambda\rho} T'_{\lambda\rho} = \frac{1}{2} G \cdot T_{00} T'_{00} \frac{1}{p^2}$

$m\neq 0$ $\quad \tilde{G}\, T_{\mu\nu} \tilde{P}_{\mu\nu\lambda\rho} T'_{\lambda\rho} = \frac{2}{3} \tilde{G}\, T_{00} T'_{00} \frac{1}{p^2-m^2}$

$$\Rightarrow \boxed{\tilde{G} = \frac{3}{4} G}$$

Now compare the predictions for the case when one of $(T'_{\mu\nu})$ describes the EM wave $\Rightarrow$ $T'^{\mu}_{\mu} = 0$.
$\Rightarrow$ third term does not contribute.

$m=0$: $\quad G\, T_{00} T'_{00} \frac{1}{p^2}$

$m\neq 0$: $\quad \tilde{G}\, T_{00} T'_{00} \frac{1}{p^2-m^2}$

Since $\tilde{G} \neq G$ $\Rightarrow$ different bendings!

Experiment supports Einstein theory.

Note: the problem is in coupling to T_μ^μ $\Rightarrow$
Generic to scalar-tensor theories.

③ Strong coupling

Does the above argument mean that graviton mass is strictly zero? Not, because there is a loophole: the linear approximation breaks at much larger distances than one would naively expect.

Vainstein [1972] has argued that the distance at which the linear theory breaks is

$$r_V = \left(\frac{M}{M_p^2 \cdot m_g^4} \right)^{1/5}$$

taking $m_g \simeq H_0 = 10^{-33}$ eV

this leads to

$$r_V = 100 \text{ kpc for the Sun!}$$

How this comes about? Again, "Goldstone" fields! The field which plays key role is the longitudinal part of ξ_μ, $\xi_\mu = \partial_\mu \phi + \ldots$
This field enters the action with 2 derivatives, $\partial_\mu \partial_\nu \phi$.
It is easy to check that Fierz-Pauli choice corresponds to the absence of terms $\int (\partial^2 \phi)^2$. Thus, the action (schematically) is

$$\int \left\{ M_p^2 (\partial h)^2 + M_p^2 m^2 (h \partial^2 \phi + h^2) + Th \right\}$$

$\Rightarrow$ kinetic term mixes $\partial^2\phi$ and h. Diagonalize them:

$$\tilde{h} = h + \alpha \cdot m^2 \phi$$

$$\rightarrow \int \left\{ M_p^2 (\partial \tilde{h})^2 + M_p^2 m^4 (\partial\phi)^2 + T\tilde{h} + m^2 T \phi + \ldots \right\}$$

Solving the equation one finds

$$\begin{cases} h = \dfrac{M}{M_p^2 r} \\ m^2 \phi = \dfrac{M}{M_p^2 r} \end{cases}$$

$\leftarrow$ ϕ is singular in the limit $m^2 \to 0$

Non-linear terms coming from the mass term:

$$\int M_p^2 m^2 \cdot (\partial^2 \phi)^3$$

If we require that this is smaller, than quadratic terms, we get the condition:

$$\frac{M}{M_p^2 m^4 r^5} \ll 1$$

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adding particular non-linear interactions may lower the scale to

$$r_* = \left(\frac{M}{M_p^2 m_g^2}\right)^{1/3} = 100 \text{ pc}$$

$H_0 = 10^{-42}$ GeV

$$\frac{10^{57}}{10^{38} \cdot 10^{-42 \cdot 2}} \frac{1}{\text{GeV}^3} =$$

$$10^{19+84} = 10^{103}$$

$10^{34} \sim 10^{20}$ cm Ok.

Lecture II.

Our goal now is to construct a model which possesses massive graviton and escapes the problems discussed in the first part

Lorentz-invariant combination which is ghost-free is unique (Fierz-Pauli model) and it is strongly coupled unacceptably close to the sources. So we either have to deal with non-linearities or to look for the Lorentz-breaking model.

Note: strong constraints on $\not{LI}$ can be escaped if it occurs only in the gravitational sector.

Generic $\not{LI}$ rotations-conserving mass term:

$$L_m = M_{pe}^2 \{ m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii} \}$$

the graviton mass.

LI: $m_1^2 = m_2^2 = \alpha$; $m_3^2 = m_4^2 = \beta$; $m_0^2 = \alpha + \beta$

FP: $\alpha = -\beta$ $[m_0^2 = 0;\ m_1^2 = m_2^2 = m_3^2 = m_4^2]$

ghost condensate: $m_1^2 = m_2^2 = m_3^2 = m_4^2 = 0$

More generally, we may add a function of metric components which is non-linear

$$S = \int d^4x \sqrt{g} \left\{ M_p^2 \cdot R + \underbrace{\Lambda^4}_{\text{cutoff scale of our effective theory.}} F(g^{00}, g^{0i}, g^{ij}) + \text{matter} \right\}$$

Analogy with the gauge field

$$S = \int d^4x \left\{ -F_{\mu\nu}^2 + m^2 A_\mu^2 \right\}$$

add a scalar field ϕ:

$$S = \int d^4x \left\{ -F_{\mu\nu}^2 + m^2 (A_\mu - \partial_\mu \phi)^2 \right\}$$

$\Rightarrow$ gauge-invariant action; we can choose the gauge $\phi = 0$ and recover the original action, but in some cases we may prefer a different gauge. For instance, if we are interested in the high-energy behavior of the theory.

Note that we have recovered the phase part of the Higgs Lagrangian. ϕ is the Goldstone boson which corresponds to the broken symmetry.

A similar trick (Stückelberg's trick) can be performed in our case.

|| Arkani-Hamed, Georgi, Schwarz (2003), Dubovsky (2004).

4 broken diffeomorphisms

$$x^\mu \to \xi^\mu(x) + x^\mu$$

$\Rightarrow$ 4 Goldstone fields ϕ^0, ϕ^i

$$\Lambda^4 F(g^{00}, g^{0i}, g^{ij})$$

$$X = g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^0 / \Lambda^4$$

$$V^i = g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^0 / \Lambda^4$$

$$Y^{ij} = g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j / \Lambda^4$$

$$\left[\text{or } W^{ij} = Y^{ij} - \frac{V^i V^j}{X}\right]$$

$$\Rightarrow \Lambda^4 F(X, V^i, W^{ij})$$

$$S = \int d^4x \sqrt{g} \left\{ M_{pe}^2 R + \Lambda^4 F(X, V^i, W^{ij}) + L_{\text{matter}} \right\}$$

Note!

1) ϕ^0, ϕ^i enter through derivatives only, just like in our example. This is how the Goldstone fields should enter the action.

2). fixing $\phi^0 = \Lambda^2 t$, $\phi^i = \Lambda^2 \cdot x^i$ } [breaks Lorentz symmetry!]

is the analog of the "unitary gauge". In this gauge the function F is a function of metric components $\Rightarrow$ this brings us to the original form of the action

3). L_{matter} does not contain couplings of ϕ^0, ϕ^i to the matter fields

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Vacuum

we require that in the absence of matter the flat Minkowski space is a solution.

Let us show that this is generically the case.

Let us start with the Einstein equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{this is } 0 \text{ for } g_{\mu\nu} = \gamma_{\mu\nu}} = M_P^2 \underbrace{T^{(G)}_{\mu\nu}}_{}$$

$\Rightarrow$ $\boxed{\text{this has to be } = 0}$!

$$\delta F = F_X \delta X + F_i \delta V^i + F_{ij} \delta Y^{ij}$$

$$\delta X = \frac{1}{\Lambda^4} \partial_\mu \phi^0 \partial_\nu \phi^0 \, \delta g^{\mu\nu}$$

$$\delta V^i = \frac{1}{\Lambda^4} \partial_\mu \phi^0 \partial_\nu \phi^i \, \delta g^{\mu\nu}$$

$$\delta Y^{ij} = \frac{1}{\Lambda^4} \partial_\mu \phi^i \partial_\nu \phi^j \, \delta g^{\mu\nu}$$

$\delta S = +\frac{1}{2} \int d^4x \sqrt{g}\, T_{\mu\nu} \, \delta g^{\mu\nu}$

$$T^{(G)}_{\mu\nu} = 2 F_X \cdot \partial_\mu \phi_0 \partial_\nu \phi_0 + (\partial_\mu \phi_i \partial_\nu \phi_0 + \partial_\nu \phi_i \partial_\mu \phi_0) \cdot F_i + 2 \partial_\mu \phi_i \partial_\nu \phi_j \cdot F_{ij} - g_{\mu\nu} \cdot \Lambda^4 F = 0$$

$\rightarrow$ this is the equation we have to solve for ϕ^0, ϕ^i

In general, these equations are overdetermined.

Take ansatz: (assume in F all indices are contracted)

$$\phi_0 = a \cdot \Lambda^2 t$$

$$\phi^i = b \cdot \Lambda^2 x^i$$

constants

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then
$$X = a^2$$
$$V_i = 0$$
$$Y_{ij} = -b^2 \delta_{ij}$$

$$F_i = 0$$
$$F_{ij} \equiv F_Y \cdot \delta_{ij}$$

$$T_{00} = \Lambda^4 \left(a^2 \cdot 2F_X - F \right)$$
$$T_{ij} = \Lambda^4 \delta_{ij} \left(b^2 \cdot 2F_Y + F \right)$$

$\Rightarrow$ we have to satisfy $\underline{\underline{2}}$ algebraic equations by adjusting 2 constants a and b:

$$\begin{cases} 2a^2 F_X(a^2, b^2) - F(a^2, b^2) = 0 \\ 2b^2 F_Y(a^2, b^2) - F(a^2, b^2) = 0. \end{cases}$$

In general, there is a solution without fine-tuning.

assume $a = b = 1$ in what follows.

The vacuum solution

$$\boxed{\begin{aligned} \phi^0 &= \Lambda^2 t \\ \phi^i &= \Lambda^2 x^i \\ g_{\mu\nu} &= \eta_{\mu\nu} \end{aligned}}$$

Note

1) if the function F possesses the global rotational symmetry $\phi^i \to O^{ij}\phi^j$, then the vacuum state is rotationally invariant (diagonal combination of two symmetries)

2) Lorentz symmetry is broken, unless F is invariant under global transformations of ϕ^0, ϕ^i leaving invariant $\phi^{0\,2} - \phi^{i\,2}$.

3) Λ is the cutoff scale of the model. Beyond this scale the theory is not known. Such a theory (called UV-completion) may not even exist. Constructing a UV completion is a major unsolved problem.

4) The cutoff scale is related to the graviton mass as
$$\Lambda = \sqrt{M_{Pl}\, m}.$$

$\boxed{\text{V Particle content}}$

Beyond vacuum solution

$$\phi^\mu = \Lambda^2 x^\mu + \pi^\mu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

perturbations.

gauge tr: $\pi^\mu \to \pi^\mu + \xi^\mu(x)$; $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Unitary gauge

$\pi^\mu = 0$

$F \to$ mass terms $m_0^2 h_{00}^2 + \dots$

Good for spectrum

Goldstone sector

$h_{\mu\nu} \to \partial_\mu \pi_\nu + \partial_\nu \pi_\mu$

good for pathology studies

1). masses -?

[graviton mass parameters $m_0^2, m_1^2 \dots$ ...] = [combinations of second derivatives of F calculated at the vacuum]

Parametrically: $S^{(2)} \sim M_p^2 (\partial h)^2 + \underbrace{\Lambda^4 h^2}_{= M_{Pl}^2 m_g^2 h^2}$

$\Rightarrow \boxed{\Lambda = \sqrt{M_p m_g}}$

2) Stability -?

L_{mass} : $h_{\mu\nu} \mapsto \partial_\mu \pi_\nu + \partial_\nu \pi_\mu$

$\Rightarrow$ quadratic Lagrangian for π_μ :

$$L = M_p^2 \Big\{ 2m_0^2 (\partial_0 \pi_0)^2 + m_1^2 (\partial_0 \pi_i)^2 + m_1^2 (\partial_i \pi_0)^2 + (4m_4^2 - 2m_1^2)\, \pi_0 \partial_0 \partial_i \pi_i - m_2^2 (\partial_i \pi_j)^2 - (m_2^2 - 2m_3^2)(\partial_i \pi_i)^2 \Big\}$$

$\to$ One has to see at which values of parameters this Lagrangian is non-pathological.

$\Downarrow$

many possibilities

- at general values 4 propagating degrees of freedom, at leas one is ghost or unstable.

- $m_0^2 = 0$: 1 scalar does not propagate.
  1 s + 2v propagate
  2 tensor + 2v + 1s = 5 polarizations of massive graviton.

$\boxed{- \; m_1^2 = 0 \quad \text{Nothing propagates.}}$

this is our interest in what follows. or $\phi^i \to \phi^i + \xi^i(\phi^0)$

! protected by the symmetry $x^i \to x^i + \xi^i(t)$.

Thus, we get a non-pathological model if the action is choosen in the form

$$\Lambda^4 F(x, w^{ij})$$

instead of general $\Lambda^4 F(x, v^i, w^{ij})$.

3). Spectrum go to the unitary gauge and solve linear equations for perturbations

Parametrization:

$$h_{00} = 2\varphi$$
$$h_{0i} = S_i - \partial_i B$$
$$h_{ij} = -\hat{h}_{ij} - \partial_i F_j - \partial_j F_i + 2(\psi\,\delta_{ij} - \partial_i\partial_j E)$$

$$10 = 1(\varphi) + 1(B) + 1(\psi) + 1(E) +$$
$$+ 2(S_i) + 2(F_i) + 2(\hat{h}_{ij})$$

$\Rightarrow$ solve linear equations with the source.

tensor sector:

$$(-\partial_0^2 + \partial_i^2 - m_2^2)\,\hat{h}_{ij} = 0$$

$\Rightarrow$ two massive tensor modes

Vector modes do not propagate.

Scalar sector

$$\Phi = \varphi + \partial_0 B - \partial_0^2 E$$

$$\Phi = \underbrace{\frac{1}{\partial_i^2}\frac{T_{00}+T_{ii}}{M_P^2} - 3\frac{\partial_0^2}{\partial_i^4}\frac{T_{00}}{M_P^2}}_{} + \underbrace{\mu^2 \cdot \frac{1}{\partial_i^4}\frac{T_{00}}{M_{Pe}^2}}_{}$$

Φ_E, standard Einstein piece giving $\frac{1}{r}$ Newton's potential + relativistic corrections (we did not assume that the source is static!!)

extra piece growing $\propto r$

coefficient proportional to some combination of m_i^2:

$\mu^2 \to 0$ as $m_i^2 \to 0$

$\Rightarrow$ NO DISCONTINUITY

Two cases:

- $\mu^2 \neq 0$ $\rightarrow$ corrections to the $1/r$ potential, non-linear effects ets.

- $\mu^2 = 0$. $\rightarrow$ Newton's potential is unchanged.

  This is our choice for now

$$\boxed{\mu^2 = 0 \text{ may be imposed by choosing } F(x, w^{ij}) = F(x^{\gamma} w^{ij})}$$

constant.

$\Rightarrow$ Solar system constraints & lab. tests are satisfied.

What are <u>experimental constraints</u> on the graviton mass ?

- They do not come from Newton's potential because it is identical to GR

- mass of the graviton changes the emission of gravity waves. However, the latter is in agreement with spin-down of the binary pulsar system which is interpreted as being due to gravitational waves. ⟹ constraints.

$$\boxed{m \ll (\text{hour})^{-1} \sim 10^{-19}\,\text{eV} \sim 10^{15}\,\text{cm}}$$

cutoff scale

$$\Lambda \sim \sqrt{M_{pe}\, m}$$

at $m \sim (10^{15}\,\text{cm})^{-1}$ $\quad \boxed{\Lambda \lesssim 10\,\text{keV}}$

⟹ high enough.

<u>Phenomenological consequences</u> (some of)

① Massive graviton – <u>DM candidate</u> !

- can be created in sufficient amounts at inflation (at least in some particular models)

- can be detected by the gravitational wave detectors

- MG as DM is not excluded at present, but may be probed in the near future

② <u>Cosmology</u>.

Cosmological ansatz (only for flat Universe!)

$$ds^2 = dt^2 - a^2(t)\, dx_i^{i\,2}$$

$$\phi^0 = \phi(t)$$

$$\phi^i = \Lambda^2 x^i$$

Note: impossible for open/closed Universe!

?. what is the meaning?

<u>Note</u>: for this ansatz $W^{ij} = -\dfrac{\delta^{ij}}{a^2}$

Einstein equations $\Rightarrow$

$$\boxed{\begin{aligned}\left(\frac{\dot a}{a}\right)^2 &= \frac{1}{M_P^2}\Big\{\rho_{matter} + \underbrace{2\Lambda^4 X\cdot F_X}_{\rho_1} - \underbrace{\Lambda^4 F}_{\rho_2}\Big\}\\ \partial_t\left(a^3\sqrt{X}\cdot F_X\right) &= 0\end{aligned}}$$

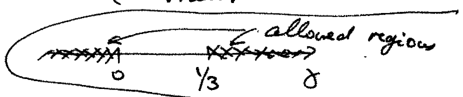
$\Rightarrow$ 2 equations for 2 variables $\underbrace{\phi(t)}_{[\text{equivalently, } X(t)]}$ and $a(t)$

$\Rightarrow$ solution is always possible (in general).

Cosmologically interesting solutions arise when $X \propto$ power of a.
Then there exists γ such that $X^\gamma / a^2 \to$ const.

Then $\quad \rho_1 = \text{Const}\,\dfrac{1}{a^{3-1/\gamma}} \quad \leftarrow$ (matter with the equation of state depending on γ.

$\rho_2 = $ const. $\quad \leftarrow$ (cosmological constant

$\Rightarrow$ 

This happens, for instance, if the function F has a particular form $F(X^\gamma \cdot W^{i})$ [i.e., depends on only one argument].

$\Rightarrow$ Condensate of the Goldstone fields behaves like a "quinessence" with the equation of state depending on γ.

An interesting situation occurs in the case when $\gamma = 1/3$. Then both contributions to the Friedmann equation scale like a cosmological constant. But first contribution is determined by the initial conditions $\Rightarrow$ cosmological constant becomes "dynamical."

③. Structure formation

Q: Do perturbations grow in the same way in the Universe filled with the Goldstone condensate as they do in the standard picture?

$\to$ in some regions of parameters – definitely yes.

in other regions – under study.

$-1 < \gamma < 0$ – Ok.

$\gamma = 1$ – Ok.

other regions – ?

④. Black holes.

Full non-linear action allows to study black holes.

Are black holes modified?

Are black holes universal as in GR?

The answer to the first question is relatively easy. One may fix the metric to the BH metric and try to find the configuration of Goldstone fields such that all the equations are satisfied.

Surprisingly, for the Schwarzschild BH this program succeeds:

$$10^9 M_\odot \Leftrightarrow 3\ h.$$

$$ds^2 = \left(1 - \frac{M}{r}\right) dt^2 - \frac{1}{1 - M/r}\, dr^2 - r^2 \gamma_{ij}\, dx^i dx^j$$

$$\phi_0 = t + 2\sqrt{rM} - M \ln \frac{\sqrt{r} + \sqrt{M}}{\sqrt{r} - \sqrt{M}}$$

For the rotating black hole the situation is quite different. Setting the metric to the Kerr metric one may show that there does not exist a configuration of the Goldstone fields which leads to $T_{\mu\nu} = 0$ for the Kerr metric

$\Rightarrow$ rotating black holes have to be different!

What happens?

The point is that the model contains an instantaneous interaction (one may view this interaction as being mediated by one of the modes with $k^2 = 0$)

This phenomenon has been observed and can be studied in the case of ED with a Lorentz-breaking mass term

$$-\frac{1}{2} m^2 A_i^2$$

Then Coulomb interactions become instantaneous

instantaneous interactions $\Rightarrow$ BH hairs

$\Rightarrow$ no universality.

The strength of the effect (relative to Newton)

~ 1 at $r \gtrsim m_g^{-1}$

$(m_g r)^2$ at $r \lesssim m_g^{-1}$

(Presumably)
$\Rightarrow$ large for very large BH's $\sim 10^9 M_\odot$

suppressed for smaller ones

Conclusions

- One may construct a meaningful model to play with where things are under control below some cutoff scale ($\Lambda \lesssim 10$ kev in our case)

- Observable effects :

  ** massive graviton can be a DM
  then it is detectable by GW detectors

  ** cosmological constant depends on initial conditions ?

  ** BH's grow hairs which are observable

- Open questions :

  ** UV completion - ?

  ** refined cosmological tests

  ** observable $\cancel{LI}$ effects ?

  ** Early Universe ?

  ** $\mu^2 \neq 0$ phenomenology